May S Day

Miss Mary S. Day

ARITHMETIC,

IN WHICH THE

PRINCIPLES OF OPERATING BY NUMBERS

ARE

ANALYTICALLY EXPLAINED,

AND

SYNTHETICALLY APPLIED;

THUS

COMBINING THE ADVANTAGES

TO BE DERIVED BOTH FROM

THE INDUCTIVE AND SYNTHETIC MODE
OF INSTRUCTING:

THE WHOLE

MADE FAMILIAR BY A GREAT VARIETY OF USEFUL AND INTERESTING EXAMPLES, CALCULATED AT ONCE TO ENGAGE THE PUPIL IN THE STUDY, AND TO GIVE HIM A FULL KNOWLEDGE OF FIGURES IN THEIR APPLICATION TO ALL THE PRACTICAL PURPOSES OF LIFE.

DESIGNED FOR THE USE OF

SCHOOLS AND ACADEMIES

IN THE UNITED STATES.

BY DANIEL ADAMS, M. D.
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KEENE, N. H.
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PREFACE.

There are two methods of teaching,—the *synthetic* and the *analytic*. In the synthetic method, the pupil is first presented with a *general* view of the science he is studying, and afterwards with the particulars of which it consists. The analytic method *reverses* this order: the pupil is first presented with the *particulars*, from which he is led, by certain natural and easy gradations, to those views which are more general and comprehensive.

The Scholar's Arithmetic, published in 1801, is synthetic. If that is a *fault* of the work, it is a fault of the *times* in which it appeared. The analytic or inductive method of teaching, as now applied to elementary instruction, is among the improvements of later years. Its introduction is ascribed to Pestalozzi, a distinguished teacher in Switzerland. It has been applied to arithmetic, with great ingenuity, by Mr. Colburn, in our own country.

The analytic is unquestionably the best method of *acquiring* knowledge; the synthetic is the best method of *recapitulating*, or *reviewing* it. In a treatise designed for school education, *both* methods are useful. Such is the plan of the present undertaking, which the author, occupied as he is with other objects and pursuits, would willingly have forborne, but that, the demand for the Scholar's Arithmetic still continuing, an obligation, incurred by long-continued and extended patronage, did not allow him to decline the labour of a revisal, which should adapt it to the present more enlightened views of teaching this science in our schools. In doing this, however, it has been necessary to make it a new work.

In the execution of this design, an *analysis* of each rule is first given, containing a familiar explanation of its various principles; after which follows a *synthesis* of these principles, with questions in form of a supplement. Nothing is taught *dogmatically;* no technical term is used till it has first been defined, nor any principle inculcated without a previous developement of its truth; and the pupil is made to understand the *reason* of each process as he proceeds.

The examples under each rule are mostly of a practical nature, beginning with those that are very easy, and gradually advancing to those more difficult, till one is introduced containing larger numbers, and which is not easily solved in the mind; then, in a plain, familiar manner, the pupil is shown how the solution may be facilitated by *figures*. In this way he is made to see at once their *use* and their *application*.

At the close of the fundamental rules, it has been thought advisable to collect into one clear view the distinguishing properties of those rules, and to give a number of examples involving one or more of them. These exercises will prepare the pupil more readily to understand the

application of these to the succeeding rules; and, besides, will serve to interest him in the science, since he will find himself able, by the application of a very few principles, to solve many curious questions.

The arrangement of the subjects is that, which to the author has appeared most natural, and may be seen by the Index. *Fractions* have received all that consideration which their importance demands. The principles of a rule called *Practice* are exhibited, but its detail of cases is omitted, as unnecessary since the adoption and general use of federal money. The *Rule of Three*, or *Proportion*, is retained, and the solution of questions involving the principles of proportion, by *analysis*, is distinctly shown.

The articles *Alligation*, *Arithmetical* and *Geometrical Progression*, *Annuities* and *Permutation*, were prepared by Mr. IRA YOUNG, a member of Dartmouth College, from whose knowledge of the subject, and experience in teaching, I have derived important aid in other parts of the work.

The numerical paragraphs are chiefly for the purpose of *reference:* these references the pupil should not be allowed to neglect. His attention also ought to be particularly directed, by his instructer, to the illustration of each particular principle, from which general rules are deduced: for this purpose recitations by classes ought to be instituted in every school where arithmetic is taught.

The supplements to the rules, and the geometrical demonstrations of the extraction of the square and cube roots, are the only traits of the *old* work preserved in the *new*.

DANIEL ADAMS.

Mont Vernon, (N. H.) *Sept.* 29, 1827.

INDEX.

MISCELLANEOUS EXAMPLES.

ARITHMETIC.

NUMERATION.

¶ 1. A SINGLE or individual thing is called a *unit*, *unity*, or *one*; one and one more are called *two*; two and one more are called *three*; three and one more are called *four*; four and one more are called *five*; five and one more are called *six*; six and one more are called *seven*; seven and one more are called *eight*; eight and one more are called *nine*; nine and one more are called *ten*, &c.

These terms, which are expressions for quantities, are called *numbers*. There are two methods of expressing numbers shorter than writing them out in words; one called the *Roman* method by letters,* and the other the *Arabic* method by figures. The latter is that in general use.

In the Arabic method, the nine first numbers have each an appropriate character to represent them. Thus,

---

* In the Roman method by letters, I represents *one*; V, *five*; X, *ten*; L, *fifty*; C, *one hundred*; D, *five hundred*; and M, *one thousand*.

As often as any letter is repeated, so many times its value is repeated, unless it be a letter representing a *less* number placed before one representing a *greater*; then the less number is taken from the greater; thus, IV represents *four*, IX, *nine*, &c., as will be seen in the following

TABLE.

One	I.	Ninety	LXXXX. or XC.
Two	II.	One hundred	C.
Three	III.	Two hundred	CC.
Four	IIII. or IV.	Three hundred	CCC.
Five	V.	Four hundred	CCCC.
Six	VI.	Five hundred	D. or IↃ.*
Seven	VII.	Six hundred	DC.
Eight	VIII.	Seven hundred	DCC.
Nine	VIIII. or IX.	Eight hundred	DCCC.
Ten	X.	Nine hundred	DCCCC.
Twenty	XX.	One thousand	M. or CIↃ.†
Thirty	XXX.	Five thousand	IↃↃ. or $\overline{V}$.‡
Forty	XXXX. or XL.	Ten thousand	CCIↃↃ. or $\overline{X}$.
Fifty	L.	Fifty thousand	IↃↃↃ.
Sixty	LX.	Hundred thousand	CCCIↃↃↃ. or $\overline{C}$.
Seventy	LXX.	One million	$\overline{M}$.
Eighty	LXXX.	Two million	$\overline{MM}$.

* IↃ is used instead of D to represent five hundred, and for every additional Ↄ annexed at the right hand, the number is increased ten times.

† CIↃ is used to represent one thousand, and for every C and Ↄ put at each end, the number is increased ten times.

‡ A line over any number increases its value *one thousand times*.

A *unit*, *unity*, or *one*, is represented by this character, 1.
Two . . . . 2.
Three . . . . 3.
Four . . . . 4.
Five . . . . 5.
Six . . . . 6.
Seven . . . . 7.
Eight . . . . 8.
Nine . . . . 9.

Ten has no appropriate character to represent it; but is considered as forming a unit of a second or higher order, consisting of *tens*, represented by the same character (1) as a unit of the first or lower order, but is written in the *second place* from the right hand, that is, on the left hand side of units; and as, in this case, there are no units to be written with it, we write, in the place of units, a cipher, (0,) which of itself signifies nothing; thus, *Ten* 10.

One ten and one unit are called . . *Eleven* 11.
One ten and two units are called . . *Twelve* 12.
One ten and three units are called . . *Thirteen* 13.
One ten and four units are called . . *Fourteen* 14.
One ten and five units are called . . *Fifteen* 15
One ten and six units are called . . *Sixteen* 16.
One ten and seven units are called . *Seventeen* 17.
One ten and eight units are called . *Eighteen* 18.
One ten and nine units are called . . *Nineteen* 19.
Two tens are called . . . . . *Twenty* 20.
Three tens are called . . . . . *Thirty* 30.
Four tens are called . . . . . *Forty* 40.
Five tens are called . . . . . *Fifty* 50.
Six tens are called . . . . . *Sixty* 60.
Seven tens are called . . . . . *Seventy* 70.
Eight tens are called . . . . . *Eighty* 80.
Nine tens are called . . . . . *Ninety* 90.

Ten tens are called a *hundred*, which forms a unit of a still higher order, consisting of *hundreds*, represented by the same character (1) as a unit of each of the foregoing orders, but is written one place *further* toward the left hand, that is, on the left hand side of *tens*; thus, ... *One hundred* 100.

One hundred, one ten, and one unit, are called
One hundred and eleven 111.

¶ 2. There are three hundred sixty-five days in a year. In this number are contained all the orders now described, viz. units, tens, and hundreds. Let it be recollected, *units* occupy the *first place* on the right hand; *tens*, the *second place* from the right hand; *hundreds*, the *third place*. This number may now be *decomposed*, that is, *separated into parts*, exhibiting each order by itself, as follows:—The highest order, or hundreds, are *three*, represented by this character, 3; but, that it may be made to occupy the third place, counting from the right hand, it must be followed by two ciphers, thus, 300, (three hundred.) The next lower order, or *tens*, are six, (six tens are sixty,) represented by this character, 6; but, that it may occupy the second place, which is the place of tens, it must be followed by one cipher, thus, 60, (sixty.) The lowest order, or *units*, are five, represented by a single character, thus, 5, (five.)

We may now combine all these parts together, first writing down the five units for the right hand figure, thus, 5; then the six tens (60) on the left hand of the units, thus, 65; then the three hundreds (300) on the left hand of the six tens, thus, 365, which number, so written, may be read three hundred, six tens, and five units; or, as is more usual, three hundred and sixty-five.

¶ 3. Hence it appears, that *figures have a different value according to the* PLACE *they occupy, counting from the right hand towards the left.*

Hund. Tens. Units.

Take for example the number 3 3 3, made by the same figure three times repeated. The 3 on the right hand, or in the *first* place, signifies 3 units; the same figure, in the *second* place, signifies 3 *tens*, or thirty; its value is now increased *ten times*. Again, the same figure, in the *third* place, signifies neither 3 *units*, nor 3 *tens*, but 3 *hundreds*, which is *ten times* the value of the same figure in the place immediately preceding, that is, in the place of *tens*; and this is a fundamental law in notation, that *a removal of one place towards the left increases the value of a figure* TEN TIMES.

Ten hundred make a *thousand*, or a unit of the *fourth* order. Then follow tens and hundreds of thousands, in the same manner as tens and hundreds of units. To thousands

succeed *millions, billions,* &c., to each of which, as to units and to thousands, are appropriated *three* places,* as exhibited in the following examples:

	of Quadrillions.	of Trillions.			of Billions.			of Millions.			of Thousands.			of Units.		
	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
EXAMPLE 1st.	3	1	7	4	5	9	2	8	3	7	4	6	3	5	1	2
EXAMPLE 2d.	3,	1	7	4,	5	9	2,	8	3	7,	4	6	3,	5	1	2,
	6th period, or period of Quadrillions.	5th period, or period of Trillions.			4th period, or period of Billions.			3d period, or period of Millions.			2d period, or period of Thousands.			1st period, or period of Units		

To facilitate the reading of large numbers, it is frequently practised to point them off into periods of *three figures each,* as in the 2d example. The names and the order of the periods being known, this division enables us to read numbers consisting of many figures as easily as we can read three figures only. Thus, the above examples are read 3 (three) Quadrillions, 174 (one hundred seventy-four) Trillions, 592 (five hundred ninety-two) Billions, 837 (eight hundred thirty-seven) Millions, 463 (four hundred sixty-three) Thousands, 512 (five hundred and twelve.)

After the same manner are read the numbers contained in the following

* This is according to the *French* method of counting. The *English,* after hundreds of millions, instead of proceeding to billions, reckon thousands, tens and hundreds of thousands of millions, appropriating *six* places, instead of three, to millions, billions, &c.

NUMERATION TABLE.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
.	.	.	.	.	.	.	.	7
.	.	.	.	.	.	.	8	6
.	.	.	.	.	.	4	3	2
.	.	.	.	.	7	0	5	4
.	.	.	.	8	6	2	0	0
.	.	.	9	0	0	3	7	1
.	.	5	0	8	6	0	0	0
.	1	0	3	0	2	0	7	0
8	0	6	1	0	5	4	0	9

Those words at the head of the table are applicable to any sum or number, and must be committed perfectly to memory, so as to be readily applied on any occasion.

Of these characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, the *nine first* are sometimes called significant figures, or digits, in distinction from the *last*, which, of itself, is of no value, yet, placed at the right hand of *another* figure, it increases the value of that figure in the same tenfold proportion as if it had been followed by any one of the significant figures.

Note. Should the pupil find any difficulty in reading the following numbers, let him first transcribe them, and point them off into periods.

5768	52831209	286297314013
34120	175264013	5203845761204
701602	3456720834	13478120673019
6539285	25037026531	341246801734526

The expressing of numbers, (as now shown,) by figures, is called *Notation.* The reading of any number set down in figures, is called *Numeration.*

After being able to read correctly all the numbers in the foregoing table, the pupil may proceed to express the following numbers by figures:

1. Seventy-six.
2. Eight hundred and seven.
3. Twelve hundred, (that is, one thousand and two hundred.)

4. Eighteen hundred.
5. Twenty-seven hundred and nineteen.
6. Forty-nine hundred and sixty.
7. Ninety-two thousand and forty-five.
8. One hundred thousand.
9. Two millions, eighty thousands, and seven hundreds.
10. One hundred millions, one hundred thousand, one hundred and one.
11. Fifty-two millions, six thousand, and twenty.
12. Six billions, seven millions, eight thousand, and nine hundred.
13. Ninety-four billions, eighteen thousand, one hundred and seventeen.
14. One hundred thirty-two billions, two hundred millions, and nine.
15. Five trillions, sixty billions, twelve millions, and ten thousand.
16. Seven hundred trillions, eighty-six billions, and seven millions.

ADDITION

OF SIMPLE NUMBERS.

¶ 4. 1. James had 5 peaches, his mother gave him 3 peaches more; how many peaches had he then?

2. John bought a slate for 25 cents, and a book for eight cents; how many cents did he give for both?

3. Peter bought a waggon for 36 cents, and sold it so as to gain 9 cents; how many cents did he get for it?

4. Frank gave 15 walnuts to one boy, 8 to another, and had 7 left; how many walnuts had he at first?

5. A man bought a chaise for 54 dollars; he expended 8 dollars in repairs, and then sold it so as to gain 5 dollars; how many dollars did he get for the chaise?

6. A man bought 3 cows; for the first he gave 9 dollars, for the second he gave 12 dollars, and for the other he gave 10 dollars; how many dollars did he give for all the cows?

7. Samuel bought an orange for 8 cents, a book for 17 cents, a knife for 20 cents, and some walnuts for 4 cents; how many cents did he spend?

8. A man had 3 calves worth 2 dollars each, 4 calves worth 3 dollars each, and 7 calves worth 5 dollars each; how many calves had he?

9. A man sold a cow for 16 dollars, some corn for 20 dollars, wheat for 25 dollars, and butter for 5 dollars; how many dollars must he receive?

The putting together two or more numbers, (as in the foregoing examples,) so as to make one *whole number*, is called *Addition*, and the whole number is called the *sum*, or *amount*.

10. One man owes me 5 dollars, another owes me 6 dollars, another 8 dollars, another 14 dollars, and another 3 dollars; what is the amount due to me?

11. What is the amount of 4, 3, 7, 2, 8, and 9 dollars?

12. In a certain school 9 study grammar, 15 study arithmetic, 20 attend to writing, and 12 study geography; what is the whole number of scholars?

SIGNS. A cross, +, one line horizontal and the other perpendicular, is the sign of addition. It shows that numbers, with this sign between them, are to be added together. It is sometimes read *plus*, which is a Latin word signifying *more*.

Two parallel, horizontal lines, =, are the sign of equality. It signifies that the number *before* it is equal to the number *after* it. Thus, $5 + 3 = 8$ is read 5 and 3 are 8; or, 5 plus (that is, more) 3 is equal to 8.

In this manner let the pupil be instructed to commit the following

ADDITION TABLE.

$2 + 0 = 2$	$3 + 0 = 3$	$4 + 0 = 4$	$5 + 0 = 5$
$2 + 1 = 3$	$3 + 1 = 4$	$4 + 1 = 5$	$5 + 1 = 6$
$2 + 2 = 4$	$3 + 2 = 5$	$4 + 2 = 6$	$5 + 2 = 7$
$2 + 3 = 5$	$3 + 3 = 6$	$4 + 3 = 7$	$5 + 3 = 8$
$2 + 4 = 6$	$3 + 4 = 7$	$4 + 4 = 8$	$5 + 4 = 9$
$2 + 5 = 7$	$3 + 5 = 8$	$4 + 5 = 9$	$5 + 5 = 10$
$2 + 6 = 8$	$3 + 6 = 9$	$4 + 6 = 10$	$5 + 6 = 11$
$2 + 7 = 9$	$3 + 7 = 10$	$4 + 7 = 11$	$5 + 7 = 12$
$2 + 8 = 10$	$3 + 8 = 11$	$4 + 8 = 12$	$5 + 8 = 13$
$2 + 9 = 11$	$3 + 9 = 12$	$4 + 9 = 13$	$5 + 9 = 14$

ADDITION TABLE—CONTINUED.

6 + 0 = 6	7 + 0 = 7	8 + 0 = 8	9 + 0 = 9
6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10
6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11
6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12
6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13
6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14
6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15
6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16
6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17
6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18

5 + 9 = how many?
8 + 7 = how many?
4 + 3 + 2 = how many?
6 + 4 + 5 = how many?
2 + 0 + 4 + 6 = how many?
7 + 1 + 0 + 8 = how many?
3 + 0 + 9 + 5 = how many?
9 + 2 + 6 + 4 + 5 = how many?
1 + 3 + 5 + 7 + 8 = how many?
1 + 2 + 3 + 4 + 5 + 6 = how many?
8 + 9 + 0 + 2 + 4 + 5 = how many?
6 + 2 + 5 + 0 + 8 + 3 = how many?

¶ 5. When the numbers to be added are *small*, the addition is readily performed in the *mind;* but it will frequently be more convenient, and even necessary, to write the numbers down before adding them.

13. Harry had 43 cents, his father gave him 25 cents more; how many cents had he then?

One of these numbers contains 4 tens and 3 units. The other number contains 2 tens and 5 units. To unite these two numbers together into one, write them down one under the other, placing the *units* of one number directly under *units* of the other, and the *tens* of one number directly under *tens* of the other, thus:

43 *cents.*
25 *cents.*

Having written the numbers in this manner, draw a line underneath.

43 *cents.* 25 *cents.* — 8	We then begin at the right hand, and add the 5 units of the lower number to the 3 units of the upper number, making 8 units, which we set down in unit's place.
43 *cents.* 25 *cents.* — *Ans.* 68 *cents.*	We then proceed to the next column, and add the 2 tens of the lower number to the 4 tens of the upper number, making 6 tens, or 60, which we set down in ten's place, and the work is done.

It now appears that Harry's whole number of cents is 6 tens and 8 units, or 68 cents; that is, $43 + 25 = 68$.

14. A farmer bought a chaise for 210 dollars, a horse for 70 dollars, and a saddle for 9 dollars; what was the whole amount?

Write the numbers as before directed, with units under units, tens under tens, &c.

OPERATION.

Chaise, 210 *dollars.* *Horse*, 70 *dollars.* *Saddle*, 9 *dollars.* — *Answer*, 289 *dollars.*	Add as before. The units will be 9, the tens 8, and the hundreds 2; that is, $210 + 70 + 9 = 289$.

After the same manner are performed the following examples:

15. A man had 15 sheep in one pasture, 20 in another pasture, and 143 in another; how many sheep had he in the three pastures? $15 + 20 + 143 =$ how many?

16. A man has three farms, one containing 500 acres, another 213 acres, and another 76 acres; how many acres in the three farms? $500 + 213 + 76 =$ how many?

17. Bought a farm for 2316 dollars, and afterwards sold it so as to gain 550 dollars; what did I sell the farm for? $2316 + 550 =$ how many?

Hitherto the amount of any one column, when added up, has not exceeded 9; consequently has been expressed by a *single* figure. But it will frequently happen that the amount of a single column will *exceed* 9, requiring *two* or *more figures* to express it.

18. There are three bags of money. The first contains 876 dollars, the second, 653 dollars, the third, 524 dollars, what is the amount contained in all the bags?

OPERATION.	
First bag,	876
Second bag,	653
Third bag,	524
Amount,	2053

Writing down the numbers as already directed, we begin with the right hand, or unit column, and find the amount to be 13, that is, 3 units and 1 ten. Setting down the 3 units, or right hand figure, in unit's place, directly under the column, we reserve the 1 ten, or left hand figure, to be added with the other tens, in the next column, saying, 1, which we reserved, to 2 makes 3, and 5 are 8, and 7 are 15, which is 5 units of its *own* order, and 1 unit of the next *higher* order, that is, 5 *tens* and 1 *hundred.* Setting down the 5 tens, or *right* hand figure, directly under the column of tens, we reserve the *left* hand figure, or 1 hundred, to be added in the column of hundreds, saying, 1 to 5 is 6, and 6 are 12, and 8 are 20, which being the last column, we set down the whole number, writing the 0, or right hand figure, directly under the column, and carrying forward the 2, or left hand figure, to the next place, or place of thousands. Wherefore, we find the whole amount of money contained in the three bags to be 2053 dollars,—the answer.

PROOF. We may reverse the order, and, beginning at the top, add the figures downward. If the two results are alike, the work is supposed to be right.

From the examples and illustrations now given, we derive the following

RULE.

I. Write the numbers to be added, one under another, placing units under units, tens under tens, &c., and draw a line underneath.

II. Begin at the right hand or unit column, and add together all the figures contained in that column: if the amount does not exceed 9, write it under the column; but if the amount exceed 9, so that it shall require two or more figures to express it, write down the unit figure only under the column; the figure or figures to the left hand of units, being *tens*, are so many units of the next higher order, which, being reserved, must be carried forward, and added to the first figure in the next column.

III. Add each succeeding column in the same manner, and set down the whole amount at the last column.

EXAMPLES FOR PRACTICE.

19. A man bought four loads of hay; one load weighed 1817 pounds, another weighed 1950 pounds, another 2156 pounds, and another 2210 pounds; what was the amount of hay purchased?

20. A person owes A 100 dollars, B 2160 dollars, C 785 dollars, D 92 dollars; what is the amount of his debts?

21. A farmer raised in one year 1200 bushels of wheat, 850 bushels of Indian corn, 1000 bushels of oats, 1086 bushels of barley, and 74 bushels of pease; what was the whole amount? *Ans.* 4210.

22. St. Paul's Cathedral, in London, cost 800,000 pounds sterling; the Royal Exchange 80,000 pounds; the Mansion-House 40,000 pounds; Black Friars Bridge 152,840 pounds; Westminster Bridge 389,000 pounds, and the Monument 13,000 pounds; what is the amount of these sums?
Ans. 1,474,840 pounds.

23. At the census in 1820, the number of inhabitants in the New England States was as follows:—Maine, 298,335; New Hampshire, 244,161; Vermont, 235,764; Massachusetts, 253,287; Rhode Island, 83,059; Connecticut, 275,248; what was the whole number of inhabitants, at that time, in those States? *Ans.* 1,389,854.

24. From the creation to the departure of the Israelites from Egypt was 2513 years; to the siege of Troy, 307 years more; to the building of Solomon's Temple, 180 years; to the building of Rome, 251 years; to the expulsion of the kings from Rome, 244 years; to the destruction of Carthage, 363 years; to the death of Julius Cæsar, 102 years; to the Christian era, 44 years; required the time from the Creation to the Christian era. *Ans.* 4004 years.

25.

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2863705421061
3107429315638
   6253034792
       247135
         8673
-------------
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26.

```
4367583021463
1752349713620
6081275306217
5652174630128
8703263472013
-------------
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27.

5364207631023
2812345672948
6057042087094
8162835906718
7604286537892

28.

9023754682135
2834967326708
9306342167321
2365478024369
8050607080900

29. What is the amount of 46723, 6742, and 986 dollars?

30. A man has three orchards; in the first there are 140 trees that bear apples, and 64 trees that bear peaches; in the second, 234 trees bear apples, and 73 bear cherries; in the third, 47 trees bear plums, 36 bear pears, and 25 bear cherries; how many trees in all the orchards?

SUPPLEMENT

TO NUMERATION AND ADDITION.

QUESTIONS.

1. What is a single or individual thing called? 2. What is notation? 3. What are the methods of notation now in use? 4. How many are the Arabic characters or figures? 5. What is numeration? 6. What is a fundamental law in notation? 7. What is addition? 8. What is the rule for addition? 9. What is the result, or number sought, called? 10. What is the sign of addition? 11. —— of equality? 12. How is addition proved?

EXERCISES.

1. Washington was born in the year of our Lord 1732; he was 67 years old when he died; in what year of our Lord did he die?

2. The invasion of Greece by Xerxes took place 481 years before Christ; how long ago is that this current year 1827?

3. There are two numbers, the less number is 8671, the difference between the numbers is 597; what is the greater number.

4. A man borrowed a sum of money, and paid in part 684 dollars; the sum left unpaid was 876 dollars; what was the sum borrowed?

5. There are four numbers, the first 317, the second 812, the third 1350, and the fourth as much as the other three; what is the sum of them all?

6. A gentleman left his daughter 16 thousand, 16 hundred and 16 dollars; he left his son 1800 more than his daughter; what was his son's portion, and what was the amount of the whole estate? *Ans.* { Son's portion, 19,416. Whole estate, 37,032.

7. A man, at his death, left his estate to his four children, who, after paying debts to the amount of 1476 dollars, received 4768 dollars each; how much was the whole estate? *Ans.* 20548.

8. A man bought four hogs, each weighing 375 pounds; how much did they all weigh? *Ans.* 1500.

9. The fore quarters of an ox weigh one hundred and eight pounds each, the hind quarters weigh one hundred and twenty-four pounds each, the hide seventy-six pounds, and the tallow sixty pounds; what is the whole weight of the ox? *Ans.* 600.

10. A man, being asked his age, said he was thirty-four years old when his eldest son was born, who was then fifteen years of age; what was the age of the father?

11. A man sold two cows for sixteen dollars each, twenty bushels of corn for twelve dollars, and one hundred pounds of tallow for eight dollars; what was his due?

SUBTRACTION

OF SIMPLE NUMBERS.

¶ 6. 1. Charles, having 18 cents, bought a book, for which he gave 6 cents; how many cents had he left?

2. John had 12 apples; he gave 5 of them to his brother; how many had he left?

3. Peter played at marbles; he had 23 when he began, but when he had done he had only 12; how many did he lose?

4. A man bought a cow for 17 dollars, and sold her again for 22 dollars; how many dollars did he gain?

5. Charles is 9 years old, and Andrew is 13; what is the difference in their ages?

6. A man borrowed 50 dollars, and paid all but 18; how many dollars did he pay? that is, take 18 from 50, and how many would there be left?

7. John bought a book and slate for 33 cents; he gave 8 cents for the book; what did the slate cost him?

8. Peter bought a waggon for 36 cents, and sold it for 45 cents; how many cents did he gain by the bargain?

9. Peter sold a waggon for 45 cents, which was 9 cents more than he gave for it; how many cents did he give for the waggon?

10. A boy, being asked how old he was, said that he was 25 years younger than his father, whose age was 33 years; how old was the boy?

The taking of a less number from a greater (as in the foregoing examples) is called *Subtraction.* The greater number is called the *minuend*, the less number the *subtrahend*, and what is left after subtraction is called the *difference*, or *remainder.*

11. If the minuend be 8, and the subtrahend 3, what is the difference or remainder? *Ans.* 5.

12. If the subtrahend be 4, and the minuend 16, what is the remainder?

13. Samuel bought a book for 20 cents; he paid down 12 cents; how many cents more must he pay?

SIGN. A short horizontal line, —, is the sign of subtraction. It is usually read *minus*, which is a Latin word signifying *less*. It shows that the number *after* it is to be taken from the number *before* it. Thus, $8 - 3 = 5$, is read 8 minus or less 3 is equal to 5; or, 3 from 8 leaves 5. The latter expression is to be used by the pupil in committing the following

SUBTRACTION TABLE.

2 — 2 = 0	6 — 3 = 3	5 — 5 = 0	7 — 7 = 0
3 — 2 = 1	7 — 3 = 4	6 — 5 = 1	8 — 7 = 1
4 — 2 = 2	8 — 3 = 5	7 — 5 = 2	9 — 7 = 2
5 — 2 = 3	9 — 3 = 6	8 — 5 = 3	10 — 7 = 3
6 — 2 = 4	10 — 3 = 7	9 — 5 = 4	8 — 8 = 0
7 — 2 = 5	4 — 4 = 0	10 — 5 = 5	9 — 8 = 1
8 — 2 = 6	5 — 4 = 1	6 — 6 = 0	10 — 8 = 2
9 — 2 = 7	6 — 4 = 2	7 — 6 = 1	9 — 9 = 0
10 — 2 = 8	7 — 4 = 3	8 — 6 = 2	10 — 9 = 1
3 — 3 = 0	8 — 4 = 4	9 — 6 = 3	
4 — 3 = 1	9 — 4 = 5	10 — 6 = 4	
5 — 3 = 2	10 — 4 = 6		

7 — 3 = how many?	18 — 7 = how many?
8 — 5 = how many?	28 — 7 = how many?
9 — 4 = how many?	22 — 13 = how many?
12 — 3 = how many?	33 — 5 = how many?
13 — 4 = how many?	41 — 15 = how many?

¶ 7. When the numbers are *small*, as in the foregoing examples, the taking of a less number from a greater is readily done in the *mind;* but when the numbers are *large*, the operation is most easily performed part at a time, and therefore it is necessary to *write* the numbers down before performing the operation.

14. A farmer, having a flock of 237 sheep, lost 114 of them by disease; how many had he left?

Here we have 4 units to be taken from 7 units, 1 ten to be taken from 3 tens, and 1 hundred to be taken from 2 hundreds. It will therefore be most convenient to write the less number under the greater, observing, as in addition, to place units under units, tens under tens, &c. thus:

OPERATION.

From 237 *the minuend,*
Take 114 *the subtrahend,*

123 *the remainder.*

We now begin with the units, saying, 4 (units) from 7, (units,) and there remain 3, (units,) which we set down directly under the column in unit's place. Then, proceeding to the next column, we say, 1 (ten) from 3, (tens,) and there remain 2, (tens,) which we set down in *ten's* place.

Proceeding to the next column, we say, 1 (hundred) from 2, (hundreds,) and there remains 1, (hundred,) which we set down in *hundred's* place, and the work is done. It now appears, that the number of sheep left was 123; that is, 237 — 114 = 123.

After the same manner are performed the following examples:

15. There are two farms; one is valued at 3750, and the other at 1500 dollars; what is the difference in the value of the two farms?

16. A man's property is worth 8560 dollars, but he has debts to the amount of 3500 dollars; what will remain after paying his debts?

17. James, having 15 cents, bought a pen-knife, for which he gave 7 cents; how many cents had he left?

OPERATION.
15 *cents.*
 7 *cents.*
—
 8 *cents left.*

A difficulty presents itself here; for we cannot take 7 from 5; but we can take 7 from 15, and there will remain 8.

18. A man bought a horse for 85 dollars, and a cow for 27 dollars; what did the horse cost him more than the cow?

OPERATION.
Horse, 85
Cow, 27
—
Difference, 58

The same difficulty meets us here as in the last example; we cannot take 7 from 5; but in the last example the larger number consisted of 1 ten and 5 units, which together make 15; we therefore took 7 from 15. Here we have 8 tens and 5 units. We can now, in the mind, suppose 1 ten taken from the 8 tens, which would leave 7 tens, and this 1 ten we can suppose joined to the 5 units, making 15. We can now take 7 from 15, as before, and there will remain 8, which we set down. The taking of 1 ten out of 8 tens, and joining it with the 5 units, is called *borrowing ten.* Proceeding to the next higher order, or tens, we must consider the upper figure, 8, from which we borrowed, 1 less, calling it 7; then, taking 2 (tens) from 7, (tens,) there will remain 5, (tens,) which we set down, making the difference 58 dollars. Or, instead of making the *upper* figure 1 *less*, calling it 7, we may make the *lower* figure one *more*, calling it 3, and the result will be the same; for 3 from 8 leaves 5, the same as 2 from 7.

19. A man borrowed 713 dollars, and paid 471 dollars; how many dollars did he then owe? 713 — 471 = how many? *Ans.* 242 dollars.

20. 1612 — 465 = how many? *Ans.* 1147.

21. 43751 — 6782 = how many? *Ans.* 36969.

¶ 8. The pupil will readily perceive, that subtraction is the *reverse* of addition.

22. A man bought 40 sheep, and sold 18 of them; how many had he left? 40 — 18 = how many? *Ans.* 22 sheep.

23. A man sold 18 sheep, and had 22 left; how many had he at first? 18 + 22 = how many? *Ans.* 40.

24. A man bought a horse for 75 dollars, and a cow for 16 dollars; what was the difference of the costs? 75 — 16 = how many? Reversed, 59 + 16 = how many?

25. 114 — 103 = how many? Reversed, 11 + 103 = how many?

26. 143 — 76 = how many? Reversed, 67 + 76 = how many?

Hence, subtraction may be proved by *addition*, as in the foregoing examples, and addition by *subtraction*.

To prove subtraction, we may add the remainder to the *subtrahend*, and, if the work is right, the *amount* will be equal to the *minuend*.

To prove addition, we may *subtract*, successively, from the amount, the *several numbers* which were added to produce it, and, if the work is right, there will be no *remainder*. Thus 7 + 8 + 6 = 21; *proof*, 21 — 6 = 15, and 15 — 8 = 7, and 7 — 7 = 0.

From the remarks and illustrations now given, we deduce the following

RULE.

I. Write down the numbers, the less under the greater, placing units under units, tens under tens, &c. and draw a line under them.

II. Beginning with units, take successively each figure in the *lower* number from the figure *over* it, and write the remainder directly below.

III. When the figure in the lower number *exceeds* the figure over it, suppose 10 to be added to the *upper* figure; but in this case we must add 1 to the *lower* figure in the next column, *before* subtracting. This is called *borrowing* 10,

EXAMPLES FOR PRACTICE.

27. If a farm and the buildings on it be valued at 10000, and the buildings alone be valued at 4567 dollars, what is the value of the land?

28. The population of New England, at the census in 1800, was 1,232,454; in 1820 it was 1,659,854; what was the increase in 20 years?

29. What is the difference between 7,648,203 and 928,671?

30. How much must you add to 358,642 to make 1,487,945?

31. A man bought an estate for 13,682 dollars, and sold it again for 15,293 dollars; did he gain or lose by it? and how much?

32. From 364,710,825,193 take 27,940,386,574.

33. From 831,025,403,270 take 651,308,604,782.

34. From 127,368,047,216,843 take 978,654,827,352.

SUPPLEMENT

TO SUBTRACTION.

QUESTIONS.

1. What is *subtraction*? 2. What is the *greater* number called? 3. —— the *less* number? 4. What is the *result or answer* called? 5. What is the *sign* of subtraction? 6. What is the *rule*? 7. What is understood by *borrowing ten*? 8. Of what is subtraction the *reverse*? 9. How is subtraction proved? 10. How is addition proved by subtraction?

EXERCISES.

1. How long from the discovery of America by Columbus, in 1492, to the commencement of the Revolutionary war in 1775, which gained our Independence?

2. Supposing a man to have been born in the year 1773, how old was he in 1827?

3. Supposing a man to have been 80 years old in the year 1826, in what year was he born?

4. There are two numbers, whose difference is 8764; the greater number is 15687; I demand the less?

5. What number is that which, taken from 3794, leaves 865?

6. What number is that to which if you add 789, it will become 6350?

7. In New York, by the census of 1820, there were 123,706 inhabitants; in Boston, 43,940; how many more inhabitants were then in New York than in Boston?

8. A man, possessing an estate of twelve thousand dollars, gave two thousand five hundred dollars to each of his two daughters, and the remainder to his son; what was his son's share?

9. From seventeen million take fifty-six thousand, and what will remain?

10. What number, together with these three, viz. 1301, 2561, and 3120, will make ten thousand?

11. A man bought a horse for one hundred and fourteen dollars, and a chaise for one hundred and eighty-seven dollars; how much more did he give for the chaise than for the horse?

12. A man borrows 7 ten dollar bills and 3 one dollar bills, and pays at one time 4 ten dollar bills and 5 one dollar bills; how many ten dollar bills and one dollar bills must he afterwards pay to cancel the debt?

Ans. 2 ten doll. bills and 8 one doll.

13. The greater of two numbers is 24, and the less is 16; what is their difference?

14. The greater of two numbers is 24, and their difference 8; what is the less number?

15. The sum of two numbers is 40, the less is 16; what is the greater?

16. A tree, 68 feet high, was broken off by the wind; the top part, which fell, was 49 feet long; how high was the stump which was left?

17. Our pious ancestors landed at Plymouth, Massachusetts, in 1620; how many years since?

18. A man carried his produce to market; he sold his pork for 45 dollars, his cheese for 38 dollars, and his butter for 29 dollars; he received, in pay, salt to the value of 17 dollars, 10 dollars worth of sugar, 5 dollars worth of molasses, and the rest in money; how much money did he receive?

Ans. 80 dollars.

19. A boy bought a sled for 28 cents, and gave 14 cents

to have it repaired; he sold it for 40 cents; did he gain or lose by the bargain? and how much?

20. One man travels 67 miles in a day, another man follows at the rate of 42 miles a day; if they both start from the same place at the same time, how far will they be apart at the close of the first day? —— of the second? —— of the third? —— of the fourth?

21. One man starts from Boston Monday morning, and travels at the rate of 40 miles a day; another starts from the same place Tuesday morning, and follows on at the rate of 70 miles a day; how far are they apart Tuesday night?

Ans. 10 miles.

22. A man, owing 379 dollars, paid at one time 47 dollars, at another time 84 dollars, at another time 23 dollars, and at another time 143 dollars; how much did he then owe? *Ans.* 82 dollars.

23. A man has property to the amount of 34764 dollars, but there are demands against him to the amount of 14297 dollars; how many dollars will be left after the payment of his debts?

24. Four men bought a lot of land for 482 dollars; the first man paid 274 dollars, the second man 194 dollars less than the first, and the third man 20 dollars less than the second; how much did the second, the third, and the fourth man pay?

Ans. The second paid 80. The third paid 60. The fourth paid 68.

25. A man, having 10,000 dollars, gave away 9 dollars; how many had he left? *Ans.* 9991.

MULTIPLICATION

OF SIMPLE NUMBERS.

¶ 9. 1. If one orange costs 5 cents, how many cents must I give for 2 oranges? —— how many cents for 3 oranges? —— for 4 oranges?

2. One bushel of apples costs 20 cents; how many cents must I give for 2 bushels? —— for 3 bushels?

3. One gallon contains 4 quarts; how many quarts in 2 gallons? —— in 3 gallons? —— in 4 gallons?

4. Three men bought a horse; each man paid 23 dollars for his share; how many dollars did the horse cost them?

5. A man has 4 farms worth 324 dollars each; how many dollars are they all worth?

6. In one dollar there are one hundred cents; how many cents in 5 dollars?

7. How much will 4 pair of shoes cost at 2 dollars a pair?

8. How much will two pounds of tea cost at 43 cents a pound?

9. There are 24 hours in one day; how many hours in 2 days? —— in 3 days? —— in 4 days? —— in 7 days?

10. Six boys met a beggar, and gave him 15 cents each; how many cents did the beggar receive?

When questions occur, (as in the above examples,) where the same number is to be added to itself several times, the operation may be much facilitated by a rule, called *Multiplication*, in which the number to be repeated is called the *multiplicand*, and the number which shows how *many times* the multiplicand is to be repeated is called the *multiplier*. The multiplicand and multiplier, when spoken of *collectively*, are called the *factors*, (producers,) and the answer is called the *product*.

11. There is an orchard in which there are 5 rows of trees, and 27 trees in each row; how many trees in the orchard?

In the first row,	27	*trees.*
.... *second*	27	....
.... *third*	27	....
.... *fourth*	27	....
.... *fifth*	27	....
In the whole orchard,	135	*trees.*

In this example, it is evident that the whole number of trees will be equal to the amount of *five* 27's added together.

In adding, we find that 7 taken five times amounts to 35. We write down the five units, and reserve the 3 tens; the amount of 2 taken five times is 10, and the 3, which we reserved, makes 13, which, written to the left of units, makes the whole number of trees 135.

If we have learned that 7 taken 5 times amounts to 35, and that 2 taken 5 times amounts to 10, it is plain we need write the number 27 but *once*, and then, setting the multi-

plier under it, we may say, 5 times 7 are 35, writing down the 5, and reserving the 3 (tens) as in addition. Again, 5 times 2 (tens) are 10, (tens,) and 3, (tens,) which we reserved, make 13, (tens,) as before.

Multiplicand,	27	*trees in each row.*
Multiplier,	5	*rows.*
Product,	135	*trees, Ans.*

¶ **10.** 12. There are on a board 3 *rows* of spots, and 4 spots in each row; how many spots on the board?

* * * *
* * * *
* * * *

A slight inspection of the figure will show, that the number of spots may be found either by taking 4 *three times*, (3 times 4 are 12,) or by taking 3 *four times*, (4 times 3 are 12;) for we may say there are 3 rows of 4 spots each, or 4 rows of 3 spots each; therefore, we may use *either* of the given numbers for a multiplier, as best suits our convenience. We generally write the numbers as in subtraction, the larger uppermost, with units under units, tens under tens, &c. Thus,

Multiplicand,	4	*spots.*
Multiplier,	3	*rows.*
Product,	12	*Ans.*

Note. 4 and 3 are the *factors*, which produce the product 12.

Hence,—*Multiplication is a short way of performing many additions;* in other words,—*It is the method of repeating any number any given number of times.*

SIGN. Two short lines, crossing each other in the form of the letter X, are the sign of multiplication. Thus, $3 \times 4 = 12$, signifies that 3 times 4 are equal to 12, or 4 times 3 are 12.

Note. Before any progress can be made in this rule, the following table must be committed perfectly to memory.

MULTIPLICATION TABLE.

2 times 0 are 0	4 × 10 = 40	7 × 7 = 49	10 × 4 = 40
2 × 1 = 2	4 × 11 = 44	7 × 8 = 56	10 × 5 = 50
2 × 2 = 4	4 × 12 = 48	7 × 9 = 63	10 × 6 = 60
2 × 3 = 6	5 × 0 = 0	7 × 10 = 70	10 × 7 = 70
2 × 4 = 8	5 × 1 = 5	7 × 11 = 77	10 × 8 = 80
2 × 5 = 10	5 × 2 = 10	7 × 12 = 84	10 × 9 = 90
2 × 6 = 12	5 × 3 = 15	8 × 0 = 0	10 × 10 = 100
2 × 7 = 14	5 × 4 = 20	8 × 1 = 8	10 × 11 = 110
2 × 8 = 16	5 × 5 = 25	8 × 2 = 16	10 × 12 = 120
2 × 9 = 18	5 × 6 = 30	8 × 3 = 24	11 × 0 = 0
2 × 10 = 20	5 × 7 = 35	8 × 4 = 32	11 × 1 = 11
2 × 11 = 22	5 × 8 = 40	8 × 5 = 40	11 × 2 = 22
2 × 12 = 24	5 × 9 = 45	8 × 6 = 48	11 × 3 = 33
3 × 0 = 0	5 × 10 = 50	8 × 7 = 56	11 × 4 = 44
3 × 1 = 3	5 × 11 = 55	8 × 8 = 64	11 × 5 = 55
3 × 2 = 6	5 × 12 = 60	8 × 9 = 72	11 × 6 = 66
3 × 3 = 9	6 × 0 = 0	8 × 10 = 80	11 × 7 = 77
3 × 4 = 12	6 × 1 = 6	8 × 11 = 88	11 × 8 = 88
3 × 5 = 15	6 × 2 = 12	8 × 12 = 96	11 × 9 = 99
3 × 6 = 18	6 × 3 = 18	9 × 0 = 0	11 × 10 = 110
3 × 7 = 21	6 × 4 = 24	9 × 1 = 9	11 × 11 = 121
3 × 8 = 24	6 × 5 = 30	9 × 2 = 18	11 × 12 = 132
3 × 9 = 27	6 × 6 = 36	9 × 3 = 27	12 × 0 = 0
3 × 10 = 30	6 × 7 = 42	9 × 4 = 36	12 × 1 = 12
3 × 11 = 33	6 × 8 = 48	9 × 5 = 45	12 × 2 = 24
3 × 12 = 36	6 × 9 = 54	9 × 6 = 54	12 × 3 = 36
4 × 0 = 0	6 × 10 = 60	9 × 7 = 63	12 × 4 = 48
4 × 1 = 4	6 × 11 = 66	9 × 8 = 72	12 × 5 = 60
4 × 2 = 8	6 × 12 = 72	9 × 9 = 81	12 × 6 = 72
4 × 3 = 12	7 × 0 = 0	9 × 10 = 90	12 × 7 = 84
4 × 4 = 16	7 × 1 = 7	9 × 11 = 99	12 × 8 = 96
4 × 5 = 20	7 × 2 = 14	9 × 12 = 108	12 × 9 = 108
4 × 6 = 24	7 × 3 = 21	10 × 0 = 0	12 × 10 = 120
4 × 7 = 28	7 × 4 = 28	10 × 1 = 10	12 × 11 = 132
4 × 8 = 32	7 × 5 = 35	10 × 2 = 20	12 × 12 = 144
4 × 9 = 36	7 × 6 = 42	10 × 3 = 30	

9 × 2 = how many?	4 × 3 × 2 = 24.
4 × 6 = how many?	3 × 2 × 5 = how many?
8 × 9 = how many?	7 × 1 × 2 = how many?
3 × 7 = how many?	8 × 3 × 2 = how many?
5 × 5 = how many?	3 × 2 × 4 × 5 = how many?

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13. What will 84 barrels of flour cost at 7 dollars a barrel? *Ans.* 588 dollars.

14. A merchant bought 273 hats at 8 dollars each; what did they cost? *Ans.* 2184 dollars.

15. How many inches are there in 253 feet, every foot being 12 inches?

OPERATION.
```
     253
      12
     ---
Ans. 3036
```

The product of 12, with each of the significant figures or digits, having been committed to memory from the multiplication table, it is just as easy to multiply by 12 as by a single figure. Thus, 12 times 3 are 36, &c.

16. What will 476 barrels of fish cost at 11 dollars a barrel? *Ans.* 5236 dollars.

17. A piece of valuable land, containing 33 acres, was sold for 246 dollars an acre; what did the whole come to?

As 12 is the largest number, the product of which, with the nine digits, is found in the multiplication table, therefore, when the multiplier *exceeds* 12, we multiply by each figure in the multiplier *separately*. Thus:

OPERATION.
```
     246 dollars, the price of 1 acre.
      33 number of acres.
    ----
     738 dollars, the price of 3 acres.
    738  dollars, the price of 30 acres.
    ----
Ans. 8118 dollars, the price of 33 acres.
```

The multiplier consists of 3 tens and 3 units. First, multiplying by the 3 units gives us 738 dollars, the price of 3 acres. We then multiply by the 3 tens, writing the first figure of the product (8) in ten's place, that is, *directly under the figure by which we multiply.* It now appears, that the product by the 3 tens consists of the same figures as the product by the three units; but there is this difference—the figures in the product by the 3 *tens* are all removed one place further toward the *left* hand, by which their value is increased *tenfold,* which is as it should be, because the price of 30 acres

is evidently ten times as much as the price of 3 acres, that is, 7380 dollars; and it is plain, that these two products, added together, give the price of 33 acres.

These examples will be sufficient to establish the following

RULE.

I. Write down the multiplicand, under which write the multiplier, placing units under units, tens under tens, &c., and draw a line underneath.

II. When the multiplier does *not* exceed 12, begin at the right hand of the multiplicand, and multiply each figure contained in it by the multiplier, setting down, and carrying, as in addition.

III. When the multiplier *exceeds* 12, multiply by each figure of the multiplier separately, first by the *units*, then by the *tens*, &c., remembering always to place the first figure of each product directly under the figure by which you multiply. Having gone through in this manner with each figure in the multiplier, add their several products together, and the sum of them will be the product required.

EXAMPLES FOR PRACTICE.

18. There are 320 rods in a mile; how many rods are there in 57 miles?

19. It is 436 miles from Boston to the city of Washington; how many rods is it?

20. What will 784 chests of tea cost, at 69 dollars a chest?

21. If 1851 men receive 758 dollars apiece, how many dollars will they all receive? *Ans.* 1403058 dollars.

22. There are 24 hours in a day; if a ship sail 7 miles in an hour, how many miles will she sail in 1 day, at that rate? how many miles in 36 days? how many miles in 1 year, or 365 days? *Ans.* 61320 miles in 1 year.

23. A merchant bought 13 pieces of cloth, each piece containing 28 yards, at 6 dollars a yard; how many yards were there, and what was the whole cost?

Ans. The whole cost was 2184 dollars.

24.	Multiply 37864	by	235.	*Product,*	8898040.
25.	 29831	...	952	....	28399112.
26.	 93956	...	8704.	....	817793024.

CONTRACTIONS IN MULTIPLICATION.

I. *When the multiplier is a composite number.*

¶ **11.** Any number, which may be produced by the multiplication of two or more numbers, is called a *composite number.* Thus, 15, which arises from the multiplication of 5 and 3, ($5 \times 3 = 15$,) is a composite number, and the numbers 5 and 3, which, multiplied together, produce it, are called *component parts*, or *factors* of that number. So, also, 24 is a composite number; its *component parts* or *factors* may be 2 and 12 ($2 \times 12 = 24$;) or they may be 4 and 6 ($4 \times 6 = 24$;) or they may be 2, 3, and 4 ($2 \times 3 \times 4 = 24$.)

1. What will 15 yards of cloth cost, at 4 dollars a yard?

$$\begin{array}{r} 4 \\ 5 \\ \hline 20 \\ 3 \\ \hline 60 \end{array}$$

15 yards are equal to 5×3 yards. The cost of 5 yards would be $5 \times 4 = 20$ dollars; and because 15 yards contain 3 times 5 yards, so the cost of 15 yards will evidently be 3 times the cost of 5 yards, that is, 20 dollars $\times 3 = 60$ dollars. *Ans.* 60 dollars.

Wherefore, *If the multiplier be a composite number*, we may, if we please, *multiply the multiplicand first by one of the component parts, that product by the other, and so on*, if the component parts be more than two; and, having in this way multiplied by each of the component parts, the *last* product will be the product *required.*

2. What will 136 tons of potashes come to, at 96 dollars per ton?

$8 \times 12 = 96$. It follows, therefore, that 8 and 12 are component parts or factors of 96. Hence,

136 dollars, the price of 1 ton.
8 one of the component parts, or factors.

1088 dollars, the price of 8 tons.
12 the other component part, or factor.

Ans. 13056 dollars, the price of 96 tons.

3. Supposing 342 men to be employed in a certain piece of work, for which they are to receive 112 dollars each, how much will they all receive?

$8 \times 7 \times 2 = 112$. *Ans.* 38304 dollars.

4. Multiply 367 by 48. *Product*, 17616.
5. .... 853 ... 56. .... 47768.
6. .... 1086 ... 72. .... 78192.

II. *When the multiplier is* 10, 100, 1000, *&c.*

¶ **12.** It will be recollected, (¶ 3.) that any figure, on being removed one place towards the *left* hand, has its value increased *tenfold;* hence, to multiply any number by 10, it is only necessary to *write a cipher on the right hand of it.* Thus, 10 times 25 are 250; for the 5, which was *units* before, is now made *tens*, and the 2, which was *tens* before, is now made *hundreds*. So, also, if any figure be removed *two* places towards the left hand, its value is increased 100 times, &c. Hence,

When the multiplier is 10, 100, 1000, or 1 *with any number of ciphers annexed*, annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the multiplicand, so increased, will be the product required. Thus,

Multiply 46 by 10, the product is 460.
.... 83 ... 100, ... 8300.
.... 95 ... 1000, ... 95000.

EXAMPLES FOR PRACTICE.

1. What will 76 barrels of flour cost, at 10 dollars a barrel?
2. If 100 men receive 126 dollars each, how many dollars will they all receive?
3. What will 1000 pieces of broadcloth cost, estimating each piece at 312 dollars?
4. Multiply 5682 by 10000.
5. .... 82134 ... 100000.

¶ **13.** On the principle suggested in the last ¶, it follows,

When there are ciphers on the right hand of the multiplicand, multiplier, either or both, we may, at first, neglect these ciphers, multiplying by the *significant figures* only; after which we must annex as many ciphers to the product as there are ciphers on the right hand of the multiplicand and multiplier, counted together.

EXAMPLES FOR PRACTICE.

1. If 1300 men receive 460 dollars apiece, how many dollars will they all receive?

OPERATION.

```
 460
1300
----
138
46
----
```

Ans. 598000 *dollars.*

The ciphers in the multiplicand and multiplier, counted together, are *three*. Disregarding these, we write the *significant* figures of the multiplier under the *significant* figures of the multiplicand, and multiply; after which we annex three ciphers to the right hand of the product, which gives the true answer.

2. The number of distinct buildings in New England, appropriated to the spinning, weaving, and printing of cotton goods, was estimated, in 1826, at 400, running, on an average, 700 spindles each; what was the whole number of spindles?

3. Multiply 357 by 6300.
4. .... 8600 .... 17.
5. .... 9340 .... 460.
6. .... 5200 .... 410.
7. .... 378 .... 204.

OPERATION.

```
  378
  204
 ----
 1512
 000
756
-----
77112
```

In the operation it will be seen, that multiplying by ciphers produces nothing. Therefore,

III. *When there are ciphers between the significant figures of the multiplier*, we may omit the ciphers, multiplying by the *significant figures only*, placing the first figure of each product directly under the figure by which we multiply.

EXAMPLES FOR PRACTICE.

8. Multiply 154326 by 3007.

OPERATION.

154326
3007
1080282
462978
Product, 464058282

9. Multiply 543 by 206.
10. .... 1620 ... 2103.
11. ....36243 ... 32004.

SUPPLEMENT
TO MULTIPLICATION.

QUESTIONS.

1. What is multiplication? 2. What is the number *to be multiplied* called? 3. —— to multiply *by* called? 4. What is the *result* or *answer* called? 5. Taken *collectively*, what are the multiplicand and multiplier called? 6. What is the *sign* of multiplication? 7. What does it show? 8. In what *order* must the given number be placed for multiplication? 9. How do you proceed when the multiplier is *less* than 12? 10. When it *exceeds* 12, what is the method of procedure? 11. What is a *composite* number? 12. What is to be understood by the *component parts*, or *factors*, of any number? 13. How may you proceed when the multiplier is a *composite number*? 14. To multiply by 10, 100, 1000, &c., what suffices? 15. Why? 16. When there are *ciphers on the right hand* of the multiplicand, multiplier, either or both, how may we proceed? 17. When there are ciphers *between* the significant figures of the multiplier, how are they to be treated?

EXERCISES.

1. An army of 10700 men, having plundered a city, took so much money, that, when it was shared among them, each man received 46 dollars; what was the sum of money taken?

2. Supposing the number of houses in a certain town to be 145, each house, on an average, containing two families, and each family 6 members, what would be the number of inhabitants in that town? *Ans.* 1740.

3. If 46 men can do a piece of work in 60 days, how many men will it take to do it in one day?

4. Two men depart from the same place, and travel in opposite directions, one at the rate of 27 miles a day, the other 31 miles a day; how far apart will they be at the end of 6 days? *Ans.* 348 miles.

5. What number is that, the factors of which are 4, 7, 6, and 20? *Ans.* 3360.

6. If 18 men can do a piece of work in 90 days, how long will it take one man to do the same?

7. What sum of money must be divided between 27 men, so that each man may receive 115 dollars?

8. There is a certain number, the factors of which are 89 and 265; what is that number?

9. What is that number, of which 9, 12, and 14 are factors?

10. If a carriage wheel turn round 346 times in running 1 mile, how many times will it turn round in the distance from New York to Philadelphia, it being 95 miles.

Ans. 32870.

11. In one minute are 60 seconds; how many seconds in 4 minutes? —— in 5 minutes? —— in 20 minutes? —— in 40 minutes?

12. In one hour are 60 minutes; how many *seconds* in an hour? —— in two hours? how many seconds from nine o'clock in the morning till noon?

13. In one dollar are 6 shillings; how many shillings in 3 dollars? —— in 300 dollars? —— in 467 dollars?

14. Two men, A and B, start from the same place at the same time, and travel the same way; A travels 52 miles a day, and B 44 miles a day; how far apart will they be at the end of 10 days?

15. If the interest of 100 cents, for one *year*, be 6 cents, how many cents will be the interest for 2 years? —— for 4 years? —— for 10 years? —— for 35 years? —— for 84 years?

16. If the interest of one dollar, for one year, be six cents, what is the interest for 2 dollars the same time? —— 5 dollars? —— 7 dollars? —— 8 dollars? —— 95 dollars?

17. A farmer sold 468 pounds of pork, at 6 cents a pound, and 48 pounds of cheese, at 7 cents a pound; how many cents must he receive in pay?

18. A boy bought 10 oranges; he kept 7 of them, and sold the others for 5 cents apiece; how many cents did he receive?

19. The component parts of a certain number are 4, 5, 7, 6, 9, 8, and 3; what is the number?

20. In 1 hogshead are 63 gallons; how many gallons in 8 hogsheads? In 1 gallon are 4 quarts; how many quarts in 8 hogsheads? In 1 quart are 2 pints; how many pints in 8 hogsheads?

DIVISION

OF SIMPLE NUMBERS.

¶ 14. 1. James divided 12 apples among 4 boys; how many did he give each boy?

2. James would divide 12 apples among 3 boys; how many must he give each boy?

3. John had 15 apples, and gave them to his playmates, who received 3 apples each; how many boys did he give them to?

4. If you had 20 cents, how many cakes could you buy at 4 cents apiece?

5. How many yards of cloth could you buy for 30 dollars, at 5 dollars a yard?

6. If you pay 40 dollars for 10 yards of cloth, what is one yard worth?

7. A man works 6 days for 42 shillings; how many shillings is that for one day?

8. How many quarts in 4 pints? —— in 6 pints? —— in 10 pints?

9. How many times is 8 contained in 88?

10. If a man can travel 4 miles an hour, how many hours would it take him to travel 24 miles?

11. In an orchard there are 28 trees standing in rows, and there are 3 trees in a row; how many rows are there?

Remark. When any one thing is divided into two equal parts, one of those parts is called a *half;* if into 3 equal parts, one of those parts is called a *third;* if into four equal parts, one part is called a *quarter* or a *fourth;* if into five, one part is called a *fifth*, and so on.

12. A boy had two apples, and gave one half an apple to each of his companions; how many were his companions?

13. A boy divided four apples among his companions, by giving them one third of an apple each; among how many did he divide his apples?

14. How many quarters in 3 oranges?

15. How many oranges would it take to give 12 boys one quarter of an orange each?

16. How much is one half of 12 apples?

17. How much is one third of 12?

18. How much is one fourth of 12?

19. A man had 30 sheep, and sold one fifth of them; how many of them did he sell?

20. A man purchased sheep for 7 dollars apiece, and paid for them all 63 dollars; what was their number?

21. How many oranges, at 3 cents each, may be bought for 12 cents?

It is plain, that as many times as 3 cents can be taken from 12 cents, so many oranges may be bought; the object, therefore, is to find how many times 3 is contained in 12.

	12 *cents.*
First orange,	3 *cents.*
	9
Second orange,	3 *cents.*
	6
Third orange,	3 *cents.*
	3
Fourth orange,	3 *cents.*
	0

We see in this example, that we may take 3 from 12 four times, after which there is no remainder; consequently, *subtraction* alone is sufficient for the operation; but we may come to the same result by a process, in most cases much shorter, called *Division.*

¶ **15.** It is plain, that the cost of one orange, (3 cents,) multiplied by the number of oranges, (4,) is equal to the cost of all the oranges, (12 cents;) 12 is, therefore, a *product*, and 3 *one* of its factors; and to find how many times 3 is contained in 12, is to find the *other* factor, which, multiplied into 3, will produce 12. This factor we find, by trial, to be 4, ($4 \times 3 = 12$;) consequently, 3 is contained in 12 4 times. *Ans.* 4 oranges.

22. A man would divide 12 oranges equally among 3 children; how many oranges would each child have?

Here the object is to divide the 12 oranges into 3 equal

parts, and to ascertain the number of oranges in each of those parts. The operation is evidently as in the last example, and consists in finding a number, which, multiplied by 3, will produce 12. This number we have already found to be 4.

Ans. 4 oranges apiece.

As, therefore, *multiplication* is a short way of performing many *additions* of the same number; so, *division* is a short way of performing many *subtractions* of the same number; and may be defined, *The method of finding how many times one number is contained in another*, and also *of dividing a number into any number of equal parts.* In all cases, the *process* of division consists in finding *one* of the factors of a given product, when the *other* factor is known.

The number given *to be divided* is called the *dividend*, and answers to the *product* in multiplication. The number given *to divide by* is called the *divisor*, and answers to *one* of the factors in multiplication. The *result*, or *answer* sought, is called the *quotient*, (from the Latin word *quoties*, how many?) and answers to the *other* factor.

SIGN. The sign for division is a short horizontal line between two dots, $\div$. It shows that the number *before* it is to be divided by the number *after* it. Thus $27 \div 9 = 3$ is read, 27 divided by 9 is equal to 3; or, to shorten the expression, 27 by 9 is 3; or, 9 in 27 3 times. In place of the *dots*, the *dividend* is often written *over* the line, and the *divisor under* it, to express division; thus, $\frac{27}{9} = 3$, read as before.

DIVISION TABLE.*

$\frac{2}{2} = 1$*	$\frac{3}{3} = 1$	$\frac{4}{4} = 1$	$\frac{5}{5} = 1$	$\frac{6}{6} = 1$	$\frac{7}{7} = 1$
$\frac{4}{2} = 2$*	$\frac{6}{3} = 2$	$\frac{8}{4} = 2$	$\frac{10}{5} = 2$	$\frac{12}{6} = 2$	$\frac{14}{7} = 2$
$\frac{6}{2} = 3$	$\frac{9}{3} = 3$	$\frac{12}{4} = 3$	$\frac{15}{5} = 3$	$\frac{18}{6} = 3$	$\frac{21}{7} = 3$
$\frac{8}{2} = 4$	$\frac{12}{3} = 4$	$\frac{16}{4} = 4$	$\frac{20}{5} = 4$	$\frac{24}{6} = 4$	$\frac{28}{7} = 4$
$\frac{10}{2} = 5$	$\frac{15}{3} = 5$	$\frac{20}{4} = 5$	$\frac{25}{5} = 5$	$\frac{30}{6} = 5$	$\frac{35}{7} = 5$
$\frac{12}{2} = 6$	$\frac{18}{3} = 6$	$\frac{24}{4} = 6$	$\frac{30}{5} = 6$	$\frac{36}{6} = 6$	$\frac{42}{7} = 6$
$\frac{14}{2} = 7$	$\frac{21}{3} = 7$	$\frac{28}{4} = 7$	$\frac{35}{5} = 7$	$\frac{42}{6} = 7$	$\frac{49}{7} = 7$
$\frac{16}{2} = 8$	$\frac{24}{3} = 8$	$\frac{32}{4} = 8$	$\frac{40}{5} = 8$	$\frac{48}{6} = 8$	$\frac{56}{7} = 8$
$\frac{18}{2} = 9$	$\frac{27}{3} = 9$	$\frac{36}{4} = 9$	$\frac{45}{5} = 9$	$\frac{54}{6} = 9$	$\frac{63}{7} = 9$

* The reading used by the pupil in committing the table may be, 2 by 2 is 1, 4 by 2 is 2, &c.; or, 2 in 2 one time, 2 in 4 two times, &c.

DIVISION TABLE—CONTINUED.

$\frac{8}{8}=1$	$\frac{9}{9}=1$	$\frac{10}{10}=1$	$\frac{11}{11}=1$	$\frac{12}{12}=1$
$\frac{16}{8}=2$	$\frac{18}{9}=2$	$\frac{20}{10}=2$	$\frac{22}{11}=2$	$\frac{24}{12}=2$
$\frac{24}{8}=3$	$\frac{27}{9}=3$	$\frac{30}{10}=3$	$\frac{33}{11}=3$	$\frac{36}{12}=3$
$\frac{32}{8}=4$	$\frac{36}{9}=4$	$\frac{40}{10}=4$	$\frac{44}{11}=4$	$\frac{48}{12}=4$
$\frac{40}{8}=5$	$\frac{45}{9}=5$	$\frac{50}{10}=5$	$\frac{55}{11}=5$	$\frac{60}{12}=5$
$\frac{48}{8}=6$	$\frac{54}{9}=6$	$\frac{60}{10}=6$	$\frac{66}{11}=6$	$\frac{72}{12}=6$
$\frac{56}{8}=7$	$\frac{63}{9}=7$	$\frac{70}{10}=7$	$\frac{77}{11}=7$	$\frac{84}{12}=7$
$\frac{64}{8}=8$	$\frac{72}{9}=8$	$\frac{80}{10}=8$	$\frac{88}{11}=8$	$\frac{96}{12}=8$
$\frac{72}{8}=9$	$\frac{81}{9}=9$	$\frac{90}{10}=9$	$\frac{99}{11}=9$	$\frac{108}{12}=9$

$28 \div 7$, or $\frac{28}{7}=$ how many? $49 \div 7$, or $\frac{49}{7}=$ how many?
$42 \div 6$, or $\frac{42}{6}=$ how many? $32 \div 4$, or $\frac{32}{4}=$ how many?
$54 \div 9$, or $\frac{54}{9}=$ how many? $99 \div 11$, or $\frac{99}{11}=$ how many?
$32 \div 8$, or $\frac{32}{8}=$ how many? $84 \div 12$, or $\frac{84}{12}=$ how many?
$33 \div 11$, or $\frac{33}{11}=$ how many? $108 \div 12$, or $\frac{108}{12}=$ how many?

¶ **16.** 23. How many yards of cloth, at 4 dollars a yard, can be bought for 856 dollars?

Here the number to be divided is 856, which therefore is the *dividend;* 4 is the number to divide by, and therefore the *divisor.* It is not evident how many times 4 is contained in so large a number as 856. This difficulty will be readily overcome, if we decompose this number, thus:

$$856 = 800 + 40 + 16.$$

Beginning with the hundreds, we readily perceive that 4 is contained in 8 2 times; consequently, in 800 it is contained 200 times. Proceeding to the tens, 4 is contained in 4 1 time, and consequently in 40 it is contained 10 times. Lastly, in 16 it is contained 4 times. We now have $200 + 10 + 4 = 214$ for the quotient, or the number of times 4 is contained in 856. *Ans.* 214 yards.

We may arrive to the same result without decomposing the dividend, except as it is done in the mind, taking it by parts, in the following manner:

	Dividend.
Divisor, 4)	856
Quotient,	214

For the sake of convenience, we write down the dividend with the divisor on the left, and draw a line between them; we also draw a line underneath. Then, beginning on the left hand,

we seek how often the divisor (4) is contained in 8, (hundreds,) the left hand figure; finding it to be 2 times, we write 2 directly under the 8, which, falling in the place of hundreds, is in reality 200. Proceeding to tens, 4 is contained in 5 (tens) 1 time, which we set down in *ten's* place, directly under the 5 (tens.) But, after taking 4 times ten out of the 5 tens, there is 1 ten left. This 1 ten we join to the 6 units, making 16. Then, 4 into 16 goes 4 times, which we set down, and the work is done.

This manner of performing the operation is called *Short Division*. The computation, it may be perceived, is carried on *partly* in the mind, which it is always easy to do when the divisor does not exceed 12.

RULE.

From the illustration of this example, we derive this general rule for dividing, when the divisor does not exceed 12:

I. Find how many times the divisor is contained in the first figure, or figures, of the dividend, and, setting it directly under the dividend, carry the remainder, if any, to the next figure as so many tens.

II. Find how many times the divisor is contained in *this* dividend, and set it down as before, continuing so to do till all the figures in the dividend are divided.

PROOF. We have seen, (¶ 15,) that the divisor and quotient are factors, whose product is the dividend, and we have also seen, that dividing the dividend by *one* factor is merely a process for finding the *other*.

Hence *division* and *multiplication* mutually prove each other.

To prove division, we may *multiply* the divisor by the quotient, and, if the work be right, the product will be the same as the dividend; or we may divide *the dividend by the quotient*, and, if the work is right, the *result* will be the same as the divisor.

To prove multiplication, we may *divide the product* by *one factor*, and, if the work be right, the *quotient* will be the *other factor*.

EXAMPLES FOR PRACTICE.

24. A man would divide 13,462,725 dollars among 5 men; how many dollars would each receive?

OPERATION.

	Dividend.
Divisor, 5)	13,462,725
Quotient,	2,692,545

In this example, as we cannot have 5 in the first figure, (1,) we take two figures, and say, 5 in 13 will go 2 times, and there are 3 over, which, joined to 4, the next figure, makes 34; and 5 in 34 will go 6 times, &c.

PROOF.

Quotient.
2,692,545
5 *divisor.*
13,462,725

In proof of this example, we multiply the quotient by the divisor, and, as the product is the same as the dividend, we conclude that the work is right. From a bare inspection of the above example and its proof, it is plain, as before stated, that division is the *reverse* of multiplication, and that the two rules mutually prove each other.

25. How many yards of cloth can be bought for 4,354,560 dollars, at 2 dollars a yard? —— at 3 dollars? —— at 4 dollars? —— at 5 dollars? —— at 6 dollars? —— at 7? —— at 8? ——at 9? —— at 10?

Note. Let the pupil be required to prove the foregoing, and all following examples.

26. Divide 1005903360 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

27. If 2 pints make a quart, how many quarts in 8 pints? —— in 12 pints? —— in 20 pints? —— in 24 pints? —— in 248 pints? —— in 3764 pints? —— in 47632 pints?

28. Four quarts make a gallon; how many gallons in 8 quarts? —— in 12 quarts? —— in 20 quarts? —— in 36 quarts? —— in 368 quarts? —— in 4896 quarts? —— in 5436144 quarts?

29. A man gave 86 apples to 5 boys; how many apples would each boy receive?

	Dividend.
Divisor, 5)	86
Quotient,	17—1 *Remainder.*

Here, dividing the number of the apples (86) by the number of boys, (5,) we find, that *each* boy's share would be 17 apples; but there is one apple left.

¶ 17. 5)86
$17\frac{1}{5}$

In order to divide *all* the apples equally among the boys, it is plain, we must divide this *one* remaining apple into 5 *equal*

parts, and give one of these parts to *each* of the boys. Then each boy's share would be 17 apples, and one fifth part of another apple; which is written thus, $17\frac{1}{5}$ apples.

Ans. $17\frac{1}{5}$ apples each.

The 17, expressing *whole* apples, are called *integers*, (that is, *whole* numbers.) The $\frac{1}{5}$ (one fifth) of an apple, expressing *part* of a broken or divided apple, is called a *fraction*, (that is, a *broken* number.)

Fractions, as we here see, are written with two numbers, one directly over the other, with a short line between them, showing that the *upper* number is to be divided by the *lower*. The upper number, or *dividend*, is, in fractions, called the *numerator*, and the lower number, or *divisor*, is called the *denominator*.

Note. A number like $17\frac{1}{5}$, composed of integers (17) and a fraction, ($\frac{1}{5}$,) is called a *mixed number*.

In the preceding example, the one apple, which was left after carrying the division as far as could be by *whole* numbers, is called the *remainder*, and is evidently a part of the *dividend* yet undivided. In order to complete the division, this remainder, as we before remarked, must be divided into 5 equal parts; but the *divisor* itself expresses the number of parts. If, now, we examine the fraction, we shall see, that it consists of the remainder (1) for its *numerator*, and the divisor (5) for its *denominator*.

Therefore, if there be a *remainder*, set it down at the right hand of the quotient for the *numerator* of a fraction, under which write the divisor for its *denominator*.

Proof of the last example.

$$\begin{array}{r} 17\frac{1}{5} \\ 5 \\ \hline 86 \end{array}$$

In proving this example, we find it necessary to multiply our fraction by 5; but this is easily done, if we consider, that the fraction $\frac{1}{5}$ expresses *one* part of an apple divided into 5 equal parts; hence, 5 times $\frac{1}{5}$ is $\frac{5}{5} = 1$, that is, one *whole* apple, which we reserve to be added to the *units*, saying, 5 times 7 are 35, and one we reserved makes 36, &c.

30. Eight men drew a prize of 453 dollars in a lottery; how many dollars did each receive?

Dividend.	
Divisor, 8) 453	
Quotient, $56\frac{5}{8}$	

Here, after carrying the division as far as possible by *whole* numbers, we have a remainder of 5 dollars, which, written as above directed, gives for the answer 56 dollars and $\frac{5}{8}$ (five eighths) of another dollar, to each man.

¶ **18.** Here we may notice, that the eighth part of 5 dollars is the same as 5 times the eighth part of 1 dollar, that is, the eighth part of 5 dollars is $\frac{5}{8}$ of a dollar. Hence, $\frac{5}{8}$ expresses the quotient of 5 divided by 8.

Proof.
$56\frac{5}{8}$
8
453

$\frac{5}{8}$ is 5 parts, and 8 times 5 is 40, that is, $\frac{40}{8}=5$, which, reserved and added to the product of 8 times 6, makes 53, &c. Hence, *to multiply a fraction, we may multiply the numerator, and divide the product by the denominator.*

Or, in proving division, we may multiply the *whole* number in the quotient *only*, and to the product *add* the remainder; and this, till the pupil shall be more particularly taught in fractions, will be more easy in practice. Thus, $56 \times 8 = 448$, and $448 + 5$, the remainder, $= 453$, as before.

31. There are 7 days in a week; how many weeks in 365 days? *Ans.* $52\frac{1}{7}$ weeks.

32. When flour is worth 6 dollars a barrel, how many barrels may be bought for 25 dollars? how many for 50 dollars? —— for 487 dollars? —— for 7631 dollars?

33. Divide 640 dollars among 4 men.

$640 \div 4$, or $\frac{640}{4} = 160$ dollars, *Ans.*

34. $678 \div 6$, or $\frac{678}{6} =$ how many? *Ans.* 113.

35. $\frac{5040}{5} =$ how many?

36. $\frac{1234}{7} =$ how many?

37. $\frac{3464}{9} =$ how many? *Ans.* $384\frac{8}{9}$.

38. $\frac{2764}{11} =$ how many?

39. $\frac{40301}{8} =$ how many?

40. $\frac{2014012}{12} =$ how many?

¶ **19.** 41. Divide 4370 dollars equally among 21 men.

When, as in this example, the divisor exceeds 12, it is evident that the computation cannot be readily carried on in the mind, as in the foregoing examples. Wherefore, it is more convenient to write down the computation *at length,* in the following manner:

OPERATION.

Divisor. Dividend. Quotient.

21) 4370 ($208\frac{2}{21}$.
42
―――
170
168
―――
2 *Remainder.*

We may write the divisor and dividend as in short division, but, instead of writing the quotient *under* the dividend, it will be found more convenient to set it to the *right hand.*

Taking the dividend *by parts*, we seek how often we can have 21 in 43 (hundreds;) finding it to be 2 times, we set down 2 on the right hand of the dividend for the highest figure in the quotient. The 43 being *hundreds*, it follows, that the 2 must also be hundreds. This, however, we need not regard, for it is to be followed by *tens* and *units*, obtained from the tens and units of the dividend, and will therefore, at the end of the operation, be in the place of hundreds, as it should be.

It is plain that 2 (hundred) times 21 dollars ought now to be taken out of the dividend; therefore, we multiply the divisor (21) by the quotient figure 2 (hundred) now found, making 42, (hundred,) which, written under the 43 in the dividend, we subtract, and to the remainder, 1, (hundred,) bring down the 7, (tens,) making 17 tens.

We then seek how often the divisor is contained in 17, (tens;) finding that it will not go, we write a cipher in the quotient, and bring down the next figure, making the whole 170. We then seek how often 21 can be contained in 170, and, finding it to be 8 times, we write 8 in the quotient, and, multiplying the divisor by this number, we set the product, 168, under the 170; then, subtracting, we find the remainder to be 2, which, written as a fraction on the right hand of the quotient, as already explained, gives $208\frac{2}{21}$ dollars, for the *answer.*

This manner of performing the operation is called *Long Division.* It consists in writing down the *whole* computation.

From the above example, we derive the following

RULE.

I. Place the divisor on the left of the dividend, separate them by a line, and draw another line on the right of the dividend to separate it from the quotient.

II. Take as many figures, on the left of the dividend, as

contain the divisor once or more; seek how many times they contain it, and place the answer on the right hand of the dividend for the first figure in the quotient.

III. Multiply the divisor by this quotient figure, and write the product under that part of the dividend taken.

IV. Subtract the product from the figures above, and to the remainder bring down the next figure in the dividend, and divide the number it makes up, as before. So continue to do, till all the figures in the dividend shall have been brought down and divided.

Note 1. Having brought down a figure to the remainder, if the number it makes up be *less* than the divisor, write a cipher in the quotient, and bring down the next figure.

Note 2. If the product of the divisor, by any quotient figure, be *greater* than the part of the dividend taken, it is an evidence that the quotient figure is *too large*, and must be diminished. If the remainder at any time be *greater* than the divisor, or equal to it, the quotient figure is *too small*, and must be increased.

EXAMPLES FOR PRACTICE.

1. How many hogsheads of molasses, at 27 dollars a hogshead, may be bought for 6318 dollars?

Ans. 234 hogsheads.

2. If a man's income be 1248 dollars a year, how much is that per week, there being 52 weeks in a year?

Ans. 24 dollars per week.

3. What will be the quotient of 153598, divided by 29?

Ans. $5296\frac{14}{29}$.

4. How many times is 63 contained in 30131?

Ans. $478\frac{17}{63}$ times; that is, 478 times, and $\frac{17}{63}$ of another time.

5. What will be the several quotients of 7652, divided by 16, 23, 34, 86, and 92?

6. If a farm, containing 256 acres, be worth 7168 dollars, what is that per acre?

7. What will be the quotient of 974932, divided by 365?

Ans. $2671\frac{17}{365}$.

8. Divide 3228242 dollars equally among 563 men; how many dollars must each man receive? *Ans.* 5734 dollars.

9. If 57624 be divided into 216, 586, and 976 equal parts, what will be the magnitude of one of each of these equal parts?

Ans. The magnitude of one of the last of these equal parts will be $59\frac{40}{576}$.

10. How many times does 1030603615 contain 3215?

Ans. 320561 times.

11. The earth, in its annual revolution round the sun, is said to travel 596088000 miles; what is that per hour, there being 8766 hours in a year?

12. $\frac{1234567890}{1307}$ = how many?

13. $\frac{40700020}{7812}$ = how many?

14. $\frac{9876540031}{9124}$ = how many?

CONTRACTIONS IN DIVISION.

I. *When the* DIVISOR *is a* COMPOSITE NUMBER.

¶ **20.** 1. Bought 15 yards of cloth for 60 dollars; how much was that per yard?

15 yards are 3 × 5 yards. If there had been but 5 yards, the cost of one yard would be $\frac{60}{5} = 12$ dollars; but, as there are 3 times 5 yards, the cost of one yard will evidently be but one *third* part of 12 dollars; that is, $\frac{12}{3} = 4$ dollars. *Ans.*

Hence, when the divisor is a composite number, we may, if we please, divide the dividend by *one* of the component parts, and the *quotient*, arising from that division, by the *other*; the last quotient will be the answer.

2. If a man can travel 24 miles a day, how many days will it take him to travel 264 miles?

It will evidently take him as many days as 264 contains 24.

OPERATION.

```
24 = 6 × 4.     6)264                24)264(11 days, Ans.
                 ----        or,        24
                4)44                    --
                 --                      24
                11 days.                 24
                                         --
```

3. Divide 576 by 48 = (8 × 6.)
4. Divide 1260 by 63 = (7 × 9.)
5. Divide 2430 by 81.
6. Divide 448 by 56.

II. *To divide by* 10, 100, 1000, &c.

¶ **21.** 1. A prize of 2478 dollars is owned by 10 men; what is each man's share?

Each man's share will be equal to the number of *tens* contained in the whole sum, and, if one of the figures be cut off at the right hand, all the figures to the left may be considered so many *tens;* therefore, each man's share will be $247\frac{8}{10}$ dollars.

It is evident, also, that if 2 figures had been cut off from the right, all the remaining figures would have been so many *hundreds;* if 3 figures, so many *thousands*, &c. Hence we derive this general RULE *for dividing by* 10, 100, 1000, &c.: Cut off from the right of the dividend so many figures as there are ciphers in the divisor; the figures to the *left* of the point will express the *quotient*, and those to the *right*, the *remainder*.

2. In one dollar are 100 cents; how many dollars in 42400 cents? *Ans.* 424 dollars.

424|00 Here the divisor is 100; we therefore cut off 2 figures on the right hand, and all the figures to the *left* (424) express the dollars.

3. How many dollars in 34567 cents? *Ans.* $345\frac{67}{100}$ dollars.

4. How many dollars in 4567840 cents?

5. How many dollars in 345600 cents?

6. How many dollars in 42604 cents? *Ans.* $426\frac{4}{100}$.

7. 1000 mills make *one* dollar; how many dollars in 4000 mills? —— in 25000 mills? —— in 845000?

8. How many dollars in 6487 mills? *Ans.* $6\frac{487}{1000}$ dollars.

9. How many dollars in 42863 mills? —— in 368456 mills? —— in 96842378 mills?

10. In one cent are 10 mills; how many cents in 40 mills? —— in 400 mills? —— in 20 mills? —— in 468 mills? —— in 4784 mills? —— in 34640 mills?

III. *When there are* CIPHERS *on the right hand of the divisor.*

¶ 22. 1. Divide 480 dollars among 40 men?

OPERATION.

4|0)48|0

12 *dolls. Ans.*

In this example, our divisor, (40,) is a composite number, ($10 \times 4 = 40$;) we may, therefore, divide by *one* component part, (10,) and that quotient by the *other*, (4;) but to divide by 10, we have seen, is but to cut off the right hand figure, leaving the figures to the left

of the point for the quotient, which we divide by 4, and the work is done. It is evident, that, if our divisor had been 400, we should have cut off 2 figures, and have divided in the same manner; if 4000, 3 figures, &c. Hence this general RULE :— *When there are ciphers at the right hand of the divisor*, cut them off, and also as many places in the dividend; divide the remaining figures in the dividend by the remaining figures in the divisor; then annex the figures, cut off from the dividend, to the remainder.

2. Divide 748346 by 8000.

Dividend.
Divisor, 8|000)748|346.

Quotient, 93.—4346 *Remainder*. *Ans.* $93\frac{4346}{8000}$.

3. Divide 46720367 by 4200000.

Dividend.
42|00000)467|20367(11$\frac{520367}{4200000}$ *Quotient.*
42
47
42
520367 *Remainder.*

4. How many yards of cloth can be bought for 346800 dollars, at 20 dollars per yard?

5. Divide 76428400 by 900000.

6. Divide 345006000 by 84000.

7. Divide 4680000 by 20, 200, 2000, 20000, 300, 4000, 50, 600, 70000, and 80.

SUPPLEMENT TO DIVISION.

QUESTIONS.

1. What is division? 2. In what does the *process* of division consist? 3. Division is the *reverse* of what? 4. What is the *number to be divided* called, and to what does it answer in multiplication? 5. What is the number to *divide by* called, and to what does it answer, &c.? 6. What is the *result* or *answer* called, &c.? 7. What is the *sign* of division, and what does it show? 8. What is the *other way* of expressing division? 9. What is *short division*, and how is

it performed? 10. How is division *proved*? 11. How is *multiplication* proved? 12. What are *integers*, or whole numbers? 13. What are *fractions*, or broken numbers? 14. What is a *mixed* number? 15. When there is any thing *left* after division, what is it called, and how is it to be written? 16. How are fractions *written*? 17. What is the upper number called? 18. —— the lower number? 19. How do you multiply a fraction? 20. To what do the numerator and the denominator of a fraction answer in division? 21. What is *long* division? 22. Rule? 23. When the divisor is a composite number, how may we proceed? 24. When the divisor is 10, 100, 1000, &c., how may the operation be contracted? 25. When there are ciphers at the right hand of the divisor, how may we proceed?

EXERCISES.

1. An army of 1500 men, having plundered a city, took 2625000 dollars; what was each man's share?

2. A certain number of men were concerned in the payment of 18950 dollars, and each man paid 25 dollars; what was the number of men?

3. If 7412 eggs be packed in 34 baskets, how many in a basket?

4. What number must I multiply by 135 that the product may be 505710?

5. Light moves with such amazing rapidity, as to pass from the sun to the earth in about the space of 8 minutes. Admitting the distance, as usually computed, to be 95,000,000 miles, at what rate per minute does it travel?

6. If the product of two numbers be 704, and the multiplier be 11, what is the multiplicand? *Ans.* 64.

7. If the product be 704, and the multiplicand 64, what is the multiplier? *Ans.* 11.

8. The divisor is 18, and the dividend 144; what is the quotient?

9. The quotient of two numbers is 8, and the dividend 144; what is the divisor?

10. A man wishes to travel 585 miles in 13 days; how far must he travel each day?

11. If a man travels 45 miles a day, in how many days will he travel 585 miles?

12. A man sold 35 cows for 560 dollars; how much was that for each cow?

13. A man, selling his cows for 16 dollars each, received for all 560 dollars; how many did he sell?

14. If 12 inches make a foot, how many feet are there in 364812 inches?

15. If 364812 inches are 30401 feet, how many inches make one foot?

16. If you would divide 48750 dollars among 50 men, how many dollars would you give to each one?

17. If you distribute 48750 dollars among a number of men, in such a manner as to give to each one 975 dollars, how many men receive a share?

18. A man has 17484 pounds of tea in 186 chests; how many pounds in each chest?

19. A man would put up 17484 pounds of tea into chests containing 94 pounds each; how many chests must he have?

20. In a certain town there are 1740 inhabitants, and 12 persons in each house; how many houses are there?—— in each house are 2 families; how many persons in each family?

21. If 2760 men can dig a certain canal in one day, how many days would it take 46 men to do the same? How many men would it take to do the work in 15 days? —— in 5 days? —— in 20 days? —— in 40 days? —— in 120 days?

22. If a carriage wheel turns round 32870 times in running from New York to Philadelphia, a distance of 95 miles, how many times does it turn in running 1 mile? *Ans.* 346.

23. Sixty seconds make one minute; how many minutes in 3600 seconds? —— in 86400 seconds? —— in 604800 seconds? —— in 2419200 seconds?

24. Sixty minutes make one hour; how many hours in 1440 minutes? —— in 10080 minutes? —— in 40320 minutes? —— in 525960 minutes?

25. Twenty-four hours make a day; how many days in 168 hours? —— in 672 hours? —— in 8766 hours?

26. How many times can I subtract forty-eight from four hundred and eighty?

27. How many times 3478 is equal to 47854?

28. A bushel of grain is 32 quarts; how many quarts must I dip out of a chest of grain to make one half ($\frac{1}{2}$) of a bushel? —— for one fourth ($\frac{1}{4}$) of a bushel? —— for one eighth ($\frac{1}{8}$) of a bushel? *Ans. to the last,* 4 quarts.

29. How many is ½ of 20? —— ½ of 48? —— ½ of 247? —— ½ of 345678? —— ½ of 204030648?
Ans. to the last, 102015324.

30. How many walnuts are one third part (⅓) of 3 walnuts? —— ⅓ of 6 walnuts? —— ⅓ of 12? —— ⅓ of 30? —— ⅓ of 45? —— ⅓ of 300? —— ⅓ of 478? —— ⅓ of 3456320?
Ans. to the last, 1152106⅔.

31. What is ¼ of 4? —— ¼ of 20? —— ¼ of 320? —— ¼ of 7843?
Ans. to the last, 1960¾.

MISCELLANEOUS QUESTIONS,

Involving the Principles of the preceding Rules.

Note. The preceding rules, viz. Numeration, Addition, Subtraction, Multiplication, and Division, are called the *Fundamental Rules of Arithmetic*, because they are the foundation of all other rules.

1. A man bought a chaise for 218 dollars, and a horse for 142 dollars; what did they both cost?

2. If a horse and chaise cost 360 dollars, and the chaise cost 218 dollars, what is the cost of the horse? If the horse cost 142 dollars, what is the cost of the chaise?

3. If the sum of 2 numbers be 487, and the greater number be 348, what is the less number? If the less number be 139, what is the greater number?

4. If the minuend be 7842, and the subtrahend 3481, what is the remainder? If the remainder be 4361, and the minuend be 7842, what is the subtrahend?

¶ **23.** When the minuend and the subtrahend are given, how do you find the remainder?

When the minuend and remainder are given, how do you find the subtrahend?

When the subtrahend and the remainder are given, how do you find the minuend?

When you have the *sum* of two numbers, and *one* of them given, how do you find the other?

When you have the *greater* of two numbers, and their *difference* given, how do you find the *less* number?

When you have the *less* of two numbers, and their *difference* given, how do you find the *greater* number?

5. The *sum* of two numbers is 48, and *one* of the numbers is 19; what is the *other?*

6. The *greater* of two numbers is 29, and their *difference* 10; what is the *less* number?

7. The *less* of two numbers is 19, and their *difference* is 10; what is the *greater?*

8. A man bought 5 pieces of cloth, at 44 dollars a piece; 974 pairs of shoes, at 3 dollars a pair; 600 pieces of calico, at 6 dollars a piece; what is the amount?

9. A man sold six cows, worth fifteen dollars each, and a yoke of oxen, for 67 dollars; in pay, he received a chaise, worth 124 dollars, and the rest in money; how much money did he receive?

10. What will be the cost of 15 pounds of butter, at 13 cents per pound?

11. How many bushels of wheat can you buy for 487 dollars, at 2 dollars per bushel?

¶ **24.** When the price of *one* pound, *one* bushel, &c. of any commodity is given, how do you find the cost of *any number* of pounds, or bushels, &c. of that commodity? If the price of the 1 pound, &c. be in cents, in what will the whole cost be? If in dollars, what? —— if in shillings? —— if in pence? &c.

When the cost of *any given number* of pounds, or bushels, &c. is given, how do you find the price of *one* pound or bushel, &c. In what kind of money will the answer be?

When the *cost of a number* of pounds, &c. is given, and also the *price of one* pound, &c., how do you find the number of pounds, &c.

12. When rye is 84 cents per bushel, what will be the cost of 948 bushels? how many dollars will it be?

13. If 648 pounds of tea cost 284 dollars, (that is, 28400 cents,) what is the price of one pound?

When the factors are given, how do you find the product?

When the product and one factor are given, how do you find the other factor?

When the divisor and quotient are given, how do you find the dividend?

When the dividend and quotient are given, how do you find the divisor?

14. What is the product of 754 and 25?

15. What number, multiplied by 25, will produce 18850?

16. What number, multiplied by 754, will produce 18850?

17. If a man save six cents a day, how many cents would he save in a year, (365 days,)? —— how many in 45 years? how many dollars would it be? how many cows could he buy with the money, at 12 dollars each?

Ans. to the last, 82 cows, and 1 dollar 50 cents remainder.

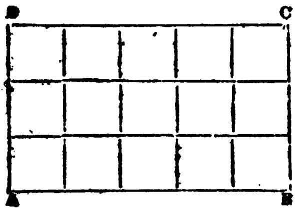
18. A boy bought a number of apples; he gave away ten of them to his companions, and afterwards bought thirty-four more, and divided one half of what he then had among four companions, who received 8 apples each; how many apples did the boy first buy?

Let the pupil take the last number of apples, 8, and reverse the process. *Ans.* 40 apples.

19. There is a certain number, to which, if 4 be added, and 7 be subtracted, and the difference be multiplied by 8, and the product divided by 3, the quotient will be 64; what is that number? *Ans.* 27.

20. A chess board has 8 rows of 8 squares each; how many squares on the board?

¶ 25. 21. There is a spot of ground 5 rods long, and 3 rods wide; how many square rods does it contain?



Note. A square rod is a square (like one of those in the annexed figure) measuring a rod on each side. By an inspection of the figure, it will be seen, that there are as many squares in a row as rods on *one* side, and that the number of rows is equal to the number of *rods* on the *other* side; therefore, $5 \times 3 = 15$, the number of squares.

Ans. 15 square rods.

A figure like A, B, C, D, having its opposite sides equal and parallel, is called a *parallelogram* or *oblong*.

22. There is an oblong field, 40 rods long, and 24 rods wide; how many square rods does it contain?

23. How many square inches in a board 12 inches long, and 12 inches broad? *Ans.* 144.

24. How many square feet in a board 14 feet long and 2 feet wide?

25. A certain township is six miles square; how many square miles does it contain? *Ans.* 36.

26. A man bought a farm for 22464 dollars; he sold one half of it for 12480 dollars, at the rate of 20 dollars per acre; how many acres did he buy? and what did it cost him per acre?

27. A boy bought a sled for 86 cents, and sold it again for 8 quarts of walnuts; he sold one half of the nuts at 12 cents a quart, and gave the rest for a penknife, which he sold for 34 cents; how many cents did he lose by his bargains?

28. In a certain school-house, there are 5 rows of desks; in each row are six seats, and each seat will accommodate 2 pupils; there are also 2 rows, of 3 seats each, of the same size as the others, and one long seat where 8 pupils may sit; how many scholars will this house accommodate? *Ans.* 80.

29. How many square feet of boards will it take for the floor of a room 16 feet long, and 15 feet wide, if we allow 12 square feet for waste?

30. There is a room 6 yards long and 5 yards wide; how many yards of carpeting, a yard wide, will be sufficient to cover the floors, if the hearth and fireplace occupy 3 square yards?

31. A board, 14 feet long, contains 28 square feet; what is its breadth?

32. How many pounds of pork, worth 6 cents a pound, can be bought for 144 cents?

33. How many pounds of butter, at 15 cents per pound, must be paid for 25 pounds of tea, at 42 cents per pound?

34. $4 + 5 + 6 + 1 + 8 =$ how many?

35. $4 + 3 + 10 - 2 - 4 + 6 - 7 =$ how many?

36. A man divides 30 bushels of potatoes among 3 poor men; how many bushels does each man receive? What is $\frac{1}{3}$ of thirty? How many are $\frac{2}{3}$ (*two* thirds) of 30?

37. How many are *one* third ($\frac{1}{3}$) of 3? —— of 6? —— of 9? —— of 282? —— of 45674312?

38. How many are *two* thirds ($\frac{2}{3}$) of 3? —— of 6? —— of 9? —— of 282? —— of 45674312?

39. How many are $\frac{1}{4}$ of 40? —— $\frac{3}{4}$ of 40? —— $\frac{1}{4}$ of 60? —— $\frac{3}{4}$ of 60? —— $\frac{1}{4}$ of 80? —— of 124? —— of 246876? —— $\frac{3}{4}$ of 246876?

40. How many is $\frac{1}{5}$ of 80? —— $\frac{4}{5}$ of 80? —— $\frac{3}{5}$ of 100?

41. An inch is one twelfth part ($\frac{1}{12}$) of a foot; how many

feet in 12 inches? —— in 24 inches? —— in 36 inches? —— in 12243648 inches?

42. If 4 pounds of tea cost 128 cents, what does 1 pound cost? —— 2 pounds? —— 3 pounds? —— 5 pounds? —— 100 pounds?

43. When oranges are worth 4 cents apiece, how many can be bought for four pistareens, (or 20 cent pieces?)

44. The earth, in moving round the sun, travels at the rate of 68000 miles an hour; how many miles does it travel in one day, (24 hours?) how many miles in one year, (365 days?) and how many days would it take a man to travel this last distance, at the rate of 40 miles a day? how many years? *Ans. to the last*, 40800 years.

45. How much can a man earn in 20 weeks, at 80 cents per day, Sundays excepted?

46. A man married at the age of 23; he lived with his wife 14 years; she then died, leaving him a daughter, 12 years of age; 8 years after, the daughter was married to a man 5 years older than herself, who was 40 years of age when the father died; how old was the father at his death? *Ans.* 60 years

47. There is a field 20 rods *long*, and 8 rods *wide*; how many square rods does it contain? *Ans.* 160 rods.

48. What is the width of a field, which is 20 rods long, and contains 160 square rods?

49. What is the length of a field, 8 rods wide, and containing 160 square rods?

50. What is the width of a piece of land, 25 rods long, and containing 400 square rods?

COMPOUND NUMBERS.

¶ **26.** A number expressing things of the same kind is called a *simple number;* thus, 100 men, 56 years, 75 cents, are each of them simple numbers; but when a number expresses things of different kinds, it is called a *compound number;* thus, 43 dollars 25 cents and 3 mills, is a compound number; so 4 years 6 months and 3 days, 46 pounds 7 shillings and 6 pence, are compound numbers.

Note. Different kinds, or names, are usually called *different denominations.*

FEDERAL MONEY.

Federal money is the coin of the United States. The kinds, or denominations, are eagles, dollars, dimes, cents, and mills.

10 mills - - - are equal to - 1 cent.
10 cents, (= 100 mills,) - - - = 1 dime.
10 dimes, (= 100 cents = 1000 mills,) - = 1 dollar.
10 dollars, (= 100 dimes = 1000 cents = 10000 mills) = 1 eagle.*

SIGN. This character, $, placed before a number, shows it to express *federal money.*

As 10 mills make a cent, 10 cents a dime, 10 dimes a dollar, &c. it is plain, that the relative value of mills, cents, dimes, dollars and eagles corresponds to the orders of units, tens, hundreds, &c. in simple numbers. Hence, they may be read either in the *lowest* denomination, or *partly* in a *higher*, and partly in the *lowest* denomination. Thus:

eagles.	dollars.	dimes.	cents.	mills.
3	4	6	5	2

may be read, 34652 mills; or 3465 cents and 2 mills; or, reckoning the eagles *tens* of dollars, and the dimes *tens* of cents, which is the usual practice, the whole may be read, 34 dollars 65 cents and 2 mills.

For ease in calculating, a point (‘) called a *separatrix*,† is placed between the dollars and cents, showing that all the figures at the *left* hand express dollars, while the *two first figures* at the *right* hand express cents, and the *third*, mills. Thus, the above example is written $ 34‘652; that is, 34 dollars 65 cents 2 mills, as above. As 100 cents make a dollar, the *cents* may be any number from 1 to 99, often requiring *two* figures to express them; for this reason, *two* places are appropriated to cents, at the right hand of the point, and if the number of cents be less than *ten*, requiring but *one* figure to express them, the *ten's* place must be filled with a cipher. Thus, 2 dollars and 6 cents are written 2‘06. 10 mills make a cent, and consequently the *mills* never exceed 9, and are always expressed by a *single* figure. Only

* The eagle is a *gold* coin, the dollar and dime are *silver* coins, the cent is a *copper* coin. The mill is only *imaginary*, there being no coin of that denomination. There are half eagles, half dollars, half dimes, and half cents, *real* coins.

† The character used for the *separatrix*, in the "Scholars' Arithmetic," was the comma; the comma *inverted* is here adopted, to distinguish it from the comma used in punctuation.

one place, therefore, is appropriated to mills, that is, the place immediately following cents, or the *third* place from the point. When there are no cents to be written, it is evident that we must write *two* ciphers to fill up the places of cents. Thus, 2 dollars and 7 mills are written 2'007. Six cents are written '06, and seven mills are written '007.

Note. Sometimes 5 mills $=\frac{1}{2}$ a cent is expressed fractionally: thus, '125 (twelve cents and five mills) is expressed $12\frac{1}{2}$, (twelve and a half cents.)

17 dollars and 8 mills are written, 17'008
4 dollars and 5 cents, - - - - - 4'05
75 cents, - - - - - - - - - '75
24 dollars, - - - - - - - - 24'
9 cents, - - - - - - - - - '09
4 mills, - - - - - - - - - '004
6 dollars 1 cent and 3 mills, - - - 6'013

Write down 470 dollars 2 cents; 342 dollars 40 cents and 2 mills; 100 dollars, 1 cent and 4 mills; 1 mill; 2 mills; 3 mills; 4 mills; $\frac{1}{2}$ cent, or 5 mills; 1 cent and 1 mill; 2 cents and 3 mills; six cents and one mill; sixty cents and one mill; four dollars and one cent; three cents; five cents; nine cents.

REDUCTION OF FEDERAL MONEY.

¶ **27.** How many mills in one cent? —— in 2 cents? in 3 cents? —— in 4 cents? —— in 6 cents? —— in 9 cents? —— in 10 cents? —— in 30 cents? —— in 78 cents? —— in 100 cents, (= 1 dollar)? —— in 2 dollars? —— in 3 dollars? —— in 4 dollars? —— in 484 cents? —— in 563 cents? —— in 1 cent and 2 mills? —— in 4 cents and 5 mills?

How many cents in 2 dollars? —— in 4 dollars? —— in 8 dollars? —— in 3 dollars and 15 cents? —— in 5 dollars and 20 cents? —— in 4 dollars and 6 cents?

How many dollars in 400 cents? —— in 600 cents? —— in 380 cents? —— in 40765 cents? How many cents in 1000 mills? How many dollars in 1000 mills? —— in 3000 mills? —— in 8000 mills? —— in 4378 mills? —— in 846732 mills?

This changing one kind of money, &c. into another kind, without altering the value, is called REDUCTION.

As there are 10 mills in one cent, it is plain that cents are changed or reduced to mills by multiplying them by 10, tha is, by merely annexing a cipher, (¶ 12.) 100 cents make a dollar; therefore dollars are changed to cents by annexing 2 ciphers, and to mills by annexing 3 ciphers. Thus, 16 dollars = 1600 cents = 16000 mills. Again, to change mills back to dollars, we have only to cut off the *three right hand figures*, (¶ 21;) and to change cents to dollars, cut off the *two right hand figures*, when all the figures to the *left* will be dollars, and the figures to the *right*, cents and mills.

Reduce 34 dollars to cents. *Ans.* 3400 cents.
Reduce 240 dollars and 14 cents to cents.
Ans. 24014 cents.
Reduce $748'143 to mills. *Ans.* 748143 mills.
Reduce 748143 mills to dollars. *Ans.* $748'143.
Reduce 3467489 mills to dollars. *Ans.* 3467'489.
Reduce 48742 cents to dollars. *Ans.* $487'42.
Reduce 1234678 mills to dollars.
Reduce 3469876 cents to dollars.
Reduce $4867'467 to mills.
Reduce 984 mills to dollars. *Ans.* $'984
Reduce 7 mills to dollars. *Ans.* $'007
Reduce $'014 to mills.
Reduce 17846 cents to dollars.
Reduce 984321 cents to mills.
Reduce 9617½ cents to dollars. *Ans.* $96'17½.
Reduce 2064½ cents, 503 cents, 106 cents, 921½ cents, 500 cents, 726½ cents, to dollars.
Reduce 86753 mills, 96000 mills, 6042 mills, to dollars.

ADDITION AND SUBTRACTION OF FEDERAL MONEY.

¶ **28.** From what has been said, it is plain, that we may readily reduce any sums in federal money to the same denomination, as to cents, or mills, and add or subtract them as simple numbers. Or, what is the same thing, we may *set down the sums, taking care to write dollars under dollars, cents under cents, and mills under mills, in such order, that the separating points of the several numbers shall fall directly under each other, and add them up as simple numbers, placing the separatrix in the amount directly under the other points.*

What is the amount of $ 487'643, $ 132'007, $ 4'04, and $ 264'102? *Ans.* $ 887'792.

OPERATION.		OPERATION.
487643 mills.	or,	$ 487'643
132007 mills.		$ 132'007
4040 mills.		$ 4'04
264102 mills.		$ 264'102
Amount, 887792 mills, = $ 887'792.		$ 887'792 *Amount.*

EXAMPLES FOR PRACTICE.

1. Bought 1 barrel of flour for 6 dollars 75 cents, 10 pounds of coffee for 2 dollars 30 cents, 7 pounds of sugar for 92 cents, 1 pound of raisins for 12½ cents, and 2 oranges for 6 cents; what was the whole amount? *Ans.* $ 10'155.

2. A man is indebted to A, $ 237'62; to B, $ 350; to C, $ 86'12½; to D, $ 9'62½; and to E, $ 0'834; what is the amount of his debts? *Ans.* $ 684'204.

3. A man has three notes specifying the following sums, viz. three hundred dollars, fifty dollars sixty cents, and nine dollars eight cents; what is the amount of the three notes? *Ans.* $ 359'68.

4. What is the amount of $ 56'18, $ 7'37½, $ 280, $ 9'287, $ 17, and $ 90'413?

5. Bought a pair of oxen for $ 76'50, a horse for $ 85, and a cow for $ 17'25; what was the whole amount?

6. Bought a gallon of molasses for 28 cents, a quarter of tea for 37½ cents, a pound of salt petre for 24 cents, 2 yards of broadcloth for 11 dollars, 7 yards of flannel for 1 dollar 62½ cents, a skein of silk for 6 cents, and a stick of twist for 4 cents; how much for the whole?

SUBTRACTION OF FEDERAL MONEY.

7. A man gave 4 dollars 75 cents for a pair of boots, and 2 dollars 12½ cents for a pair of shoes; how much did the boots cost him more than the shoes?

OPERATION.		OPERATION.
4750 mills.	or,	$ 4'75
2125 mills.		$ 2'125
2625 mills = $ 2'625 *Ans.*		$ 2'625 *Ans.*

8. A man bought a cow for eighteen dollars, and sold her again for twenty-one dollars thirty-seven and a half cents; how much did he gain? *Ans.* $ 3‘375.

9. A man bought a horse for 82 dollars, and sold him again for seventy-nine dollars seventy-five cents; did he gain or lose? and how much? *Ans.* He lost $ 2‘25.

10. A merchant bought a piece of cloth for $ 176, which proving to have been damaged, he is willing to lose on it $ 16‘50; what must he have for it? *Ans.* $ 159‘50.

11. A man sold a farm for $ 5400, which was $ 725‘37½ more than he gave for it; what did he give for the farm?

12. A man, having $ 500, lost 83 cents; how much had he left?

13. A man's income is $ 1200 a year, and he spends $ 800‘35; how much does he lay up?

14. Subtract half a cent from seven dollars.

15. How much must you add to $ 16‘82 to make $ 25?

16. How much must you subtract from $ 250, to leave $ 87‘14?

17. A man bought a barrel of flour for $ 6‘25, 7 pounds of coffee for $ 1‘41; he paid a ten dollar bill; how much must he receive back in change?

MULTIPLICATION OF FEDERAL MONEY.

¶ 29. 1. What will 3 yards of cloth cost, at $ 4‘62½ a yard?

OPERATION.

$ 4‘625
3
———
$ 13‘875, *the answer.*

$ 4‘625 are 4625 mills, which multiplied by 3, the product is 13875 mills. 13875 mills may now be reduced to dollars by placing a point between the third and fourth figures, that is, between the hundreds and thousands, which is pointing off as many places for cents and mills, in the product, as there are places of cents and mills in the sum given to be multiplied. This is evident; for, as 1000 mills make 1 dollar, consequently the thousands in 13875 mills must be so many dollars.

2. At 16 cents a pound, what will 123 pounds of butter cost?

OPERATION.

123, *the number of pounds.*
16 *cents, the price per pound.*

738
123

$ 19'68, *the answer.*

As the product of any two numbers will be the same, whichever of them be made the multiplier, therefore the quantity, being the larger number, is made the multiplicand, and the price the multiplier.

123 times 16 cents is 1968 cents, which, reduced to dollars, is $ 19'68.

RULE.

From the foregoing examples it appears, that the multiplication of federal money does not differ from the multiplication of simple numbers. *The product will be the answer in the lowest denomination contained in the given sum*, which may then be reduced to dollars.

EXAMPLES FOR PRACTICE.

3. What will 250 bushels of rye come to, at $ 0'88½ per bushel? *Ans.* $ 221'25.
4. What is the value of 87 barrels of flour, at $ 6'37½ a barrel?
5. What will be the cost of a hogshead of molasses, containing 63 gallons, at 28½ cents a gallon? *Ans.* $ 17'955.
6. If a man spend 12½ cents a day, what will that amount to in a year of 365 days? what will it amount to in 5 years? *Ans.* It will amount to $ 228'12½ in 5 years.
7. If it cost $ 36'75 to clothe a soldier 1 year, how much will it cost to clothe an army of 17800 men? *Ans.* $ 654150.
8. Multiply $ 367 by 46.
9. Multiply $ 0'273 by 8600.
10. What will be the cost of 4848 yards of calico, at 25 cents, or one quarter of a dollar, per yard? *Ans.* $ 1212.

Note. As 25 cents is just ¼ of a dollar, the operation in the above example may be contracted, or made shorter; for, at *one dollar* per yard, the cost would be as many *dollars* as there are yards, that is, $ 4848; and at one quarter (¼) of a dollar per yard, it is plain, the cost would be *one quarter* (¼) as many dollars as there are yards, that is, $\frac{4848}{4} =$ $ 1212.

When one quantity is contained in another exactly 2, 3, 4, 5, &c. times, it is called an *aliquot* or *even* part of that quantity; thus, 25 cents is an *aliquot part* of a *dollar*, because 4 times 25 cents is just equal to 1 dollar; and 6 pence is an aliquot part of a shilling, because 2 times six pence just make 1 shilling. The following table exhibits some of the aliquot parts of a *dollar:*

TABLE.

Cts.		
50	$= \frac{1}{2}$	*of a dollar.*
$33\frac{1}{3}$	$= \frac{1}{3}$	*of a dollar.*
25	$= \frac{1}{4}$	*of a dollar.*
20	$= \frac{1}{5}$	*of a dollar.*
$12\frac{1}{2}$	$= \frac{1}{8}$	*of a dollar.*
$6\frac{1}{4}$	$= \frac{1}{16}$	*of a dollar.*
5	$= \frac{1}{20}$	*of a dollar.*

From the illustration of the last example, it appears, that, when the price per *yard*, *pound*, &c. is one of these aliquot parts of a dollar, the cost may be found, by dividing the *given number* of *yards*, *pounds*, &c. by that number which it takes of the price to make 1 dollar. If the price be 50 cents, we divide by 2; if 25 cts. by 4; if $12\frac{1}{2}$ cts. by 8, &c.

This manner of calculating the cost of articles, by taking aliquot parts, is usually called *Practice.*

11. What is the value of 14756 yards of cotton cloth, at $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of a dollar, per yard?

By practice.

```
8)14756
  -----
Ans. $ 1844'50
```

By multiplication.

```
    14756
     '125
    -----
    73780
   29512
  14756
  -------
$ 1844'500  Ans. as before.
```

12. What is the cost of 18745 pounds of tea, at \$ '50, $= \frac{1}{2}$ dollar, per pound? *Ans.* \$ 9372'50

13. What is the value of 9366 bushels of potatoes, at $33\frac{1}{3}$ cents, or $\frac{1}{3}$ of a dollar, per bushel? $\frac{9366}{3} =$ \$ 3122 *Ans.*

14. What is the value of 48240 pounds of cheese, at \$ '06$\frac{1}{4}$, $= \frac{1}{16}$ of a dollar, per pound? *Ans.* \$ 3015.

15. What cost 4870 oranges, at 5 cents, $= \frac{1}{20}$ of a dollar, apiece? *Ans.* \$ 243'50

16. What is the value of 151020 bushels of apples, at 20 cents, $= \frac{1}{5}$ of a dollar, per bushel? *Ans.* \$ 30204.

17. What will 264 pounds of butter cost, at $12\frac{1}{2}$ cents per pound? *Ans.* \$ 33.

18. What cost 3740 yards of cloth, at \$ 1'25 per yard?

4) $ 3740 = cost at $ 1' per yard.
935 = cost at $ '25 per yard.

Ans. $ 4675 = cost at $ 1'25 per yard.

19. What is the cost of 8460 hats, at $ 1'12½ apiece? —— at $ 1'50 apiece? —— at $ 3'20 apiece? —— at $ 4'06¼ apiece?

Ans. $ 9517'50. $ 12690. $ 27072. $ 34368'75.

¶ 30. *To find the value of articles sold by the* 100, or 1000.

1. What is the value of 865 feet of timber, at $ 5 per hundred?

OPERATION.
865
5
$ 4325 = *value at* $ 5 per *foot.*

Were the price $ 5 per *foot*, it is plain, the value would be 865 × $ 5 = $ 4325; but the price is $ 5 for 100 feet; consequently, $ 4325 is 100 times the true value of the timber; and therefore, if we divide this number ($ 4325) by 100, we shall obtain the true value; but to divide by 100 is but to cut off the two right hand figures, or, in federal money, *to remove the separatrix two figures to the left.* *Ans.* $ 43'25.

It is evident, that, were the price so much per *thousand*, the same remarks would apply, with the exception of cutting off *three* figures instead of two. Hence we derive the general RULE for *finding the value of articles sold by the* 100, or 1000:—Multiply the number by the price, and, if it be reckoned by the 100, cut off the *two* right hand figures, and the product will be the answer, in the same kind or denomination as the price. If the article be reckoned by the 1000, cut off the *three* right hand figures.

EXAMPLES FOR PRACTICE.

2. What is the value of 4250 feet of boards, at $ 14 per 1000? *Ans.* 59 dollars and 50 cents.

OPERATION.
4250
$ 14
17000
4250
$ 59'500

In this example, because the price is at so much per 1000 feet, we divide by 1000, or cut off three figures.

3. What will 3460 feet of timber come to, at $ 4 per hundred?

4. What will 24650 bricks come to, at 5 dollars per 1000?

5. What will 4750 feet of boards come to, at $ 12'25 per 1000? *Ans.* 58'187.

6. What will 38600 bricks cost, at $ 4'75 per 1000?

7. What will 46590 feet of boards cost, at $ 10'625 per 1000?

8. What will 75 feet of timber cost, at $ 4 per 100?

9. What is the value of 4000 bricks, at 3 dollars per 1000?

DIVISION OF FEDERAL MONEY.

¶ **31.** 1. If 3 yards of cloth cost $ 5'25, what is that a yard?

OPERATION.
3)5'25
Answer, 175 *cents*, = $ 1'75.

$ 5'25 is 525 cents, which divided by 3, the quotient is 175 cents, which, reduced to dollars, is $ 1'75, the answer.

2. Bought 4 bushels of corn for $ 3; what was that a bushel?

4 is not contained in 3; we may, however, reduce the $ 3 to cents, by annexing two ciphers, thus:

OPERATION.
4)3'00
Ans. '75 *cents.*

300 cents divided by 4, the quotient is 75 cents, the price of each bush. of corn.

3. Bought 18 gallons of brandy, for $ 42'75; what did it cost a gallon?

OPERATION.

```
18)42'75(2375 mills, = $ 2'375, the answer.
   36
   --
    67
    54
    ---
    135
    126
    ---
      90
      90
      --
      ..
```

$ 42'75 is 4275 cents. After bringing down the last figure in the dividend, and dividing, there is a remainder of 9 cents, which, by annexing a cipher, is reduced to mills, (90,) in which the divisor is contained 5 times, which is 5 mills, and there is no remainder. Or, we might have reduced $ 42'75 to mills, *before* dividing, by annexing a cipher, 42750 mills, which, divided by 18, would have given the same result, 2375 mills, which, reduced to dollars, is $ 2'375, the answer.

4. Divide $ 59'387 by 8.

OPERATION.
8)59'387

Quotient, 7'423$\frac{3}{8}$, that is, 7 dollars, 42 cents, 3 mills, and $\frac{3}{8}$ of another mill. The $\frac{3}{8}$ is the remainder, after the last division, written over the divisor, and expresses such fractional part of another mill. For all purposes of business, it will be sufficiently exact to carry the quotient only to mills, as the *parts* of a mill are of so little value as to be disregarded. Sometimes the sign of addition (+) is annexed, to show that there is a remainder, thus, $ 7'423 +.

RULE.

From the foregoing examples, it appears, that division of federal money does not differ from division of simple numbers. *The quotient will be the answer in the lowest denomination in the given sum,* which may then be reduced to dollars.

Note. If the sum to be divided contain only dollars, or dollars and cents, it may be reduced to mills, by annexing ciphers before dividing; or, we may first divide, annexing ciphers to the remainder, if there shall be any, till it shall be reduced to mills, and the result will be the same.

EXAMPLES FOR PRACTICE.

5. If I pay $ 468'75 for 750 pounds of wool, what is the value of 1 pound? *Ans.* $ 0'625; or thus, $ 0'62$\frac{1}{2}$.

6. If a piece of cloth, measuring 125 yards, cost $ 181'25, what is that a yard? *Ans.* $ 1'45.

7. If 536 quintals of fish cost $ 1913'52, how much is that a quintal? *Ans.* $ 3'57.

8. Bought a farm, containing 84 acres, for $ 3213; what did it cost me per acre? *Ans.* $ 38'25.

9. At $ 954 for 3816 yards of flannel, what is that a yard? *Ans.* $ 0'25.

10. Bought 72 pounds of raisins for $ 8; what was that a pound? $\frac{8}{72}$ = how much? *Ans.* $ 0'111$\frac{1}{9}$; or, $ 0'111+.

11. Divide $ 12 into 200 equal parts; how much is *one* of the parts? $\frac{12}{200}$ = how much? *Ans.* $ 0'006.

12. Divide $ 30 by 750. $\frac{30}{750}$ = how much?

13. Divide $ 60 by 1200. $\frac{60}{1200}$ = how much?

14. Divide $ 215 into 86 equal parts; how much will one of the parts be? $\frac{215}{86}$ = how much?

15. Divide $176 equally among 250 men; how much will each man receive? $\frac{176}{250}$ = how much?

SUPPLEMENT TO FEDERAL MONEY.

QUESTIONS.

1. What is understood by *simple* numbers? 2. —— by *compound* numbers? 3. —— by different *denominations*? 4. What is federal money? 5. What are the denominations used in federal money? 6. How are dollars distinguished from cents? 7. Why are two places assigned for cents, while only one place is assigned for mills? 8. To what does the relative value of mills, cents, and dollars correspond? 9. How are mills reduced to dollars? 10. ——to cents? 11. Why? 12. How are dollars reduced to cents? 13. —— to mills? 14. Why? 15. How is the addition of federal money performed? 16. —— subtraction? 17. —— multiplication? 18. —— division? 19. Of what name is the *product* in multiplication, and the *quotient* in division? 20. In case dollars *only* are given to be divided, what is to be done? 21. When is one number or quantity said to be an *aliquot part* of another? 22. What are some of the aliquot parts of a *dollar*? 23. When the *price* is an *aliquot part* of a dollar, how may the cost be found? 24. What is this manner of operating called? 25. How do you find the cost of articles, sold by the 100 or 1000?

EXERCISES.

1. Bought 23 firkins of butter, each containing 42 pounds, for 16½ cents a pound; what would that be a firkin, and how much for the whole? *Ans.* $159‘39 for the whole.

2. A man killed a beef, which he sold as follows, viz. the hind quarters, weighing 129 pounds each, for 5 cents a pound; the fore quarters, one weighing 123 pounds, and the other 125 pounds, for 4½ cents a pound; the hide and tallow, weighing 163 pounds, for 7 cents a pound; to what did the whole amount? *Ans.* $35‘47.

3. A farmer bought 25 pounds of clover seed at 11 cents a pound, 3 pecks of herds grass seed for $2‘25, a barrel of flour for $6‘50, 13 pounds of sugar at 12½ cents a pound; for which he paid 3 cheeses, each weighing 27 pounds, at 8¼ cents a pound, and 5 barrels of cider at $1‘25 a barrel. The balance between the articles bought and sold is 1 cent is it *for*, or *against* the farmer?

4. A man dies, leaving an estate of $71600; there are demands against the estate, amounting to $39876'74; the residue is to be divided between 7 sons; what will each one receive?

5. How much coffee, at 25 cents a pound, may be had for 100 bushels of rye, at 87 cents a bushel? *Ans.* 348 pounds.

6. At 12½ cents a pound, what must be paid for 3 boxes of sugar, each containing 126 pounds?

7. If 650 men receive $86'75 each, what will they all receive?

8. A merchant sold 275 pounds of iron, at 6¼ cents a pound, and took his pay in oats, at $0'50 a bushel; how many bushels did he receive?

9. How many yards of cloth, at $4'66 a yard, must be given for 18 barrels of flour, at $9'32 a barrel?

10. What is the price of three pieces of cloth, the first containing 16 yards, at $3'75 a yard; the second, 21 yards, at $4'50 a yard; and the third, 35 yards, at $5'12½ a yard?

¶ **32.** It is usual, when goods are sold, for the seller to deliver to the buyer, with the goods, a bill of the articles and their prices, with the amount cast up. Such bills are sometimes called *bills of parcels.*

Boston, January 6, 1827.

Mr. Abel Atlas

Bought of Benj. Burdett

12½ yards figured Satin,	at $2'50	a yard,	$31'25
8 .... sprigged Tabby, ...	1'25	....	10'00
Received payment,			$41'25

BENJ. BURDETT.

Salem, June 4, 1827.

Mr. James Paywell

Bought of Simeon Thrifty

3 hogsheads new Rum, 118 gal. each,	at $0'31	a gal.	
2 pipes French Brandy, 126 and 132 gal. ..	1'12½	.. ..	
1 hogshead brown Sugar, 9¾ cwt.	.. 10'34	.. cwt.	
3 casks of Rice, 269 lb. each,	.. '05	.. lb.	
5 bags Coffee, 75 lb. each,	.. '23	.. ....	
1 chest hyson Tea, 86 lb.	.. '92	.. ....	
Received payment,			$706'52¼

For Simeon Thrifty,

PETER FAITHFUL.

Wilderness, February 8, 1827.

Mr. Peter Carpenter
(See ¶ 30.)

Bought of Asa Falltree

5682 feet Boards,		at	$ 6	per M.
2000	....	..	8'34	....
800	Thick Stuff,	..	12'64	....
1500	Lathing,	..	4'	....
650	Plank,	..	10'	....
879	Timber,	..	2'50	..... C.
236	....	..	2'75	....

Received payment, $ 101'849

ASA FALLTREE.

Note. M. stands for the Latin *mille*, which signifies 1000, and C. for the Latin word *centum*, which signifies 100.

REDUCTION.

¶ **33.** We have seen, that, in the United States, money is reckoned in dollars, cents, and mills. In England, it is reckoned in pounds, shillings, pence, and farthings, called denominations of money. Time is reckoned in years, months, weeks, days, hours, minutes, and seconds, called denominations of time. Distance is reckoned in miles, rods, feet, and inches, called denominations of measure, &c.

The relative value of these denominations is exhibited in tables, which the pupil must commit to memory.

ENGLISH MONEY.

The denominations are pounds, shillings, pence, and farthings.

TABLE.

4 farthings (qrs.)	make	1 penny,	marked d.
12 pence	- - - -	1 shilling,	- - s.
20 shillings	- - - -	1 pound,	- - £.

Note. Farthings are often written as the fraction of a penny; thus, 1 farthing is written $\frac{1}{4}$ d., 2 farthings, $\frac{1}{2}$ d., 3 farthings, $\frac{3}{4}$ d.

How many farthings in 1 penny? —— in 2 pence? —— in 3 pence? —— in 6 pence? —— in 8 pence? —— in 9 pence? —— in 12 pence? —— in 1 shilling? —— in 2 shillings?

How many pence in 4 farthings? —— in 8 farthings? —— in 12 farthings? —— in 24 farthings? —— in 32 farthings? —— in 36 farthings? —— in 48 qrs.? How many shillings in 48 qrs.? —— in 96 qrs.?

How many pence in 2 shillings? —— in 3 s.? —— in 4 s.? —— in 6 s.? —— in 8 s.? —— in 10 s.? —— in 2 shillings and 2 pence? —— in 2 s. 3 d.? —— in 2 s. 4 d.? —— in 2 s. 6 d.? —— in 3 s. 6 d.? —— in 4 s. 3 d.?

How many shillings in 24 pence? —— in 36 d.? —— in 48 d.? —— in 72 d.? —— in 96 d.? —— in 120 d.? —— in 26 d.? —— in 27 d.? —— in 28 d.? —— in 30 d.? —— in 42 d.? —— in 51 d.?

How many shillings in 1 pound? —— in 2£.? —— in 3£.? —— in 4£.? —— in 4£. 6 s.? —— in 6£. 8 s.? —— in 3£. 10 s.? —— in 2£. 15 s.?

How many pounds in 20 shillings? —— in 40 s.? —— in 60 s.? —— in 80 s.? —— in 86 s.? —— in 128 s.? —— in 70 s.? —— in 55 s.?

It has already been remarked, that the changing of *one* kind, or denomination, into *another* kind, or denomination, without altering their value, is called *Reduction.* (¶ 27.) Thus, when we change shillings into pounds, or pounds into shillings, we are said to *reduce* them. From the foregoing examples, it is evident, that, when we reduce a denomination of *greater* value into a denomination of *less* value, the reduction is performed by *multiplication;* and it is then called *Reduction Descending.* But when we reduce a denomination of *less* value into one of *greater* value, the reduction is performed by *division;* it is then called *Reduction Ascending.* Thus, to reduce pounds to shillings, it is plain, we must *multiply* by 20. And again, to reduce shillings to pounds, we must *divide* by 20. It follows, therefore, that *reduction descending and ascending reciprocally prove each other.*

1. In 17£. 13 s. 6¾ d. how many farthings?

OPERATION.

£.	s.	d.	qrs
17	13	6	3

20 s.

353 s. in 17£. 13 s.
12 d.

4242 d.
4 q.

16971 *qrs. the Ans.*

In the above example, because 20 shillings make 1 pound, therefore we multiply 17£. by 20, increasing the product by the addition of the given shillings, (13,) which, it is evident, must always be done in like cases; then, because 12 pence make 1 shilling, we multiply the shillings (353) by 12, adding in the given pence, (6.) Lastly, because 4 farthings make 1 penny, we multiply the pence (4242) by 4, adding in the given farthings, (3.) We then find, that in 17£. 13 s. 6¾ d., are contained 16971 farthings.

2. In 16971 farthings, how many pounds?

OPERATION.

Farthings in a penny, 4)16971. 3 *qrs.*
Pence in a shilling, 12)4242 6 *d.*
Shillings in a pound, 2|0)35|3 13 *s.*
17£.

Ans. 17£. 13*s.* 6¾*d.*

Farthings will be reduced to pence, if we divide them by 4, because every 4 farthings make 1 penny. Therefore, 16971 farthings divided by 4, the quotient is 4242 pence, and a remainder of 3, which is farthings, of the same name as the dividend. We then divide the pence (4242) by 12, reducing them to shillings; and the shillings (353) by 20, reducing them to pounds. The last quotient, 17£., with the several remainders, 13 s. 6 d. 3 qrs. constitute the answer.

Note. In dividing 353 s. by 20, we cut off the cipher, &c., as taught ¶ 22.

¶ **34.** The process in the foregoing examples, if carefully examined, will be found to be as follows, viz.

To reduce high denominations to lower,—Multiply the highest denomination by that number which it takes of the next less to make 1 of this higher, (increasing the product by the number given, if any, of that less denomina-

To reduce low denominations to higher,—Divide the lowest denomination given by that number which it takes of the same to make 1 of the next higher. Proceed in the same manner with each succeeding denomination, until you have

tion.) Proceed in the same manner with each succeeding denomination, until you have brought it to the denomination required.

brought it to the denomination required.

EXAMPLES FOR PRACTICE.

3. Reduce 32£. 15 s. 8 d. to farthings.

5. In 29 guineas, at 28 s. each, how many farthings?

7. Reduce $163, at 6 s. each, to pence?

9. In 15 guineas, how many pounds?

4. Reduce 31472 farthings to pounds.

6. In 38976 farthings, how many guineas?

8. Reduce 11736 pence to dollars.

10. Reduce 21£. to guineas.

Note. We cannot reduce guineas *directly* to pounds, but we may reduce the guineas to *shillings*, and then the shillings to pounds.

TROY WEIGHT.

By Troy weight are weighed gold,* silver, jewels, and all liquors. The denominations are pounds, ounces, pennyweights, and grains.

TABLE.

24 grains (grs.)	make	1 pennyweight, marked	pwt.
20 pennyweights	- -	1 ounce, - - - - - -	oz.
12 ounces	- - - -	1 pound, - - - - - -	lb.

11. Bought a silver tankard, weighing 3 lb. 5 oz., paying at the rate of $1'08 an ounce; what did it cost?

13. Reduce 210 lb. 8 oz. 12 pwt. to pennyweights.

15. In 7 lb. 11 oz. 3 pwt. 9 grs. of silver, how many grains?

12. Paid $44'28 for a silver tankard, at the rate of $1'08 an ounce; what did it weigh?

14. In 50572 pwt. how many pounds?

16. Reduce 45681 grains to pounds.

* The fineness of gold is tried by fire, and is reckoned in *carats*, by which is understood the 24th part of any quantity; if it lose nothing in the trial, it is said to be 24 carats fine; if it lose 2 carats, it is then 22 carats fine, which is the standard for gold.

Silver which abides the fire without loss is said to be 12 ounces fine. The standard for silver coin is 11 oz. 2 pwts. of fine silver, and 18 pwts. of copper together

APOTHECARIES' WEIGHT.

Apothecaries' weight* is used by apothecaries and physicians, in compounding medicines. The denominations are pounds, ounces, drams, scruples, and grains.

TABLE.

20 grains, (grs.)	make	1 scruple,	marked ℈.
3 scruples	- - -	1 dram,	- - - ʒ.
8 drams	- - - -	1 ounce,	- - - ℥.
12 ounces	- - - -	1 pound,	- - - ℔.

17. In 9℔. 8℥. 1ʒ. 2℈. 19 grs., how many grains.

18. Reduce 55799 grs. to pounds.

AVOIRDUPOIS WEIGHT.†

By avoirdupois weight are weighed all things of a coarse and drossy nature, as tea, sugar, bread, flour, tallow, hay, leather, medicines, (in buying and selling,) and all kinds of metals, except gold and silver. The denominations are tons, hundreds, quarters, pounds, ounces, and drams.

TABLE.

16 drams, (drs.)	make	1 ounce,	- marked -	oz.
16 ounces	- - - - - -	1 pound,	- - - - - - -	lb.
28 pounds	- - - - - -	1 quarter,	- - - - - - -	qr.
4 quarters	- - - - - -	1 hundred weight,	- -	cwt.
20 hundred weight	- -	1 ton,	- - - - - - - -	T.

Note 1. In this kind of weight, the words *gross* and *net* are used. Gross is the weight of the goods, together with the box, bale, bag, cask, &c., which contains them. Net weight is the weight of the goods only, after deducting the weight of the box, bale, bag, or cask, &c., and all other allowances.

Note 2. A hundred weight, it will be perceived, is 112 lb. Merchants at the present time, in our principal sea-ports, buy and sell by the 100 pounds.

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* The pound and ounce apothecaries' weight, and the pound and ounce Troy, are the same, only differently *divided*, and *subdivided*.

† 175 oz. Troy = 192 oz. avoirdupois, and 175 lb. Troy = 144 lb. avoirdupois. 1 lb. Troy = 5760 grains, and 1 lb. avoirdupois = 7000 grains Troy.

19. What will 5 cwt. 3 qrs. 17 lb. of sugar come to, at $12\frac{1}{2}$ cents a pound.

20. How much sugar, at $12\frac{1}{2}$ cents a pound, may be bought for $ 82'625?

21. A merchant would put 109 cwt. 0 qrs. 12 lb. of raisins into boxes, containing 26 lb. each; how many boxes will it require?

22. In 470 boxes of raisins, containing 26 lb. each, how many cwt.?

23. In 12 tons, 15 cwt. 1 qr. 19 lb. 6 oz. 12 dr. how many drams?

24. In 7323500 drams, how many tons?

25. In 28 lb. avoirdupois, how many pounds Troy?

26. In 34 lb. 0 oz. 6 pwt. 16 grs. Troy, how many pounds avoirdupois?

CLOTH MEASURE.

Cloth measure is used in selling cloths and other goods, sold by the yard, or ell. The denominations are ells, yards, quarters, and nails.

TABLE.

4 nails, (na.) or 9 inches, make	1 quarter,	marked	qr.
4 quarters, or 36 inches, -	1 yard,	- - - -	yd.
3 quarters, - - - - - - -	1 ell Flemish,	- -	E. Fl.
5 quarters, - - - - - - -	1 ell English,	- -	E. E.
6 quarters, - - - - - - -	1 ell French,	- -	E. Fr.

27. In 573 yds. 1 qr. 1 na. how many nails?

28. In 9173 nails, how many yards?

29. In 151 ells Eng. how many yards?

Note. Consult ¶ 34, ex. 9.

30. In $188\frac{3}{4}$ yards, how many ells English?

LONG MEASURE.

Long measure is used in measuring distances, or other things, where *length* is considered without regard to *breadth*. The denominations are degrees, leagues, miles, furlongs, rods, yards, feet, inches, and barley-corns.

TABLE.

3	barley-corns, (bar.) make	1 inch,	marked	in.
12	inches, - - - - - -	1 foot,	- - - - - -	ft.
3	feet, - - - - - - -	1 yard,	- - - - - -	yd.
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet, -	1 rod, perch, or pole,	-	r. p.
40	rods, or 220 yards, - -	1 furlong,	- - - - -	fur.
8	furlongs, or 320 rods, -	1 mile,	- - - - - -	M.
3	miles, - - - - - -	1 league,	- - - - -	L.
60	geographical, or $69\frac{1}{2}$ statute miles, - -	1 degree,	- -	deg. or °
360	degrees, - - - - -	a great circle, or circumference of the earth.		

31. How many barley-corns will reach round the globe, it being 360 degrees?

Note. To multiply by 2, is to take the multiplicand 2 times; to multiply by 1, is to take the multiplicand 1 time; to multiply by $\frac{1}{2}$, is to take the multiplicand half a time, that is, the half of it. Therefore, to reduce 360 degrees to statute miles, we multiply first by the whole number, 69, and to the product add half the multiplicand. Thus:

```
½)360
   69½
 ─────
  3240
 2160
   180 half of the multiplicand.
 ─────
 25020 statute miles in 360 degrees.
```

32. In 4755801600 barley-corns, how many degrees?

Note. The barley-corns being divided by 3, and that quotient by 12, we have 132105600 feet, which are to be reduced to rods. We cannot easily divide by $16\frac{1}{2}$ on account of the fraction $\frac{1}{2}$; but $16\frac{1}{2}$ *feet* = 33 *half feet*, in 1 rod; and 132105600 *feet* = 264211200 *half feet*, which, divided by 33, gives 8006400 *rods*.

Hence, when the divisor is encumbered with a fraction, $\frac{1}{2}$ or $\frac{1}{4}$, &c., we may reduce the divisor to *halves*, or *fourths*, &c., and reduce the dividend to the same; then the quotient will be the true answer.

33. How many inches from Boston to the city of Washington, it being 482 miles?

34. In 30539520 inches, how many miles?

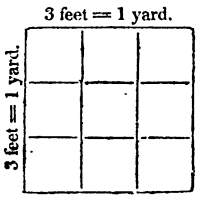
35. How many times will a wheel, 16 feet and 6 inches in circumference, turn round in the distance from Boston to Providence, it being 40 miles?

36. If a wheel, 16 feet 6 inches in circumference, turn round 12800 times in going from Boston to Providence, what is the distance?

LAND OR SQUARE MEASURE.

Square measure is used in measuring land, and any other thing, where *length* and *breadth* are considered. The denominations are miles, acres, roods, perches, yards, feet and inches.

¶ 35. 3 feet in length make a yard in long measure; but it requires 3 feet in length and 3 feet in *breadth* to make a yard in *square* measure; 3 feet in length and *one* foot wide make 3 square feet; 3 feet in length and 2 feet wide make 2 times 3, that is, 6 square feet; 3 feet in length and 3 feet wide make 3 times 3, that is, 9 square feet. This will clearly appear from the annexed figure.



It is plain, also, that a square foot, that is, a square 12 inches in length and 12 inches in breadth, must contain $12 \times 12 = 144$ square inches.

TABLE.

144	square inches $= 12 \times 12$; that is, 12 inches in length and 12 inches in breadth - - - - - - -	make	1 square foot.
9	square feet $= 3 \times 3$; that is, 3 feet in length and 3 feet in breadth	-	1 square yard.
$30\frac{1}{4}$	square yards $= 5\frac{1}{2} \times 5\frac{1}{2}$, or $272\frac{1}{4}$ square feet $= 16\frac{1}{2} \times 16\frac{1}{2}$, -	-	1 square rod, perch or pole.
40	square rods, - - - - - - - - -	-	1 rood.
4	roods, or 160 square rods,	- -	1 acre.
640	acres, - - - - - - - - - -	-	1 square mile.

Note. Gunter's chain, used in measuring land, is 4 rods in length. It consists of 100 links, each link being $7\frac{92}{100}$ inches in length; 25 links make 1 rod, long measure, and 625 square links make 1 *square* rod.

37. In 17 acres 3 roods 12 rods, how many square feet?

Note. In reducing rods to feet, the multiplier will be $272\frac{1}{4}$. To multiply by $\frac{1}{4}$, is to take a fourth part of the multiplicand. The principle is the same as shown ¶ 34, ex. 31.

38. In 776457 square feet, how many acres?

Note. Here we have 776457 square feet to be divided by $272\frac{1}{4}$. Reduce the divisor to *fourths*, that is, to the lowest denomination contained in it; then reduce the dividend to *fourths*, that is, to the same denomination, as shown ¶ 32, ex. 34.

39. Reduce 64 square miles o square feet?

40. In 1,784,217,600 square feet, how many square miles?

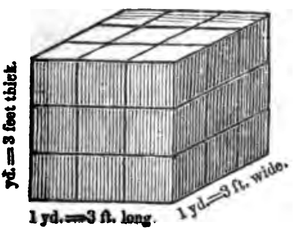
41. There is a town 6 miles square; how many square miles in that town? how many acres?

42. Reduce 23040 acres to square miles.

SOLID OR CUBIC MEASURE.

Solid or cubic measure is used in measuring things that have length, breadth, and *thickness;* such as timber, wood, stone, bales of goods, &c. The denominations are cords, tons, yards, feet, and inches.

¶ **36.** It has been shown, that a square yard contains $3 \times 3 = 9$ square feet. A cubic yard is 3 feet long, 3 feet wide, and 3 feet thick. Were it 3 feet long, 3 feet wide, and *one* foot thick, it would contain 9 cubic feet; if 2 feet thick, it would contain $2 \times 9 = 18$ cubic feet; and, as it is 3 feet thick, it does contain $3 \times 9 = 27$ cubic feet. This will clearly appear from the annexed figure.



It is plain, also, that a cubic foot, that is, a solid, 12 inches in length, 12 inches in breadth, and 12 inches in thickness, will contain $12 \times 12 \times 12 = 1728$ solid or cubic inches.

TABLE.

1728 solid inches, $= 12 \times 12 \times 12$, that is, 12 inches in length, 12 in breadth, 12 in thickness,	make 1 solid foot.
27 solid feet, $= 3 \times 3 \times 3$ - - - - -	1 solid yard.
40 feet of round timber, or 50 feet of hewn timber, - - - - - - -	1 ton or load.
128 solid feet, $= 8 \times 4 \times 4$, that is, 8 feet in length, 4 feet in width, and 4 feet in height, - -	1 cord of wood.

Note. What is called a *cord foot*, in measuring wood, is 16 solid feet; that is, 4 feet in length, 4 feet in width, and 1 foot in height, and 8 such feet, that is, 8 *cord feet* make 1 cord.

43. Reduce 9 tons of round timber to cubic inches.

44. In 622080 cubic inches, how many tons of round timber?

45. In 37 cord feet of wood, how many solid feet?

46. In 592 solid feet of wood, how many cord feet?

47. Reduce 64 cord feet of wood to cords.

48. In 8 cords of wood, how many cord feet?

49. In 16 cords of wood, how many cord feet? how many solid feet?

50. 2048 solid feet of wood, how many cord feet? how many cords?

WINE MEASURE.

Wine measure is used in measuring all spirituous liquors, ale and beer excepted; also vinegar and oil. The denominations are tuns, pipes, hogsheads, barrels, gallons, quarts, pints, and gills.

TABLE.

4	gills (gi.) - make - -	1 pint,	marked	pt.
2	pints - - - - - - - - -	1 quart,	- - -	qt.
4	quarts - - - - - - - -	1 gallon,	- - -	gal.
$31\frac{1}{2}$	gallons - - - - - - - -	1 barrel,	- - -	bar.
63	gallons - - - - - - - -	1 hogshead,	- -	hhd.
2	hogsheads - - - - - -	1 pipe,	- - -	P.
2	pipes, or 4 hogsheads -	1 tun,	- - - -	T.

Note. A gallon, wine measure, contains 231 cubic inches.

51. Reduce 12 pipes of wine to pints.

53. In 9 P. 1 hhd. 22 gals. 3 qts. how many gills?

55. In a tun of cider, how many gallons?

52. In 12096 pints of wine, how many pipes?

54. Reduce 39032 gills to pipes.

56. Reduce 252 gallons to tuns.

ALE OR BEER MEASURE.

Ale or beer measure is used in measuring ale, beer, and milk. The denominations are hogsheads, barrels, gallons, quarts, and pints.

TABLE.

2 pints (pts.)	make	1 quart,	marked	qt.
4 quarts	- - - - -	1 gallon,	- - - - -	gal.
36 gallons	- - - -	1 barrel,	- - - - -	bar.
54 gallons	- - - - -	1 hogshead,	- - - -	hhd.

Note. A gallon, beer measure, contains 282 cubic inches.

57. Reduce 47 bar. 18 gal. of ale to pints.

59. In 29 hhds. of beer, how many pints?

58. In 13680 pints of ale, how many barrels?

60. Reduce 12528 pints to hogsheads.

DRY MEASURE.

Dry measure is used in measuring all dry goods, such as grain, fruit, roots, salt, coal, &c. The denominations are chaldrons, bushels, pecks, quarts, and pints.

TABLE.

2 pints (pts.)	make -	1 quart,	- marked -	qt.
8 quarts	- - - - -	1 peck,	- - - - - -	pk.
4 pecks	- - - - -	1 bushel,	- - - - -	bu.
36 bushels	- - - - -	1 chaldron,	- - - -	ch.

Note. A gallon, dry measure, contains 268$\frac{4}{5}$ cubic inches. A Winchester bushel is 18$\frac{1}{2}$ inches in diameter, 8 inches deep, and contains 2150$\frac{2}{5}$ cubic inches.

61. In 75 bushels of wheat, how many pints?

62. In 4800 pints, how many bushels?

63. Reduce 42 chaldrons of coals to pecks.

64. In 6048 pecks, how many chaldrons?

TIME.

The denominations of time are years, months, weeks, days, hours, minutes, and seconds.

TABLE.

60 seconds (s.)	make	1 minute,	marked	m.
60 minutes	- - - - - - -	1 hour,	- - - -	h.
24 hours	- - - - - - - -	1 day,	- - - -	d.
7 days	- - - - - - - -	1 week,	- - - -	w.
4 weeks	- - - - - - -	1 month,	- - - -	mo.
13 months, 1 day and 6 hours, or 365 days and 6 hours,		1 common, or Julian year,	-	yr.

¶ 37. The year is also divided into 12 calendar months, which, in the order of their succession, are numbered as follows, viz.

January,	1st month,	has	31 days.
February,	2d,	- - -	28
March,	3d,	- - -	31
April, -	4th,	- - -	30
May, -	5th,	- - -	31
June, -	6th,	- - -	30
July, -	7th,	- - -	31
August,	8th,	- - -	31
September,	9th,	- - -	30
October,	10th,	- - -	31
November,	11th,	- - -	30
December,	12th,	- - -	31

Note. When any year can be divided by 4 without a remainder, it is called leap year, in which February has 29 days.

The number of days in each month may be easily fixed in the mind by committing to memory the following lines:

Thirty days hath September,
April, June, and November,
February twenty-eight alone;
All the rest have thirty-one.

The first seven letters of the alphabet, A, B, C, D, E, F, G, are used to mark the several days of the week, and they are disposed in such a manner, for every year, that the letter A shall stand for the 1st day of January, B for the 2d, &c. In pursuance of this order, the letter which shall stand for *Sunday*, in any year, is called the *Dominical* letter for that year. The Dominical letter being known, the day of the week on which each month comes in may be readily calculated from the following couplet:

At Dover Dwells George Brown, Esquire,
Good Carlos Finch And David Fryer.

These words correspond to the 12 months of the year, and the *first letter* in each word marks the day of the week on which each corresponding month comes in; whence any other day may be easily found. For example, let it be required to find on what day of the week the 4th day of July falls, in the year 1827, the Dominical letter for which year is G. *Good* answers to July; consequently, July comes in on a Sunday; wherefore the 4th day of July falls on Wednesday.

Note. There are *two* Dominical letters in *leap* years, *one* for January and February, and *another* for the rest of the year.

65. Supposing your age to be 15 y. 19 d. 11 h. 37 m. 45 s., how many seconds old are you, allowing 365 days 6 hours to the year?

66. Reduce 475047465 seconds to years.

67. How many minutes from the 1st day of January to the 14th day of August, inclusively?

68. Reduce 325440 minutes to days.

69. How many minutes from the commencement of the war between America and England, April 19th, 1775, to the settlement of a general peace, which took place Jan. 20th, 1783?

70. In 4079160 minutes, how many years?

CIRCULAR MEASURE, OR MOTION.

Circular measure is used in reckoning latitude and longitude; also in computing the revolution of the earth and other planets round the sun. The denominations are circles, signs, degrees, minutes, and seconds.

TABLE.

60 seconds (″)	make	1 minute,	marked	′
60 minutes	- - - - -	1 degree,	- - - - -	°
30 degrees	- - - - -	1 sign,	- - - - -	s.
12 signs, or 360 degrees,	-	1 circle of the zodiac.		

Note. Every circle, whether great or small, is divisible into 360 equal parts, called degrees.

71. Reduce 9 s. 13° 25′ to seconds.

72. In 1020300″, how many degrees?

The following are *denominations* of things not included in the Tables:—

12 particular things	make	1 dozen.
12 dozen	- - - - -	1 gross.
12 gross, or 144 dozen,	- - - - -	1 great gross.

Also,

20 particular things - make - 1 score.

6 points make 1 line, 12 lines - - 1 inch, } used in measuring the length of the rods of clock pendulums.

4 inches - - 1 hand, } used in measuring the height of horses.

6 feet - - 1 fathom, used in measuring depths at sea.

112 pounds - - make - - 1 quintal of fish.

24 sheets of paper	make	1 quire.
20 quires	- - - - -	1 ream.

SUPPLEMENT TO REDUCTION.

QUESTIONS.

1. What is reduction? 2. Of how many varieties is reduction? 3. What is understood by *different denominations,* as of money, weight, measure, &c.? 4. How are high de-

nominations brought into lower? 5. How are low denominations brought into higher? 6. What are the denominations of English money? 7. What is the use of Troy weight, and what are the denominations? 8. —— avoirdupois weight? the denominations? 9. What distinction do you make between *gross* and *net* weight? 10. What distinctions do you make between long, square, and cubic measure? 11. What are the denominations in long measure? 12. —— in square measure? 13. —— in cubic measure? 14. How do you multiply by ½? 15. When the divisor contains a fraction, how do you proceed? 16. How is the superficial contents of a square figure found? 17. How is the solid contents of any body found in cubic measure? 18. How many solid or cubic feet of wood make a cord? 128 19. What is understood by a *cord foot*? 16 20. How many such feet make a cord? 8 21. What are the denominations of dry measure? 22. —— of wine measure? 23. —— of time? 24. —— of circular measure? 25. For what is circular measure used? 26. How many rods in length is Gunter's chain? 4 of how many links does it consist? how many links make a rod? 25 27. How many rods in a mile? 320 28. How many square rods in an acre? 160 29. How many pounds make 1 cwt.? 112

EXERCISES.

1. In 46£. 4 s., how many dollars? *Ans.* $154.

2. In 36 guineas, how many crowns, at 6 s. 7 d. each? *Ans.* 153 crowns, and 9 d.

3. How many rings, each weighing 5 pwt. 7 grs., may be made of 3 lb. 5 oz. 16 pwt. 2 grs. of gold? *Ans.* 158.

4. Suppose West Boston bridge to be 212 rods in length, how many times will a chaise wheel, 18 feet 6 inches in circumference, turn round in passing over it? *Ans.* $189\frac{18}{222}$ times.

5. In 470 boxes of sugar, each 26 lb., how many cwt.?

6. In 10 lb. of silver, how many spoons, each weighing 5 oz. 10 pwt.?

7. How many shingles, each covering a space 4 inches one way and 6 inches the other, would it take to cover 1 square foot? How many to cover a roof 40 feet long, and 24 feet wide? (*See* ¶ 25.) *Ans. to the last*, 5760 shingles.

8. How many cords of wood in a pile 26 feet long, 4 feet wide, and 6 feet high? *Ans.* 4 cords, and 7 cord feet.

9. There is a room 18 feet in length, 16 feet in width, and 8 feet in height; how many rolls of paper, 2 feet wide, and containing 11 yards in each roll, will it take to cover the walls? *Ans.* $8\frac{16}{66}$.

10. How many cord feet in a load of wood $6\frac{1}{2}$ feet long, 2 feet wide, and 5 feet high? *Ans.* $4\frac{1}{16}$ cord feet.

11. If a ship sail 7 miles an hour, how far will she sail, at that rate, in 3 w. 4 d. 16 h.? 4312

12. A merchant sold 12 hhds. of brandy, at $ 2'75 a gallon; how much did each hogshead come to, and to how much did the whole amount? 173-25

13. How much cloth, at 7 s. a yard, may be bought for 29£. 1 s.? 83 yds

14. A goldsmith sold a tankard for 10£. 8 s. at the rate of 5 s. 4 d. per ounce; how much did it weigh?

15. An ingot of gold weighs 2 lb. 8 oz. 16 pwt.; how much is it worth at 3 d. per pwt.?

16. At $ 0'18 a pound, what will 1 T. 2 cwt. 3 qrs. 16 lb. of lead come to?

17. Reduce 14445 ells Flemish to ells English.

18. There is a house, the roof of which is $44\frac{1}{2}$ feet in length, and 20 feet in width, on each of the two sides; if 3 shingles in width cover one foot in length, how many shingles will it take to lay one course on this roof? if 3 courses make one foot, how many courses will there be on one side of the roof? how many shingles will it take to cover one side? —— to cover both sides? *Ans.* 16020 shingles.

19. How many steps, of 30 inches each, must a man take in travelling $54\frac{1}{2}$ miles?

20. How many seconds of time would a person redeem in 40 years, by rising each morning $\frac{1}{2}$ hour earlier than he now does?

21. If a man lay up 4 shillings each day, Sundays excepted, how many dollars would he lay up in 45 years?

22. If 9 candles are made from 1 pound of tallow, how many dozen can be made from 24 pounds and 10 ounces?

23. If one pound of wool make 60 knots of yarn, how many skeins, of ten knots each, may be spun from 4 pounds 6 ounces of wool?

ADDITION

OF COMPOUND NUMBERS.

¶ 38. 1. A boy bought a knife for 9 pence, and a comb for 3 pence; how much did he give for both? *Ans.* 1 shilling.

2. A boy gave 2 s. 6 d. for a slate, and 4 s. 6 d. for a book; how much did he give for both?

3. Bought one book for 1 s. 6 d., another for 2 s. 3 d., another for 7 d.; how much did they all cost? *Ans.* 4 s. 4 d.

4. How many gallons are 2 qts. + 3 qts. + 1 qt.?

5. How many gallons are 3 qts. + 2 qts. + 1 qt. + 3 qts. + 2 qts.?

6. How many shillings are 2 d. + 3 d. + 5 d. + 6 d. + 7 d.?

7. How many pence are 1 qr. + 2 qrs. + 3 qrs. + 2 qrs. + 1 qr.?

8. How many pounds are 4 s. + 10 s. + 15 s. + 1 s.?

9. How many minutes are 30 sec. + 45 sec. + 20 sec.?

10. How many hours are 40 min. + 25 min. + 6 min.?

11. How many days are 4 h. + 8 h. + 10 h. + 20 h.?

12. How many yards in length are 1 f. + 2 f. + 1 f.?

13. How many feet are 4 in. + 8 in. + 10 in. + 2 in. + 1 in.?

14. How much is the amount of 1 yd. 2 ft. 6 in. + 2 yds. 1 ft. 8 in.?

15. What is the amount of 2 s. 6 d. + 4 s. 3 d. + 7 s. 8 d.?

16. A man has two bottles, which he wishes to fill with wine; one will contain 2 gal. 3 qts. 1 pt., and the other 3 qts.; how much wine can he put in them?

17. A man bought a horse for 15£. 14 s. 6 d., a pair of oxen for 20£. 2 s. 8 d., and a cow for 5£. 6 s. 4 d.; what did he pay for all?

When the numbers are large, it will be most convenient to write them down, placing those of the same kind, or denomination, directly under each other, and, beginning with those of the least value, to add up each kind separately.

OPERATION.

	£.	*s.*	*d.*
	15	14	6
	20	2	8
	5	6	4
Ans.	41	3	6

In this example, adding up the column of pence, we find the amount to be 18 pence, which being = 1 s. 6 d., it is plain, that we may write down the 6 d. under the column of pence, and reserve the 1 s. to be added in with the *other* shillings.

Next, adding up the column of shillings, together with the 1 s. which we reserved, we find the amount to be 23 s. = 1 £. 3 s. Setting the 3 s. under its *own* column, we add the 1 £. with the *other* pounds, and, finding the amount to be 41 £., we write it down, and the work is done.

Ans. 41 £. 3 s. 6 d.

Note. It will be recollected, that, to reduce a lower into a higher denomination, we divide by the number which it takes of the lower to make one of the higher denomination. In addition, this is usually called *carrying* for that number: thus, between pence and shillings, we carry for 12, and between shillings and pounds, for 20, &c.

The above process may be given in the form of a general RULE *for the Addition of Compound Numbers:*

I. Write the numbers to be added so that those of the same denomination may stand directly under each other.

II. Add together the numbers in the column of the lowest denomination, and carry for that number which it takes of the same to make one of the next *higher* denomination. Proceed in this manner with all the denominations, till you come to the last, whose amount is written as in simple numbers.

Proof. The same as in addition of simple numbers.

EXAMPLES FOR PRACTICE.

£.	s.	d.	qr.
46	11	3	2
16	7	4	0
538	19	7	1

£.	s.	d.
72	9	$6\frac{1}{2}$
18	0	$10\frac{1}{4}$
36	16	$5\frac{3}{4}$

£.	s.	d.
183	19	4
8	17	10
	15	4

TROY WEIGHT.

lb.	oz.	pwt.	gr.
36	7	10	11
42	6	9	13
81	7	16	15

oz.	pwt.	gr.
6	14	9
8	6	16
3	11	10

oz.	pwt.	gr.
		18
	13	16
3	7	4

Bought a silver tankard, weighing 2 lb. 3 oz., a silver cup, weighing 3 oz. 10 pwt., and a silver thimble, weighing 2 pwt. 13 grs.; what was the weight of the whole?

Avoirdupois Weight.

T.	cwt.	qr.	lb.	oz.	dr.
14	11	1	16	5	10
25	0	2	11	9	15
7	18	0	25	11	9

cwt.	qr.	lb.	oz.	dr.
16	3	18	6	14
	2	16	8	12
		22	11	10

A man bought 5 loads of hay, weighing as follows, viz. 23 cwt. (= 1 T. 3 cwt.) 2 qrs. 17 lb.; 21 cwt. 1 qr. 16 lb.; 19 cwt. 0 qr. 24 lb.; 24 cwt. 3 qrs.; 11 cwt. 0 qr. 1 lb.; how many tons in the whole?

Cloth Measure.

yds.	qr.	na.
36	1	2
41	2	3
65	3	1

E. Fl.	qr.	na.
41	1	2
18	2	3
57	0	1

E. En.	qr.	na.
75	4	2
31	1	0
28	3	1

There are four pieces of cloth, which measure as follows, viz. 36 yds. 2 qrs. 1 na.; 18 yds. 1 qr. 2 na.; 46 yds. 3 qrs. 3 na.; 12 yds. 0 qr. 2 na.; how many yards in the whole?

Long Measure.

Deg.	mi.	fur.	r.	ft.	in.	bar.
59	46	6	29	15	10	2
216	39	1	36	14	6	1
678	53	7	24	9	8	1

Mi.	fur.	pol.
3	7	
8	6	27

Land or Square Measure.

Pol.	ft.	in.
36	179	137
19	248	119
12	96	75

A.	rood	pol.	ft.	in.
56	3	37	245	228
29	1	28	93	25
416	2	31	128	119

There are 3 fields, which measure as follows, viz. 17 A. 3 r. 16 p.; 28 A. 5 r. 18 p.; 11 A. 0 r. 25 p.; how much land in the three fields?

SOLID OR CUBIC MEASURE.

Ton.	*ft.*	*in.*
29	36	1229
12	19	64
8	11	917

yds.	*ft.*	*in.*
75	22	1412
9	26	195
3	19	1091

cords.	*ft.*
37	119
9	110
48	127

WINE MEASURE.

Hhd.	*gal.*	*qts.*	*pts.*
51	53	1	1
27	39	3	0
9	13	0	1

Tun.	*hhd.*	*gal.*	*qts.*
37	2	37	2
19	1	59	1
28	2	0	0

A merchant bought two casks of brandy, containing as follows, viz. 70 gal. 3 qts.; 67 gal. 1 qt.; how many hogsheads, of 63 gal. each, in the whole?

DRY MEASURE.

Bus.	*p.*	*qt.*	*pt.*
36	2	5	1
19	3	7	0

Ch.	*bus.*	*p.*	*qts.*
48	27	3	5
6	29	1	7

TIME.

Y.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
57	11	3	6	23	55	11
84	9	2	0	16	42	18
32	6	0	5	5	18	5

Y.	*m.*	*w.*	*d.*
40	3	1	5
16	7	0	4
27	5	2	0

SUBTRACTION

OF COMPOUND NUMBERS.

¶ **39.** 1. A boy bought a knife for 9 cents, and sold it for 17 cents; how much did he gain by the bargain?

2. A boy bought a slate for 2 s. 6 d., and a book for 3 s. 6 d.; how much more was the cost of the book than of the slate?

3. A boy owed his playmate 2 s.; he paid him 1 s. 6 d.; how much did he then owe him?

4. Bought two books; the price of one was 4 s. 6 d., the price of the other 3 s. 9 d.; what was the difference of their costs?

5. A boy lent 5 s. 3 d.; he received in payment 2 s. 6 d.; how much was then due?

6. A man has a bottle of wine containing 2 gallons and 3 quarts; after turning out 3 quarts, how much remained?

7. How much is 4 gal. less 3 gal.? 4 gal. — (less) 2 qts.? 4 gal. — 1 qt.? 4 gal. — 1 gal. 1 qt.? 4 gal. — 1 gal. 2 qts.? 4 gal. — 1 gal. 3 qts.? 4 gal. — 2 gal. 3 qts.? 4 gal. 1 qt. — 1 gal. 3 qts.?

8. How much is 1 ft. — (less) 6 in.? 1 ft. — 8 in.? 6 ft. 3 in. — 1 ft. 6 in.? 7 ft. 8 in. — 4 ft. 2 in.? 7 ft. 8 in. — 5 ft. 10 in.?

9. What is the difference between 4£. 6 s. and 1£. 8 s.?

10. How much is 3£. — (less) 1 s.? 3£. — 2 s.? 3£. — 3 s.? 3£. — 15 s.? 3£. 4 s. — 2£. 6 s.? 10£. 4 s. — 5£. 8. s?

11. A man bought a horse for 30£. 4 s. 8 d., and a cow for 5£. 14 s. 6 d.; what is the difference of their costs?

OPERATION.

	£.	*s.*	*d.*
Minuend,	30	4	8
Subtrahend,	5	14	6
Ans.	24	10	2

As the two numbers are large, it will be convenient to write them down, the less under the greater, pence under pence, shillings under shillings, &c. We may now take 6 d. from 8 d., and there will remain 2 d. Proceeding to the shillings, we cannot take 14 s. from 4 s., but we may borrow, as in simple numbers, 1 from the pounds, = 20 s., which joined to the 4 s. makes 24 s., from which taking 14 s. leaves 10 s., which we set down. We must now carry 1 to the 5£., making 6£., which taken from 30£. leaves 24£., and the work is done.

Note. The most convenient way in borrowing is, to sub-

tract the subtrahend from the figure borrowed, and add the difference to the minuend. Thus, in the above example, 14 from 20 leaves 6, and 4 is 10.

The process in the foregoing example may be presented in the form of a RULE *for the Subtraction of Compound Numbers:*

I. Write down the sums or quantities, the less under the greater, placing those numbers which are of the same denomination directly under each other.

II. Beginning with the least denomination, take successively the lower number in each denomination from the upper, and write the remainder underneath, as in subtraction of simple numbers.

III. If the lower number of any denomination be greater than the upper, borrow as many units as make *one* of the next higher denomination, subtract the lower number therefrom, and to the remainder add the upper number, remembering always to add 1 to the next higher denomination for that which you borrowed.

Proof. Add the remainder and the subtrahend together, as in subtraction of simple numbers; if the work be right, the amount will be equal to the minuend.

EXAMPLES FOR PRACTICE.

1. A merchant sold goods to the amount of 136£. 7 s. 6½ d., and received in payment 50£. 10 s. 4¾ d; how much remained due? *Ans.* 85£. 17 s. 1¾ d.

2. A man bought a farm for 1256£. 10 s., and, in selling it, lost 87£. 10 s. 6 d.; how much did he sell it for? *Ans.* 1168£. 19 s. 6 d.

3. A man bought a horse for 27£. and a pair of oxen for 19£. 12 s. 8½ d.; how much was the horse valued more than the oxen?

4. A merchant drew from a hogshead of molasses, at one time, 13 gal. 3 qts.; at another time, 5 gal. 2 qts. 1 pt.; what quantity was there left? *Ans.* 43 gal. 2 qts. 1 pt.

5. A pipe of brandy, containing 118 gal. sprang a leak, when it was found only 97 gal. 3 qts. 1 pt. remained in the cask; how much was the leakage?

6. There was a silver tankard which weighed 3 lb. 4 oz.; the lid alone weighed 5 oz. 7 pwt. 13 grs.; how much did the tankard weigh without the lid?

7. From 15 lb. 2 oz. 5 pwt. take 9 oz. 8 pwt. 10 grs.

8. Bought a hogshead of sugar, weighing 9 cwt. 2 qrs. 17 lb.; sold at three several times as follows, viz. 2 cwt. 1 qr. 11 lb. 5 oz.; 2 qrs. 18 lb. 10 oz.; 25 lb. 6 oz.; what was the weight of sugar which remained unsold?

Ans. 6 cwt. 1 qr. 17 lb. 11 oz.

9. Bought a piece of black broadcloth, containing 36 yds. 2 qrs.; two pieces of blue, one containing 10 yds. 3 qrs. 2 na., the other, 18 yds. 3 qrs. 3 na.; how much more was there of the black than of the blue?

10. From 28 miles, 5 fur. 16 r. take 15 m. 6 fur. 26 r. 12 ft.

11. A farmer has two mowing fields; one containing 13 acres 6 roods; the other, 14 acres 3 roods: he has two pastures also; one containing 26 A. 2 r. 27 p.; the other, 45 A. 5 r. 33 p.: how much more has he of pasture than of mowing?

12. From 64 A. 2 r. 11 p. 29 ft. take 26 A. 5 r. 34 p. 132 ft.

13. From a pile of wood, containing 21 cords, was sold, at one time, 8 cords 76 cubic feet; at another time, 5 cords 7 cord feet; what was the quantity of wood left?

14. How many days, hours and minutes of any year will be future time on the 4th day of July, 20 minutes past 3 o'clock, P. M.? *Ans.* 180 days, 8 hours. 40 minutes.

15. On the same day, hour and minute of July, given in the above example, what will be the difference between the past and future time of that month?

16. A note, bearing date Dec. 28th, 1826, was paid Jan. 2d, 1827; how long was it at interest?

The distance of time from one date to that of another may be found by subtracting the first date from the last, observing to number the months according to their order. (¶ 37.)

OPERATION.

A. D.	1827.	1st m.	2d day.
	1826.	12	28 ....
Ans.	0	0	4 days.

Note. In casting interest, *each* month is reckoned 30 days.

17. A note, bearing date Oct. 20th, 1823, was paid April 25th, 1825; how long was the note at interest?

18. What is the difference of time from Sept. 29, 1816, to April 2d, 1819? *Ans.* 2 y. 6 m. 3 d.

19. London is 51° 32′, and Boston 42° 23′ N. latitude; what is the difference of latitude between the two places?

Ans. 9° 9′.

20. Boston is 71° 3′, and the city of Washington is 77° 43′ W. longitude; what is the difference of longitude between the two places? *Ans.* 6° 40′.

21. The island of Cuba lies between 74° and 85° W. longitude; how many degrees in longitude does it extend?

¶ **40.** 1. When it is 12 o'clock at the most easterly extremity of the island of Cuba, what will be the hour at the most westerly extremity, the difference in longitude being 11°?

Note. The circumference of the earth being 360°, and the earth performing one entire revolution in 24 hours, it follows, that the motion of the earth, on its surface, from west to east, is

15° of motion in 1 hour of time; consequently,
1° of motion in 4 minutes of time, and
1′ of motion in 4 seconds of time.

From these premises it follows, that, when there is a difference in longitude between two places, there will be a corresponding difference in the hour, or time of the day. The difference in longitude being 15°, the difference in time will be 1 hour, the place *easterly* having the time of the day 1 hour *earlier* than the place *westerly*, which must be particularly regarded.

If the difference in longitude be 1°, the difference in time will be 4 minutes, &c.

Hence,—If the difference in longitude, in degrees and minutes, between two places, be multiplied by 4, the product will be the difference in time, in minutes and seconds, which may be reduced to hours.

We are now prepared to answer the above question.

$$\begin{array}{r} 11^\circ \\ 4 \\ \hline 44 \textit{ minutes.} \end{array}$$

Hence, when it is 12 o'clock at the most easterly extremity of the island, it will be 16 minutes past 11 o'clock at the most western extremity.

2. Boston being 6° 40′ E. longitude from the city of Washington, when it is 3 o'clock at the city of Washington, what is the hour at Boston?

Ans. 26 minutes 40 seconds past 3 o'clock.

3. Massachusetts being about 72°, and the Sandwich Islands about 155° W. longitude, when it is 28 minutes past 6 o'clock, A. M. at the Sandwich Islands, what will be the hour in Massachusetts? *Ans.* 12 o'clock at noon.

MULTIPLICATION & DIVISION

OF COMPOUND NUMBERS.

¶ 41. 1. A man bought 2 yards of cloth, at 1 s. 6 d. per yard; what was the cost?

2. If 2 yards of cloth cost 3 shillings, what is that per yard?

3. A man has three pieces of cloth, each measuring 10 yds. 3 qrs.; how many yards in the whole?

4. If 3 equal pieces of cloth contain 32 yds. 1 qr., how much does each piece contain?

5. A man has five bottles, each containing 2 gal. 1 qt. 1 pt.; how much wine do they all contain?

6. A man has 11 gal. 3 qts. 1 pt. of wine, which he would divide equally into five bottles; how much must he put into each bottle?

7. How many shillings are 3 times 8 d.? —— 3×9 d.? —— 3×10 d.? —— 4×7 d.? —— 7×6 d.? —— 10×9 d.? —— 2×3 qrs.? —— 5×2 qrs.?

8. How much is one third of 2 shillings? —— $\frac{1}{3}$ of 2 s. 3 d.? —— $\frac{1}{3}$ of 2 s. 6 d.? —— $\frac{1}{3}$ of 2 s. 4 d.? —— $\frac{1}{5}$ of 3 s. 6 d.? —— $\frac{1}{10}$ of 7 s. 6 d.? —— $\frac{1}{2}$ of $1\frac{1}{2}$ d.? —— $\frac{1}{2}$ of $2\frac{1}{2}$ d.?

9. At 1£. 5 s. $8\frac{3}{4}$ d. per yard, what will 6 yards of cloth cost?

10. If 6 yards of cloth cost 7£. 14 s. $4\frac{1}{2}$ d., what is the price per yard?

Here, as the numbers are large, it will be most convenient to write them down before multiplying and dividing.

OPERATION.

£.	s.	d.	qr.	
1	5	8	3	*price of* 1 *yard.*
			6	*number of yards.*
Ans. 7	14	4	2	*cost of* 6 *yards.*

6 times 3 qrs. are 18 qrs. = 4 d. and 2 qrs. over; we set down the 2 qrs.; then, 6 times 8 d. are 48 d., and 4 to carry makes 52 d. = 4 s. and 4 d. over, which we write down; again, 6 times 5 s. are 30 s.

OPERATION.

	£.	s.	d.	qr.	
6)	7	14	4	2	*cost of* 6 *yards.*
	1	5	8	3	*price of* 1 *yard.*

Proceeding after the manner of short division, 6 is contained in 7£. 1 time, and 1£. over; we write down the quotient, and reduce the remainder (1£.) to shillings, (20 s.,) which, with the given shillings, (14 s.,) make 34 s.;

and 4 to carry makes 34 s. = 1£. and 14 s. over; 6 times 1£. are 6£., and 1 to carry makes 7£., which we write down, and it is plain, that the united products arising from the several denominations is the real product arising from the whole compound number.

6 in 34 s. goes 5 times, and 4 s. over; 4 s. reduced to pence = 48 d., which, with the given pence, (4 d.,) make 52 d.; 6 in 52 d. goes 8 times, and 4 d. over; 4 d. = 16 qrs., which, with the given qrs. (2) = 18 qrs.; 6 in 18 qrs. goes 3 times; and it is plain, that the united quotients arising from the several denominations, is the real quotient arising from the whole compound number.

11. Multiply 3£. 4 s. 6 d. by 7.

12. Divide 22£. 11 s. 6 d. by 7.

13. What will be the cost of 5 pairs of shoes at 10 s. 6 d. a pair?

14. At 2£. 12 s. 6 d. for 5 pairs of shoes, what is that a pair?

15. In 5 barrels of wheat, each containing 2 bu. 3 pks. 6 qts., how many bushels?

16. If 14 bu. 2 pks. 6 qts. of wheat be equally divided into 5 barrels, how many bushels will each contain?

17. How many yards of cloth will be required for 9 coats, allowing 4 yds. 1 qr. 3 na. to each?

18. If 9 coats contain 39 yds. 3 qrs. 3 na., what does 1 coat contain?

19. In 7 bottles of wine, each containing 2 qts. 1 pt. 3 gills, how many gallons?

20. If 5 gal. 1 gill of wine be divided equally into 7 bottles, how much will each contain?

21. What will be the weight of 8 silver cups, each weighing 5 oz. 12 pwt. 17 grs.?

22. If 8 silver cups weigh 3 lb. 9 oz. 1 pwt. 16 grs., what is the weight of each?

23. How much sugar in 12 hogsheads, each containing 9 cwt. 3 qrs. 21 lb.?

24. If 119 cwt. 1 qr. of sugar be divided into 12 hogsheads, how much will each hogshead contain?

25. In 15 loads of hay, each weighing 1 T. 3 cwt. 2 qrs., how many tons?

26. If 15 teams be loaded with 17 T. 12 cwt. 2 qrs. of hay, how much is that to each team?

When the multiplier, or divisor, exceeds 12, the operations of multiplying and dividing are not so easy, unless they be composite numbers; in that case, we may make use of the *component parts*, or *factors*, as was done in simple numbers.

Thus 15, in the example above, is a composite number produced by the multiplication of 3 and 5, ($3 \times 5 = 15$.) We may, therefore, multiply 1 T. 3 cwt. 2 qrs. by one of those component parts, or factors, and that product by the other, which will give the true answer, as has been already taught, (¶ 11.)

OPERATION.

T.	*cwt.*	*qr.*	
1	3	2	
		3	*one of the factors.*
3	10	2	
		5	*the other factor.*
17	12	2	*the answer.*

15 being a composite number, and 3 and 5 its component parts, or factors, we may divide 17 T. 12 cwt. 2 qrs. by one of these component parts, or factors, and the quotient thence arising by the other, which will give the true answer, as already taught, (¶ 20.)

OPERATION.

		T.	*cwt.*	*qr.*
One factor,	3)	17	12	2
The other factor,	5)	5	17	2
	Ans.	1	3	2

27. What will 24 barrels of flour cost, at 2 £. 12 s. 4 d. a barrel?

28. Bought 24 barrels of flour for 62 £. 16 s.; how much was that per barrel?

29. What will 112 lb. of sugar cost, at $7\frac{1}{4}$ d. per lb.?

Note. 8, 7, and 2, are factors of 112.

30. If 1 cwt. of sugar cost 3 £. 7 s. 8 d., what is that per lb.?

31. How much brandy in 84 pipes, each containing 112 gal. 2 qts. 1 pt. 3 g.?

32. Bought 84 pipes of brandy, containing 9468 gal. 1 qt. 1 pt.; how much in a pipe?

33. What will 139 yards of cloth cost, at 3 £. 6 s. 5 d. per yard?

139 is not a composite number. We may, however, decompose this number thus, $139 = 100 + 30 + 9$.

We may now multiply the

34. Bought 139 yards of cloth for 461 £. 11 s. 11 d.; what was that per yard?

When the divisor is such a number as cannot be produced by the multiplication of small numbers, the better way is to divide after the manner of

price of 1 yard by 10, which will give the price of 10 yards, and this product again by 10, which will give the price of 100 yards.

We may then multiply the price of 10 yards by 3, which will give the price of 30 yards, and the price of 1 yard by 9, which will give the price of 9 yards, and these three products, added together, will evidently give the price of 139 yards; thus:

£.	*s.*	*d.*	
3	6	5	*price of* 1 *yd.*
		10	
33	4	2	*price of* 10 *yds.*
		10	
332	1	8	*price of* 100 *yds.*
99	12	6	*price of* 30 *yds.*
29	17	9	*price of* 9 *yds.*
461	11	11	*price of* 139 *yds.*

Note. In multiplying the price of 10 yards (33£. 4 s. 2 d.) by 3, to get the price of 30 yards, and in multiplying the price of 1 yard (3£. 6 s. 5 d.) by 9, to get the price of 9 yards, the multipliers, 3 and 9, need not be written down, but may be carried in the mind.

long division, setting down the work of dividing and reducing in manner as follows:

```
       £.    s.   d.
139 ) 461    11   11 ( 3£.
      417
      ---
       44
       20
      ---
      891 ( 6 s.
      834
      ---
       57
       12
      ---
      695 ( 5 d.
      695
      ---
```

The divisor, 139, is contained in 461 £. 3 times, (3£.,) and a remainder of 44£., which must now be reduced to shillings, multiplying it by 20, and bringing in the given shillings, (11 s.,) making 891 s., in which the divisor is contained 6 times, (6 s.,) and a remainder of 57 s., which must be reduced to pence, multiplying it by 12, and bringing in the given pence, (11 d.,) together making 695 d., in which the divisor is contained 5 times, (5 d.,) and no remainder.

The several quotients, 3£, 6 s., 5 d., evidently make the answer.

The processes in the foregoing examples may now be presented in the form of a

Rule *for the Multiplication of Compound Numbers.*

I. When the multiplier does *not* exceed 12, multiply successively the numbers of each denomination, beginning with the least, as in multiplication of simple numbers, and carry as in addition of compound numbers, setting down the whole product of the highest denomination.

II. If the multiplier *exceed* 12, and be a *composite* number, we may multiply first by *one* of the component parts, that product by another, and so on, if the component parts be more than two; the last product will be the product required.

III. When the multiplier exceeds 12, and is *not* a composite, multiply first by 10, and this product by 10, which will give the product for 100; and if the hundreds in the multiplier be *more* than one, multiply the product of 100 by the *number* of hundreds; for the *tens*, multiply the product of 10 by the number of tens; for the *units*, multiply the *multiplicand;* and these several products will be the product required.

Rule *for the Division of Compound Numbers.*

I. When the divisor does *not* exceed 12, in the manner of short division, find how many times it is contained in the highest denomination, under which write the quotient, and, if there be a remainder, reduce it to the next less denomination, adding thereto the number given, if any, of that denomination, and divide as before; so continue to do through all the denominations, and the several quotients will be the answer.

II. If the divisor *exceed* 12, and be a *composite,* we may divide first by one of the component parts, that quotient by another, and so on, if the component parts be more than two; the last quotient will be the quotient required.

III. When the divisor exceeds 12, and is *not* a composite number, divide after the manner of long division, setting down the work of dividing and reducing.

EXAMPLES FOR PRACTICE.

1. What will 359 yards of cloth cost, at 4 s. $7\frac{1}{2}$ d. per yard?

2. Bought 359 yards of cloth for 83 £. 0 s. $4\frac{1}{2}$ d.; what was that a yard?

3. In 241 barrels of flour, each containing 1 cwt. 3 qr. 9 lb.; how many cwt.?

4. If 441 cwt. 13 lb. of flour be contained in 241 barrels, how much in a barrel?

5. How many bushels of wheat in 135 bags, each containing 2 bu. 3 pks.?

$3 \times 9 \times 5 = 135.$

6. If 371 bu. 1 pk. of wheat be divided equally into 135 bags, how much will each bag contain?

7. What will 35 cwt. of tobacco cost, at 3 s. $10\frac{1}{2}$ d. per lb.?

8. At 759 £. 10 s. for 35 cwt. of tobacco, what is that per lb.?

9. If 14 men build 12 rods 6 feet of wall in one day, how many rods will they build in $7\frac{1}{2}$ days?

10. If 14 men build 92 rods 12 feet of stone wall in $7\frac{1}{2}$ days, how much is that per day?

¶ **42.** 1. At 10 s. per yard, what will 17849 yards of cloth cost?

Note. Operations in multiplication of pounds, shillings, pence, or of *any* compound numbers, may be facilitated by taking *aliquot parts* of a *higher denomination*, as already explained in "*Practice*" of Federal Money, ¶ 29, ex. 10. Thus, in this last example, the price 10 s. $= \frac{1}{2}$ of a pound; therefore, $\frac{1}{2}$ of the number of yards will be the cost in pounds. $\frac{17849}{2} = 8924$ £. 10 s. *Ans.*

2. What cost 34648 yards of cloth, at 10 s. or $\frac{1}{2}$ £. per yard? —— at 5 s. $= \frac{1}{4}$ £. per yard? —— at 4 s. $= \frac{1}{5}$ £. per yard? —— at 3 s. 4 d. $= \frac{1}{6}$ £. per yard? —— at 2 s. $= \frac{1}{10}$ £. per yard? *Ans. to last*, 3464 £. 16 s.

3. What cost 7430 pounds of sugar, at 6 d. $= \frac{1}{2}$ s. per lb? —— at 4 d. $= \frac{1}{3}$ s. per lb.? —— at 3 d. $= \frac{1}{4}$ s. per lb.? —— at 2 d. $= \frac{1}{6}$ s. per lb.? —— at $1\frac{1}{2}$ d. $= \frac{1}{8}$ s. per lb.?

Ans. to the last, $\frac{7430}{8}$ s. $=$ 928 s. 9 d. $=$ 46 £. 8 s. 9 d.

4. At \$18'75 per cwt., what will 2 qrs. $= \frac{1}{2}$ cwt. cost? —— what will 1 qr. $= \frac{1}{4}$ cwt. cost? —— what will 16 lb. $= \frac{1}{7}$ cwt. cost? —— what will 14 lbs. $= \frac{1}{8}$ cwt. cost? —— what will 8 lbs. $= \frac{1}{14}$ cwt. cost? *Ans. to the last*, \$1'339.

5. What cost 340 yards of cloth, at 12 s. 6 d. per yard?

12 s. 6 d. = 10 s. (= $\frac{1}{2}$ £.) and 2 s. 6 d. (= $\frac{1}{8}$ £.); therefore,

$\frac{1}{2}$) $\frac{1}{8}$) 340

	170 £.	= cost at 10 s. per yard.
	42 £. 10 s.	= at 2 s. 6 d. per yard.
Ans.	212 £ 10 s.	= at 12 s. 6 d. per yard.

Or,

10 s. = $\frac{1}{2}$ £.) 340

2 s. 6 d. = $\frac{1}{4}$ of 10 s.)	170 £.	at 10 s. per yard.
	42 £. 10 s.	at 2 s. 6 d. per yard.
Ans.	212 £. 10 s.	at 12 s. 6 d. per yard.

SUPPLEMENT TO THE ARITHMETIC OF COMPOUND NUMBERS.

QUESTIONS.

1. What distinction do you make between simple and compound numbers? (¶ 26.) 2. What is the rule for addition of compound numbers? 3. —— for subtraction of, &c.? 4. There are *three* conditions in the rule given for multiplication of compound numbers; what are they, and the methods of procedure under each? 5. The same questions in respect to the division of compound numbers? 6. When the multiplier or divisor is encumbered with a fraction, how do you proceed? 7. How is the distance of time from one date to another found? 8. How many degrees does the earth revolve from west to east in 1 hour? 9. In what time does it revolve 1°? Where is the time or hour of the day earlier—at the place most easterly or most westerly? 10. The difference in longitude between two places being known, how is the difference in time calculated? 11. How may operations, in the multiplication of compound numbers, be facilitated? 12. What are some of the aliquot parts of 1 £.? —— of 1 s.? —— of 1 cwt.? 13. What is this manner of operating usually called?

EXERCISES.

1. A gentleman is possessed of $1\frac{1}{2}$ dozen of silver spoons, each weighing 3 oz. 5 pwt.; 2 doz. of tea spoons, each weighing 15 pwt. 14 gr.; 3 silver cans, each 9 oz. 7 pwt.; 2 silver tankards, each 21 oz. 15 pwt.; and 6 silver porringers, each 11 oz. 18 pwt.; what is the weight of the whole?

Ans. 18 lb. 4 oz. 3 pwt.

Note. Let the pupil be required to reverse and prove the following examples:

2. An English guinea should weigh 5 pwt. 6 gr.; a piece of gold weighs 3 pwt. 17 gr.; how much is that short of the weight of a guinea?

3. What is the weight of 6 chests of tea, each weighing 3 cwt. 2 qrs. 9 lb.?

4. In 35 pieces of cloth, each measuring 27 yards, how many yards?

5. How much brandy in 9 casks, each containing 45 gal. 3 qts. 1 pt.?

6. If 31 cwt. 2 qrs. 20 lb. of sugar be distributed equally into 4 casks, how much will each contain?

7. At $4\frac{1}{2}$ d. per lb., what costs 1 cwt. of rice? —— 2 cwt.? —— 3 cwt.?

Note. The pupil will recollect, that 8, 7 and 2 are factors of 112, and may be used in place of that number.

8. If 800 cwt. of cocoa cost 18 £. 13 s. 4 d., what is that per cwt.? what is it per lb.?

9. What will $9\frac{1}{4}$ cwt. of copper cost at 5 s. 9 d. per lb.?

10. If $6\frac{1}{2}$ cwt. of chocolate cost 72 £. 16 s., what is that per lb.?

11. What cost 456 bushels of potatoes, at 2 s. 6 d. per bushel?

Note. 2 s. 6 d. is $\frac{1}{8}$ of 1 £. (See ¶ 42.)

12. What cost 86 yards of broadcloth, at 15 s. per yard?

Note. Consult ¶ 42, ex. 5.

13. What cost 7846 pounds of tea, at 7 s. 6 d. per lb.? —— at 14 s. per lb.? —— at 13 s. 4 d.?

14. At $ 94'25 per cwt., what will be the cost of 2 qrs. of tea? —— of 3 qrs.? —— of 14 lbs.? —— of 21 lbs.? —— of 16 lbs.? —— of 24 lbs.?

Note. Consult ¶ 42, ex. 4 and 5.

15. What will be the cost of 2 pks. and 4 qts. of wheat, at $ 1'50 per bushel?

16. Supposing a meteor to appear so high in the heavens as to be visible at Boston, 71° 3′, at the city of Washington, 77° 43′, and at the Sandwich Islands, 155° W. longitude, and that its appearance at the city of Washington be at 7 minutes past 9 o'clock in the evening; what will be the hour and minute of its appearance at Boston and at the Sandwich Islands?

FRACTIONS.

¶ **43.** We have seen, (¶ 17,) that numbers expressing *whole* things are called *integers*, or *whole* numbers; but that, in division, it is often necessary to *divide* or *break* a whole thing into *parts*, and that these parts are called *fractions*, or *broken* numbers.

It will be recollected, (¶ 14, ex. 11,) that when a thing or unit is divided into 3 parts, the parts or fractions are called *thirds;* when into four parts, *fourths;* when into six parts, *sixths;* that is, the fraction takes its *name* or *denomination* from the *number of parts*, into which the *unit* is divided. Thus, if the unit be divided into 16 parts, the parts are called *sixteenths*, and 5 of these parts would be 5 *sixteenths*, expressed thus, $\frac{5}{16}$. The number below the short line, (16,) as before taught, (¶ 17,) is called the *denominator*, because it gives the name or *denomination* to the parts; the number above the line is called the *numerator*, because it *numbers* the parts.

The *denominator* shows how many parts it takes to make a *unit* or *whole thing;* the numerator shows how many of these parts are expressed by the *fraction*.

1. If an orange be cut into 5 equal parts, by what fraction is 1 part expressed? —— 2 parts? —— 3 parts? —— 4 parts? —— 5 parts? how many parts make unity or a whole orange?

2. If a pie be cut into 8 equal pieces, and 2 of these pieces be given to Harry, what will be his fraction of the pie? if 5 pieces be given to John, what will be his fraction? what fraction or part of the pie will be left?

It is important to bear in mind, that fractions arise from *division*, (¶ 17,) and that the *numerator* may be considered a

dividend, and the *denominator* a *divisor*, and the *value* of the fraction is the *quotient*; thus, $\frac{1}{2}$ is the quotient of 1 (the numerator) divided by 2, (the denominator;) $\frac{1}{4}$ is the quotient arising from 1 divided by 4, and $\frac{3}{4}$ is 3 times as much, that is, 3 divided by 4; thus, one fourth part of 3 is the same as 3 fourths of 1.

Hence, in all cases, a fraction is always expressed by the *sign of division.*

$\frac{3}{4}$ expresses the quotient, of which $\left\{ \begin{array}{l} 3 \text{ is the } \textit{dividend}, \text{ or } \textit{numerator}. \\ 4 \text{ is the } \textit{divisor}, \text{ or } \textit{denominator}. \end{array} \right.$

3. If 4 oranges be equally divided among 6 boys, what part of an orange is each boy's share?

A sixth part of 1 orange is $\frac{1}{6}$, and a sixth part of 4 oranges is 4 such pieces, $= \frac{4}{6}$. *Ans.* $\frac{4}{6}$ of an orange.

4. If 3 apples be equally divided among 5 boys, what part of an apple is each boy's share? if 4 apples, what? if 2 apples, what? if 5 apples, what?

5. What is the quotient of 1 divided by 3? —— of 2 by 3? —— of 1 by 4? —— of 2 by 4? —— of 3 by 4? —— of 5 by 7? —— of 6 by 8? —— of 4 by 5? —— of 2 by 14?

6. What part of an orange is a third part of 2 oranges? —— one *fourth* of 2 oranges? —— $\frac{1}{4}$ of 3 oranges? —— $\frac{1}{5}$ of 3 oranges? —— $\frac{1}{5}$ of 4? —— $\frac{1}{6}$ of 2? —— $\frac{1}{7}$ of 5? —— $\frac{1}{7}$ of 3? —— $\frac{1}{8}$ of 2?

A Proper Fraction. Since the denominator shows the number of parts necessary to make a whole thing, or 1, it is plain, that, when the *numerator* is *less* than the denominator, the fraction is less than a *unit*, or *whole thing;* it is then called a *proper* fraction. Thus, $\frac{1}{8}$, $\frac{3}{8}$, &c. are proper fractions.

An Improper Fraction. When the numerator *equals* or *exceeds* the denominator, the fraction *equals* or *exceeds* unity, or 1, and is then called an *improper* fraction. Thus, $\frac{6}{6}$, $\frac{3}{3}$, $\frac{8}{7}$, $\frac{10}{2}$, are improper fractions.

A Mixed Number, as already shown, is one composed of a whole number and a fraction. Thus, $14\frac{1}{2}$, $13\frac{7}{8}$, &c. are mixed numbers.

7. A father bought 4 oranges, and cut each orange into 6 equal parts; he gave to Samuel 3 pieces, to James 5 pieces, to Mary 7 pieces, and to Nancy 9 pieces; what was each one's fraction?

Was James's fraction *proper*, or *improper*? Why?

Was Nancy's fraction proper, or improper? Why?

To change an improper fraction to a whole or mixed number.

¶ **44.** It is evident, that every improper fraction must contain one or more whole ones, or integers.

1. How many *whole* apples are there in 4 halves ($\frac{4}{2}$) of an apple? —— in $\frac{6}{2}$? —— in $\frac{8}{2}$? —— in $\frac{10}{2}$? —— in $\frac{20}{2}$? —— in $\frac{48}{2}$? —— in $\frac{120}{2}$? —— in $\frac{284}{2}$?

2. How many yards in $\frac{3}{3}$ of a yard? —— in $\frac{6}{3}$ of a yard? —— in $\frac{8}{3}$? —— in $\frac{9}{3}$? —— in $\frac{10}{3}$? —— in $\frac{11}{3}$? —— in $\frac{15}{3}$? —— in $\frac{17}{3}$? —— in $\frac{20}{3}$? —— in $\frac{48}{3}$?

3. How many bushels in 8 pecks? that is, in $\frac{8}{4}$ of a bushel? —— in $\frac{10}{4}$? —— in $\frac{11}{4}$? —— in $\frac{13}{4}$? —— in $\frac{24}{4}$? —— in $\frac{100}{4}$? —— in $\frac{31}{4}$?

This finding how many integers, or whole things, are contained in any improper fraction, is called *reducing an improper fraction to a whole or mixed number.*

4. If I give 27 children $\frac{1}{4}$ of an orange each, how many oranges will it take? It will take $\frac{27}{4}$; and it is evident, that dividing the numerator, 27, (= the number of parts contained in the fraction,) by the denominator, 4, (= the number of parts in 1 orange,) will give the number of *whole* oranges.

OPERATION.

4)27

Ans. 6$\frac{3}{4}$ *oranges.*

Hence, *To reduce an improper fraction to a whole or mixed number,*—RULE: Divide the numerator by the denominator; the quotient will be the whole or mixed number.

EXAMPLES FOR PRACTICE.

5. A man, spending $\frac{1}{6}$ of a dollar a day, in 83 days would spend $\frac{83}{6}$ of a dollar; how many dollars would that be?
Ans. \$13$\frac{5}{6}$.

6. In $\frac{1417}{60}$ of an hour, how many whole hours?

The 60th part of an hour is 1 minute: therefore the question is evidently the same as if it had been, In 1417 minutes, how many hours? *Ans.* 23$\frac{37}{60}$ hours.

7. In $\frac{8763}{12}$ of a shilling, how many units or shillings?
Ans. 730$\frac{3}{12}$ shillings.

8. Reduce $\frac{14678}{648}$ to a whole or mixed number.

9. Reduce $\frac{36}{20}$, $\frac{706}{40}$, $\frac{875}{100}$, $\frac{4786}{1000}$, $\frac{3465}{450}$, to whole or mixed numbers.

To reduce a whole or mixed number to an improper fraction.

¶ **45.** We have seen, that an improper fraction may be changed to a whole or mixed number; and it is evident, that, by reversing the operation, a whole or mixed number may be changed to the form of an improper fraction.

1. In 2 *whole* apples, how many *halves* of an apple? *Ans.* 4 halves; that is, $\frac{4}{2}$. In 3 apples, how many halves? in 4 apples? in 6 apples? in 10 apples? in 24? in 60? in 170? in 492?

2. Reduce 2 yards to *thirds.* *Ans.* $\frac{6}{3}$. Reduce $2\frac{2}{3}$ yards to thirds. *Ans.* $\frac{8}{3}$. Reduce 3 yards to thirds. —— $3\frac{1}{3}$ yards. —— $3\frac{2}{3}$ yards. —— 5 yards. —— $5\frac{2}{3}$ yards. —— $6\frac{2}{3}$ yards.

3. Reduce 2 bushels to *fourths.* —— $2\frac{2}{4}$ bu. —— 6 bushels. —— $6\frac{1}{4}$ bushels. —— $7\frac{3}{4}$ bushels. —— $25\frac{2}{4}$ bushels.

4. In $16\frac{5}{12}$ dollars, how many $\frac{1}{12}$ of a dollar?

$\frac{12}{12}$ make 1 dollar: if, therefore, we multiply 16 by 12, that is, multiply the *whole number* by the *denominator*, the product will be the number of 12ths in 16 dollars: $16 \times 12 = 192$, and this, increased by the numerator of the fraction, (5,) evidently gives the whole number of 12ths; that is, $\frac{197}{12}$ of a dollar, *Answer.*

OPERATION.

$16\frac{5}{12}$ dollars.
12

192 = 12ths in 16 dollars, or the *whole* number.
5 = 12ths contained in the *fraction.*

197 = $\frac{197}{12}$, the *answer.*

Hence, *To reduce a mixed number to an improper fraction,*—

RULE: Multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the result over the denominator.

EXAMPLES FOR PRACTICE.

5. What is the improper fraction equivalent to $23\frac{27}{60}$ hours?

Ans. $\frac{1417}{60}$ of an hour.

6. Reduce $730\frac{3}{12}$ shillings to 12ths.

As $\frac{1}{12}$ of a shilling is equal to 1 penny, the question is evidently the same as, In 730 s. 3 d., how many pence?

Ans. $\frac{8763}{12}$ of a shilling; that is, 8763 pence.

7. Reduce $1\frac{16}{20}$, $17\frac{28}{40}$, $8\frac{75}{100}$, $4\frac{703}{1000}$, and $7\frac{316}{450}$ to improper fractions.

8. In $156\frac{17}{24}$ days, how many 24ths of a day?

Ans. $\frac{3761}{24}$ = 3761 hours.

9. In $342\frac{3}{4}$ gallons, how many 4ths of a gallon?

Ans. $\frac{1371}{4}$ of a gallon = 1371 quarts.

To reduce a fraction to its lowest or most simple terms.

¶ **46.** The numerator and the denominator, taken together, are called the *terms of the fraction.*

If $\frac{1}{2}$ of an apple be divided into 2 equal parts, it becomes $\frac{2}{4}$. The effect on the fraction is evidently the same as if we had multiplied both of its terms by 2. In either case, *the parts are made* 2 *times as* MANY *as they were before; but they are only* HALF AS LARGE; for it will take 2 times as many *fourths* to make a whole one as it will take *halves;* and hence it is that $\frac{2}{4}$ is the same in value or quantity as $\frac{1}{2}$.

$\frac{2}{4}$ is 2 parts; and if each of these parts be again divided into 2 equal parts, that is, if both terms of the fraction be multiplied by 2, it becomes $\frac{4}{8}$. Hence, $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, and the reverse of this is evidently true, that $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$.

It follows therefore, *by multiplying or dividing both terms of the fraction by the same number, we change its terms without altering its value.*

Thus, if we reverse the above operation, and divide both terms of the fraction $\frac{4}{8}$ by 2, we obtain its equal, $\frac{2}{4}$; dividing again by 2, we obtain $\frac{1}{2}$, which is the *most simple* form of the fraction, because the terms are the *least* possible by which the fraction can be expressed.

The process of changing $\frac{4}{8}$ into its equal $\frac{1}{2}$ is called *reducing the fraction to its lowest terms.* It consists *in dividing both terms of the fraction by any number which will divide them both without a remainder, and the quotient thence arising in the same manner, and so on, till it appears that no number greater than* 1 *will again divide them.*

A number, which will divide two or more numbers without a remainder, is called a *common divisor,* or *common measure* of those numbers. The greatest number that will do this is called the *greatest common divisor.*

1. What part of an acre are 128 rods?

One rod is $\frac{1}{160}$ of an acre, and 128 rods are $\frac{128}{160}$ of an acre. Let us reduce this fraction to its *lowest terms.* We find, by trial, that 4 will exactly measure both 128 and 160, and, dividing, we change the fraction to its equal $\frac{32}{40}$. Again, we find that 8 is a divisor common to both terms, and, dividing, we reduce the fraction to its equal $\frac{4}{5}$, which is now in its lowest terms, for no greater number than 1 will again measure them. The operation may be presented thus:

$$4\Big)\frac{128}{160} = {}^{8)}\frac{32}{40} = \frac{4}{5} \text{ of an acre, } \textit{Answer.}$$

2. Reduce $\frac{450}{900}$, $\frac{99}{297}$, $\frac{140}{160}$, and $\frac{1644}{2192}$ to their lowest terms.
Ans. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$, and $\frac{3}{4}$.

Note. If any number ends with a cipher, it is evidently divisible by 10. If the two right hand figures are divisible by 4, the whole number is also. If it ends with an even number, it is divisible by 2; if with a 5 or 0, it is divisible by 5.

3. Reduce $\frac{400}{500}$, $\frac{45}{600}$, $\frac{165}{275}$, and $\frac{21}{35}$ to their lowest terms.

¶ **47.** Any fraction may evidently be reduced to its lowest terms by a single division, if we use the *greatest* common divisor of the two terms. The greatest common *measure* of any two numbers may be found by a *sort of trial* easily made. Let the numbers be the two terms of the fraction $\frac{128}{160}$. The common divisor cannot exceed the *less number*, for it must measure it. We will try, therefore, if the *less* number, 128, which measures itself, will also divide or measure 160.

```
128)160(1
    128
    ---
     32)128(4
        128
        ---
```

128 in 160 goes 1 time, and 32 *remain;* 128, therefore, is not a divisor of 160. We will now try whether this *remainder* be not the divisor sought; for if 32 be a divisor of 128, the former divisor, it must also be a divisor of 160, which consists of 128 + 32. 32 in 128 goes 4 times, *without any remainder.* Consequently, 32 *is* a divisor of 128 and 160. And it is evidently the *greatest* common divisor of these numbers; for it must be contained at least *once more* in 160 than in 128, and no number greater than their difference, that is, greater than 32, can do it.

Hence *the rule for finding the greatest common divisor of two numbers*:—Divide the greater number by the less, and that divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain. The last *divisor* will be the greatest common divisor required.

Note. It is evident, that, when we would find the greatest common divisor of *more* than *two* numbers, we may first find the greatest common divisor of *two* numbers, and then of *that* common divisor and one of the *other* numbers, and so on to the last number. Then will the greatest common divisor *last* found be the answer.

4. Find the greatest common divisor of the terms of the fraction $\frac{21}{35}$, and, by it, reduce the fraction to its *lowest* terms.

OPERATION.

```
                 21)35(1
                    21
                    --
                    14)21(1
                       14
                       --
Greatest divis.         7)14(2.
                          14
                          --
```

Then, $7)\frac{21}{35} = \frac{3}{5}$ *Ans.*

5. Reduce $\frac{96}{544}$ to its lowest terms. *Ans.* $\frac{3}{17}$.

Note. Let these examples be wrought by both methods; by several divisors, and also by finding the greatest common divisor.

6. Reduce $\frac{384}{1152}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
7. Reduce $\frac{114}{285}$ to its lowest terms. *Ans.* $\frac{2}{5}$.
8. Reduce $\frac{468}{1184}$ to its lowest terms. *Ans.* $\frac{117}{296}$.
9. Reduce $\frac{1429}{2858}$ to its lowest terms. *Ans.* $\frac{1}{2}$.

To divide a fraction by a whole number.

¶ **48.** 1. If 2 yards of cloth cost $\frac{2}{3}$ of a dollar, what does 1 yard cost? how much is $\frac{2}{3}$ divided by 2?

2. If a cow consume $\frac{3}{4}$ of a bushel of meal in 3 days, how much is that per day? $\frac{3}{4} \div 3 =$ how much?

3. If a boy divide $\frac{4}{5}$ of an orange among 2 boys, how much will he give each one? $\frac{4}{5} \div 2 =$ how much?

4. A boy bought 5 cakes for $\frac{10}{12}$ of a dollar; what did 1 cake cost? $\frac{10}{12} \div 5 =$ how much?

5. If 2 bushels of apples cost $\frac{2}{8}$ of a dollar, what is that per bushel?

1 bushel is the half of 2 bushels; the half of $\frac{2}{8}$ is $\frac{1}{8}$.

Ans. $\frac{1}{8}$ dollar.

6. If 3 horses consume $\frac{12}{13}$ of a ton of hay in a month, what will 1 horse consume in the same time?

$\frac{12}{13}$ are 12 parts; if 3 horses consume 12 such parts in a month, as many times as 3 are contained in 12, so many parts 1 horse will consume. *Ans.* $\frac{4}{13}$ of a ton.

7. If $\frac{25}{28}$ of a barrel of flour be divided equally among 5 families, how much will each family receive?

$\frac{25}{28}$ is 25 parts; 5 into 25 goes 5 times. *Ans.* $\frac{5}{28}$ of a barrel.

The process in the foregoing examples is evidently dividing a fraction by a whole number; and consists, as may be seen, in dividing the *numerator*, (when it can be done without a remainder,) and under the quotient writing the *denominator.* But it not unfrequently happens, that the numerator will not contain the whole number without a remainder.

8. A man divided $\frac{1}{2}$ of a dollar equally among 2 persons; what part of a dollar did he give to each?

$\frac{1}{2}$ of a dollar divided into 2 equal parts will be 4ths.

Ans. He gave $\frac{1}{4}$ of a dollar to each.

9. A mother divided $\frac{1}{2}$ a pie among 4 children; what part of the pie did she give to each? $\frac{1}{2} \div 4 =$ how much?

10. A boy divided $\frac{1}{3}$ of an orange equally among 3 of his companions; what was each one's share? $\frac{1}{3} \div 3 =$ how much?

11. A man divided $\frac{3}{4}$ of an apple equally between 2 children; what part did he give to each? $\frac{3}{4}$ divided by $2 =$ what part of a whole one?

$\frac{3}{4}$ is 3 parts: if each of these parts be divided into 2 equal parts, they will make 6 parts. He may now give 3 parts to one, and 3 to the other: but 4ths divided into 2 equal parts, become 8ths. The parts are now *twice so many*, but they are only *half so large;* consequently, $\frac{3}{8}$ is only half so much as $\frac{3}{4}$. *Ans.* $\frac{3}{8}$ of an apple.

In these last examples, the fraction has been divided by *multiplying the denominator*, without changing the numerator. The reason is obvious; for, by multiplying the denominator by any number, the parts are made so many times *smaller*, since it will take so many more of them to make a whole

one; and if no more of these *smaller* parts be taken than were before taken of the *larger*, that is, if the numerator be not changed, the value of the fraction is evidently made so many times less.

¶ **49.** Hence, we have TWO *ways to divide a fraction by a whole number:*—

I. *Divide the numerator* by the whole number, (if it will contain it without a remainder,) and under the quotient write the denominator.—Otherwise,

II. *Multiply the denominator* by the whole number, and over the product write the numerator.

EXAMPLES FOR PRACTICE.

1. If 7 pounds of coffee cost $\frac{21}{25}$ of a dollar, what is that per pound? $\frac{21}{25} \div 7 =$ how much? *Ans.* $\frac{3}{25}$ of a dollar.

2. If $\frac{19}{20}$ of an acre produce 24 bushels, what part of an acre will produce 1 bushel? $\frac{19}{20} \div 24 =$ how much?

3. If 12 skeins of silk cost $\frac{10}{11}$ of a dollar, what is that a skein? $\frac{10}{11} \div 12 =$ how much?

4. Divide $\frac{8}{9}$ by 16.

Note. When the divisor is a composite number, the intelligent pupil will perceive, that he can first divide by *one* component part, and the quotient thence arising by the *other*; thus he may frequently shorten the operation. In the last example, $16 = 8 \times 2$, and $\frac{8}{9} \div 8 = \frac{1}{9}$, and $\frac{1}{9} \div 2 = \frac{1}{18}$. *Ans.* $\frac{1}{18}$.

5. Divide $\frac{4}{10}$ by 12. Divide $\frac{7}{40}$ by 21. Divide $\frac{36}{48}$ by 24.

6. If 6 bushels of wheat cost \$ $4\frac{7}{8}$, what is it per bushel?

Note. The mixed number may evidently be reduced to an improper fraction, and divided as before.

Ans. $\frac{39}{48} = \frac{13}{16}$ of a dollar, expressing the fraction in its *lowest* terms. (¶ 46.)

7. Divide \$ $4\frac{11}{13}$ by 9. *Quot.* $\frac{7}{13}$ of a dollar.

8. Divide $12\frac{6}{7}$ by 5. *Quot.* $\frac{18}{7} = 2\frac{4}{7}$.

9. Divide $14\frac{3}{4}$ by 8. *Quot.* $1\frac{27}{32}$.

10. Divide $184\frac{1}{2}$ by 7. *Ans.* $26\frac{5}{14}$.

Note. When the mixed number is *large*, it will be most convenient, first, to divide the *whole* number, and then reduce the remainder to an improper fraction; and, after dividing, annex the quotient of the fraction to the quotient of

the whole number; thus, in the last example, dividing $184\frac{1}{2}$ by 7, as in whole numbers, we obtain 26 integers, with $2\frac{1}{2} = \frac{5}{2}$ remainder, which, divided by 7, gives $\frac{5}{14}$, and $26 + \frac{5}{14} = 26\frac{5}{14}$, *Ans.*

11. Divide $2786\frac{1}{4}$ by 6. *Ans.* $464\frac{3}{8}$.
12. How many times is 24 contained in $7646\frac{11}{24}$? *Ans.* $318\frac{347}{576}$.
13. How many times is 3 contained in $462\frac{1}{3}$? *Ans.* $154\frac{1}{9}$.

To multiply a fraction by a whole number.

¶ **50.** 1. If 1 yard of cloth cost $\frac{1}{3}$ of a dollar, what will 2 yards cost? $\frac{1}{3} \times 2 =$ how much?

2. If a cow consume $\frac{1}{4}$ of a bushel of meal in 1 day, how much will she consume in 3 days? $\frac{1}{4} \times 3 =$ how much?

3. A boy bought 5 cakes, at $\frac{2}{7}$ of a dollar each; what did he give for the whole? $\frac{2}{7} \times 5 =$ how much?

4. How much is 2 times $\frac{1}{3}$? —— 3 times $\frac{1}{4}$? —— 2 times $\frac{2}{5}$?

5. Multiply $\frac{2}{7}$ by 3. —— $\frac{3}{8}$ by 2. —— $\frac{1}{8}$ by 7.

6. If a man spend $\frac{3}{8}$ of a dollar per day, how much will he spend in 7 days?

$\frac{3}{8}$ is 3 parts. If he spend 3 such parts in 1 day, he will evidently spend 7 times 3, that is, $\frac{21}{8} = 2\frac{5}{8}$ in 7 days. Hence, we perceive, a fraction *is multiplied by multiplying the numerator, without changing the denominator.*

But it has been made evident, (¶ 49,) that *multiplying the denominator* produces the same effect on the *value* of the fraction, as *dividing the numerator:* hence, also, *dividing the denominator* will produce the same effect on the value of the fraction, as *multiplying the numerator.* In all cases, therefore, where *one of the terms* of the fraction is to be *multiplied*, the same result will be effected by *dividing the other;* and where *one term is to be divided*, the same result may be effected by *multiplying the other.*

This principle, borne distinctly in mind, will frequently enable the pupil to shorten the operations of fractions. Thus, in the following example:

At $\frac{5}{88}$ of a dollar for 1 pound of sugar, what will 11 pounds cost?

Multiplying the numerator by 11, we obtain for the product $\frac{55}{88} = \frac{5}{8}$ of a dollar for the answer.

¶ 51. But, by applying the above principle, and *dividing* the *denominator*, instead of *multiplying* the *numerator*, we at once come to the answer, $\frac{5}{8}$, in its lowest terms. Hence, *there are* TWO *ways to multiply a fraction by a whole number:*—

I. *Divide the denominator* by the whole number, (when it can be done without a remainder,) and over the quotient write the numerator.—Otherwise,

II. *Multiply the numerator* by the whole number, and under the product write the denominator. If then it be an improper fraction, it may be reduced to a whole or mixed number.

EXAMPLES FOR PRACTICE.

1. If 1 man consume $\frac{5}{36}$ of a barrel of flour in a month, how much will 18 men consume in the same time? —— 6 men? —— 9 men? *Ans. to the last*, $1\frac{1}{4}$ barrels.

2. What is the product of $\frac{71}{120}$ multiplied by 40? $\frac{71}{120} \times 40 =$ how much? *Ans.* $23\frac{2}{3}$.

3. Multiply $\frac{13}{144}$ by 12. —— by 18. —— by 21. —— by 36. —— by 48. —— by 60.

Note. When the multiplier is a composite number, the pupil will recollect, (¶ 11,) that he may first multiply by one component part, and that product by the other. Thus, in the last example, the multiplier 60 is equal to 12×5; therefore, $\frac{13}{144} \times 12 = \frac{13}{12}$, and $\frac{13}{12} \times 5 = \frac{65}{12} = 5\frac{5}{12}$, *Ans.*

4. Multiply $5\frac{3}{4}$ by 7. *Ans.* $40\frac{1}{4}$.

Note. It is evident, that the mixed number may be reduced to an improper fraction, and multiplied, as in the preceding examples; but the operation will usually be shorter, to multiply the fraction and whole number *separately*, and add the results together. Thus, in the last example, 7 times 5 are 35; and 7 times $\frac{3}{4}$ are $\frac{21}{4} = 5\frac{1}{4}$, which, added to 35, make $40\frac{1}{4}$, *Ans.*

Or, we may multiply the *fraction* first, and, writing down the fraction, reserve the integers, to be carried to the product of the whole number.

5. What will $9\frac{13}{20}$ tons of hay come to at \$17 per ton? *Ans.* \$$164\frac{1}{20}$.

6. If a man travel $2\frac{6}{10}$ miles in 1 hour, how far will he

travel in 5 hours? —— in 8 hours? —— in 12 hours? —— in 3 days, supposing he travel 12 hours each day?

Ans. to the last, $77\frac{2}{5}$ miles.

Note. The fraction is here reduced to its lowest terms; the same will be done in all following examples.

To multiply a whole number by a fraction.

¶ **52.** 1. If 36 dollars be paid for a piece of cloth, what costs $\frac{3}{4}$ of it? $36 \times \frac{3}{4} =$ how much?

$\frac{3}{4}$ of the quantity will cost $\frac{3}{4}$ of the price; $\frac{3}{4}$ a time 36 dollars, that is, $\frac{3}{4}$ of 36 dollars, implies that 36 be first divided into 4 equal parts, and then that 1 of these parts be taken 3 times; 4 into 36 goes 9 times, and 3 times 9 is 27.

Ans. 27 dollars.

From the above example, it plainly appears, that *the object in multiplying by a fraction, whatever may be the multiplicand, is, to take out of the multiplicand a part, denoted by the multiplying fraction;* and that this operation is composed of *two* others, namely, a *division* by the denominator of the multiplying fraction, and a *multiplication* of the quotient by the numerator. It is matter of indifference, as it respects the *result*, which of these operations precedes the other, for $36 \times 3 \div 4 = 27$, the same as $36 \div 4 \times 3 = 27$.

Hence,—*To multiply by a fraction, whether the multiplicand be a whole number or a fraction,*—

RULE.

Divide the multiplicand by the denominator of the multiplying fraction, and multiply the quotient by the numerator; or, (which will often be found more convenient in practice,) first multiply by the numerator, and divide the product by the denominator.

Multiplication, therefore, when applied to fractions, does not always imply augmentation or increase, as in whole numbers; for, when the multiplier is less than *unity*, it will always require the product to be less than the *multiplicand*, to which it would be only equal if the multiplier were 1.

We have seen, (¶ 10,) that, when two numbers are multiplied together, either of them may be made the multiplier, without affecting the result. In the last example, therefore, instead of multiplying 16 by $\frac{3}{4}$, we may multiply $\frac{3}{4}$ by 16, (¶ 50,) and the result will be the same.

EXAMPLES FOR PRACTICE.

2. What will 40 bushels of corn come to at $\frac{3}{4}$ of a dollar per bushel? $40 \times \frac{3}{4} =$ how much?

3. What will 24 yards of cloth cost at $\frac{3}{8}$ of a dollar per yard? $24 \times \frac{3}{8} =$ how much?

4. How much is $\frac{1}{2}$ of 90? —— $\frac{2}{3}$ of 369? —— $\frac{7}{10}$ of 45?

5. Multiply 45 by $\frac{7}{10}$. Multiply 20 by $\frac{1}{2}$.

To multiply one fraction by another.

¶ **53.** 1. A man, owning $\frac{4}{5}$ of a ticket, sold $\frac{2}{3}$ of his share; what part of the whole ticket did he sell? $\frac{2}{3}$ of $\frac{4}{5}$ is how much?

We have just seen, (¶ 52,) that, to multiply by a fraction, is to *divide the multiplicand* by the *denominator*, and to *multiply* the *quotient* by the numerator. $\frac{4}{5}$ divided by 3, the denominator of the multiplying fraction, (¶ 49,) is $\frac{4}{15}$, which, multiplied by 2, the numerator, (¶ 51,) is $\frac{8}{15}$, *Ans.*

The process, if carefully considered, will be found to consist in *multiplying together the two numerators for a new numerator, and the two denominators for a new denominator.*

EXAMPLES FOR PRACTICE.

2. A man, having $\frac{3}{4}$ of a dollar, gave $\frac{2}{3}$ of it for a dinner; what did the dinner cost him? *Ans.* $\frac{1}{2}$ dollar.

3. Multiply $\frac{7}{8}$ by $\frac{6}{7}$. Multiply $\frac{9}{10}$ by $\frac{2}{7}$. *Product,* $\frac{9}{35}$.

4. How much is $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{3}{4}$?

Note. Fractions like the above, connected by the word *of*, are sometimes called *compound fractions.* The word OF *implies their continual multiplication into each other.*

Ans. $\frac{168}{480} = \frac{7}{20}$.

When there are several fractions to be multiplied continually together, as the *several numerators* are *factors* of the new numerator, and the *several denominators* are *factors* of the new denominator, the operation may be shortened *by dropping those factors which are the same in both terms*, on the principle explained in ¶ 46. Thus, in the last example, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{3}{4}$, we find a 4 and a 3 both among the numerators and among the denominators; therefore we drop them, multiplying together only the remaining numerators, $2 \times 7 = 14$, for a new numerator, and the remaining denominators, $5 \times 8 = 40$, for a new denominator, making $\frac{14}{40} = \frac{7}{20}$, *Ans.* as before.

5. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{7}{8}$ of $\frac{8}{9}$ = how much? *Ans.* $\frac{3}{10}$.

6. What is the continual product of 7, $\frac{1}{2}$, $\frac{2}{7}$ of $\frac{5}{8}$ and $3\frac{1}{3}$?

Note. The integer 7 may be reduced to the form of an improper fraction by writing a unit under it for a denominator, thus, $\frac{7}{1}$. *Ans.* $2\frac{1}{12}$.

7. At $\frac{6}{25}$ of a dollar a yard, what will $\frac{7}{8}$ of a yard of cloth cost?

8. At $6\frac{3}{8}$ dollars per barrel for flour, what will $\frac{7}{16}$ of a barrel cost?

$6\frac{3}{8} = \frac{51}{8}$; then $\frac{51}{8} \times \frac{7}{16} = \frac{357}{128} = \$2\frac{101}{128}$, *Ans.*

9. At $\frac{5}{6}$ of a dollar per yard, what cost $7\frac{3}{4}$ yards? *Ans.* $\$6\frac{11}{24}$.

10. At $\$2\frac{1}{4}$ per yard, what cost $6\frac{5}{8}$ yards? *Ans.* $\$14\frac{29}{32}$.

11. What is the continued product of 3, $\frac{2}{5}$, $\frac{5}{9}$ of $\frac{3}{4}$, $2\frac{5}{7}$, and $\frac{11}{12}$ of $\frac{6}{7}$ of $\frac{4}{5}$? *Ans.* $\frac{209}{245}$.

¶ **54.** *The* RULE *for the multiplication of fractions may now be presented at one view:*—

I. *To multiply a fraction by a whole number, or a whole number by a fraction,—Divide* the *denominator* by the whole number, when it can be done without a remainder; otherwise, *multiply* the *numerator* by it, and under the product write the denominator, which may then be reduced to a whole or mixed number.

II. *To multiply a mixed number by a whole number,*—Multiply the fraction and integers, *separately*, and add their products together.

III. *To multiply one fraction by another,*—Multiply together the *numerators* for a new numerator, and the *denominators* for a new denominator.

Note. If either or both are *mixed numbers*, they may first be reduced to *improper fractions*.

EXAMPLES FOR PRACTICE.

1. At $\$\frac{3}{4}$ per yard, what cost 4 yards of cloth? —— 5 yds.? —— 6 yds.? —— 8 yds.? —— 20 yds.? *Ans. to the last,* $15.

2. Multiply 148 by $\frac{1}{2}$. —— by $\frac{7}{8}$. —— by $\frac{3}{20}$. —— by $\frac{3}{10}$. *Last product,* $44\frac{4}{10}$.

3. If $2\frac{9}{10}$ tons of hay keep 1 horse through the winter,

how much will it take to keep 3 horses the same time? —— 7 horses? —— 13 horses? *Ans. to last*, $37\frac{7}{10}$ tons.

4. What will $8\frac{7}{12}$ barrels of cider come to, at $3 per barrel?

5. At $$14\frac{3}{4}$ per cwt., what will be the cost of 147 cwt.?

6. A owned $\frac{3}{5}$ of a ticket; B owned $\frac{6}{15}$ of the same; the ticket was so lucky as to draw a prize of $1000; what was each one's share of the money?

7. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{4}{5}$. *Product*, $\frac{1}{5}$.

8. Multiply $7\frac{1}{2}$ by $2\frac{1}{15}$. *Product*, $15\frac{1}{2}$.

9. Multiply $\frac{7}{8}$ by $2\frac{2}{3}$. *Product*, $2\frac{1}{3}$.

10. Multiply $\frac{3}{4}$ of 6 by $\frac{2}{9}$. *Product*, 1.

11. Multiply $\frac{3}{4}$ of 2 by $\frac{1}{2}$ of 4. *Product*, 3.

12. Multiply continually together $\frac{1}{8}$ of 8, $\frac{2}{3}$ of 7, $\frac{3}{8}$ of 9, and $\frac{1}{7}$ of 10. *Product*, 20.

13. Multiply 1000000 by $\frac{5}{9}$. *Product*, $555555\frac{5}{9}$.

To divide a whole number by a fraction.

¶ **55.** We have already shown (¶ 49) how to divide a fraction by a whole number; we now proceed to show how to divide a whole number by a fraction.

1. A man divided $9 among some poor people, giving them $\frac{3}{4}$ of a dollar each; how many were the persons who received the money? $9 \div \frac{3}{4} =$ how many?

1 dollar is $\frac{4}{4}$, and 9 dollars is 9 times as many, that is, $\frac{36}{4}$; then $\frac{3}{4}$ is contained in $\frac{36}{4}$ as many times as 3 is contained in 36. *Ans.* 12 persons.

That is,—*Multiply the dividend by the denominator of the dividing fraction*, (thereby reducing the dividend to parts of the same magnitude as the divisor,) *and divide the product by the numerator.*

2. How many times is $\frac{3}{5}$ contained in 8? $8 \div \frac{3}{5} =$ how many?

OPERATION.

8 *Dividend.*
5 *Denominator.*

Numerator, 3)40

Quotient, $13\frac{1}{3}$ *times, the Answer.*

To multiply by a fraction, we have seen, (¶ 52,) implies two operations—*a division and a multiplication;* so, also, to divide by a fraction implies two operations—*a multiplication and a division.*

¶ **56.** **Division is the reverse of multiplication.**

To multiply by a fraction, whether the multiplicand be a whole number or a fraction, as has been already shown, (¶ 52,) we *divide* by the denominator of the multiplying fraction, and *multiply the quotient* by the numerator.

To divide by a fraction, whether the dividend be a whole number or a fraction, we *multiply* by the denominator of the dividing fraction, and *divide the product* by the numerator.

Note. In either case, it is matter of indifference, as it respects the result, which of these operations precedes the other; but in *practice* it will frequently be more convenient, that the multiplication precede the division.

12 multiplied by $\frac{3}{4}$, the product is 9.

In multiplication, the multiplier being *less* than unity, or 1, will require the product to be *less* than the multiplicand, (¶ 52,) to which it is only equal when the multiplier is 1, and greater when the multiplier is *more* than 1.

12 divided by $\frac{3}{4}$, the quotient is 16.

In division, the divisor being *less* than unity, or 1, will be contained *a greater number of times;* consequently will require the quotient to be *greater* than the dividend, to which it will be equal when the divisor is 1, and *less* when the divisor is *more* than 1.

EXAMPLES FOR PRACTICE.

1. How many times is $\frac{1}{2}$ contained in 7? $7 \div \frac{1}{2} =$ how many?

2. How many times can I draw $\frac{1}{4}$ of a gallon of wine out of a cask containing 26 gallons?

3. Divide 3 by $\frac{3}{4}$. —— 6 by $\frac{2}{3}$. —— 10 by $\frac{2}{5}$.

4. If a man drink $\frac{9}{16}$ of a quart of rum a day, how long will 3 gallons last him?

5. If $2\frac{3}{4}$ bushels of oats sow an acre, how many acres will 22 bushels sow? $22 \div 2\frac{3}{4} =$ how many times?

Note. Reduce the mixed number to an improper fraction, $2\frac{3}{4} = \frac{11}{4}$. *Ans.* 8 acres.

6. At \$$4\frac{2}{5}$ a yard, how many yards of cloth may be bought for \$37? *Ans.* $8\frac{9}{22}$ yards.

7. How many times is $\frac{96}{103}$ contained in 84? *Ans.* $90\frac{1}{8}$ times.

8. How many times is $\frac{36}{5}$ contained in 6?

Ans. $\frac{5}{6}$ of 1 time.

9. How many times is $8\frac{3}{8}$ contained in 53?

Ans. $6\frac{22}{67}$ times.

10. At $\frac{5}{6}$ of a dollar for building 1 rod of stone wall, how many rods may be built for \$87? $87 \div \frac{5}{6} =$ how many times?

To divide one fraction by another.

¶ **57.** 1. At $\frac{2}{3}$ of a dollar per bushel, how much rye may be bought for $\frac{3}{5}$ of a dollar? $\frac{2}{3}$ is contained in $\frac{3}{5}$ how many times?

Had the rye been 2 *whole* dollars per bushel, instead of $\frac{2}{3}$ of a dollar, it is evident, that $\frac{3}{5}$ of a dollar must have been divided by 2, and the quotient would have been $\frac{3}{10}$; but the divisor is 3ds, and 3ds will be contained 3 times where a like number of whole ones are contained 1 time; consequently the quotient $\frac{3}{10}$ is 3 times too *small*, and must therefore, in order to give the true answer, be multiplied by 3, that is, by the denominator of the divisor; 3 times $\frac{3}{10} = \frac{9}{10}$ bush. *Ans.*

The process is that already described, ¶ 55 and ¶ 56. If carefully considered, it will be perceived, that the *numerator* of the divisor is multiplied into the denominator of the dividend, and the *denominator* of the divisor into the *numerator* of the dividend; wherefore, in practice, it will be more convenient to *invert the divisor;* thus, $\frac{2}{3}$ inverted becomes $\frac{3}{2}$; then *multiply together the two upper terms for a numerator, and the two lower terms for a denominator*, as in the multiplication of one fraction by another. Thus, in the above example, $\frac{3 \times 3}{2 \times 5} = \frac{9}{10}$, as before.

EXAMPLES FOR PRACTICE.

2. At $\frac{1}{4}$ of a dollar per bushel for apples, how many bushels may be bought for $\frac{7}{8}$ of a dollar? How many times is $\frac{1}{4}$ contained in $\frac{7}{8}$? *Ans.* $3\frac{1}{2}$ bushels.

3. If $\frac{7}{8}$ of a yard of cloth cost $\frac{3}{5}$ of a dollar, what is that per yard? It will be recollected, (¶ 24,) that when the *cost* of any quantity is given to find the *price* of a unit, we *divide* the *cost* by the *quantity*. Thus, $\frac{3}{5}$ (the cost) divided by $\frac{7}{8}$ (the quantity) will give the price of 1 yard.

Ans. $\frac{24}{35}$ of a dollar per yard.

PROOF. If the work be right, (¶ 16, "Proof,") the product of the quotient into the divisor will be equal to the dividend; thus, $\frac{24}{35} \times \frac{7}{8} = \frac{3}{5}$. This, it will be perceived, is multiplying the price of one yard ($\frac{24}{35}$) by the quantity ($\frac{7}{8}$) to find the cost ($\frac{3}{5}$;) and is, in fact, reversing the question, thus, If the price of 1 yard be $\frac{24}{35}$ of a dollar, what will $\frac{7}{8}$ of a yard cost? *Ans.* $\frac{3}{5}$ of a dollar.

Note. Let the pupil be required to reverse and prove the succeeding examples in the same manner.

4. How many bushels of apples, at $\frac{3}{16}$ of a dollar per bushel, may be bought for $\frac{7}{8}$ of a dollar? *Ans.* $4\frac{2}{3}$ bushels.

5. If $4\frac{3}{5}$ pounds of butter serve a family 1 week, how many weeks will $36\frac{7}{8}$ pounds serve them?

The mixed numbers, it will be recollected, may be reduced to improper fractions. *Ans.* $8\frac{3}{184}$ weeks.

6. Divide $\frac{1}{2}$ by $\frac{1}{2}$. *Quot.* 1. Divide $\frac{1}{2}$ by $\frac{1}{4}$. *Quot.* 2.

7. Divide $\frac{3}{4}$ by $\frac{1}{4}$. *Quot.* 3. Divide $\frac{7}{8}$ by $\frac{9}{10}$. *Quot.* $\frac{35}{36}$.

8. Divide $2\frac{1}{4}$ by $1\frac{1}{2}$. *Quot.* $1\frac{1}{2}$. Divide $10\frac{3}{8}$ by $2\frac{1}{8}$. *Quot.* $4\frac{15}{17}$.

9. How many times is $\frac{1}{10}$ contained in $\frac{2}{5}$? *Ans.* 4 times.

10. How many times is $\frac{3}{7}$ contained in $4\frac{7}{8}$? *Ans.* $11\frac{3}{8}$ times.

11. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{7}{8}$ of $\frac{1}{7}$. *Quot.* 4.

¶ 58. *The* RULE *for division of fractions may now be presented at one view:—*

I. *To divide a fraction by a whole number,—Divide* the *numerator* by the whole number, when it can be done without a remainder, and under the quotient write the denominator; otherwise, *multiply* the *denominator* by it, and over the product write the numerator.

II. *To divide a whole number by a fraction,—Multiply* the dividend by the *denominator* of the fraction, and *divide* the product by the *numerator.*

III. *To divide one fraction by another,*—Invert the *divisor,* and multiply together the two upper terms for a numerator, and the two lower terms for a denominator.

Note. If either or both are mixed numbers, they may be reduced to improper fractions.

EXAMPLES FOR PRACTICE.

1. If 7 lb. of sugar cost $\frac{63}{100}$ of a dollar, what is it per pound? $\frac{63}{100} \div 7 =$ how much? $\frac{1}{7}$ of $\frac{63}{100}$ is how much?

2. At \$ $\frac{1}{3}$ for $\frac{3}{8}$ of a barrel of cider, what is that per barrel?

3. If 4 pounds of tobacco cost $\frac{7}{8}$ of a dollar, what does 1 pound cost?

4. If $\frac{7}{8}$ of a yard cost \$ 4, what is the price per yard?

5. If $14\frac{3}{8}$ yards cost \$ 75, what is the price per yard? *Ans.* $5\frac{5}{23}$.

6. At $31\frac{1}{2}$ dollars for $10\frac{1}{2}$ barrels of cider, what is that per barrel? *Ans.* \$ 3.

7. How many times is $\frac{3}{8}$ contained in 746? *Ans.* $1989\frac{1}{3}$.

8. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{4}$. *Quot.* $\frac{4}{9}$. Divide $\frac{7}{8}$ by $\frac{4}{7}$ of $\frac{2}{5}$. *Quot.* $3\frac{53}{64}$.

9. Divide $\frac{1}{2}$ of $\frac{4}{5}$ by $\frac{5}{6}$ of $\frac{2}{3}$. *Quot.* $\frac{18}{25}$.

10. Divide $\frac{1}{5}$ of 4 by $\frac{4}{15}$. *Quot.* 3.

11. Divide $4\frac{5}{9}$ by $\frac{5}{9}$ of 4. *Quot.* $2\frac{1}{20}$.

12. Divide $\frac{5}{9}$ of 4 by $4\frac{5}{9}$. *Quot.* $\frac{20}{41}$.

ADDITION AND SUBTRACTION OF FRACTIONS.

¶ 59. 1. A boy gave to one of his companions $\frac{2}{8}$ of an orange, to another $\frac{4}{8}$, to another $\frac{1}{8}$; what part of an orange did he give to all? $\frac{2}{8} + \frac{4}{8} + \frac{1}{8} =$ how much? *Ans.* $\frac{7}{8}$.

2. A cow consumes, in 1 month, $\frac{2}{15}$ of a ton of hay; a horse, in the same time, consumes $\frac{4}{15}$ of a ton; and a pair of oxen, $\frac{6}{15}$; how much do they all consume? how much more does the horse consume than the cow? —— the oxen than the horse? $\frac{2}{15} + \frac{4}{15} + \frac{6}{15} =$ how much? $\frac{4}{15} - \frac{2}{15} =$ how much? $\frac{6}{15} - \frac{4}{15} =$ how much?

3. $\frac{1}{3} + \frac{2}{3} + \frac{1}{3} =$ how much? $\frac{3}{4} - \frac{1}{4} =$ how much?

4. $\frac{1}{20} + \frac{7}{20} + \frac{9}{20} + \frac{14}{20} + \frac{2}{20} =$ how much? $\frac{14}{18} - \frac{3}{18} =$ how much?

5. A boy, having $\frac{3}{4}$ of an apple, gave $\frac{1}{4}$ of it to his sister; what part of the apple had he left? $\frac{3}{4} - \frac{1}{4} =$ how much?

When the denominators of two or more fractions are *alike*, (as in the foregoing examples,) they are said to have a *common* denominator. The parts are then in the same denomination, and, consequently, of the same magnitude or value. It is evident, therefore, that they may be added or subtracted, by adding or subtracting their *numerators*, that is, the number of their parts, care being taken to write under the result their proper denominator. Thus, $\frac{4}{17}+\frac{8}{17}=\frac{12}{17}$; $\frac{3}{8}-\frac{2}{8}=\frac{1}{8}$.

6. A boy, having an orange, gave $\frac{3}{4}$ of it to his sister, and $\frac{1}{8}$ of it to his brother; what part of the orange did he give away?

4ths and 8ths, being parts of *different* magnitudes, or value, cannot be added together. We must therefore first reduce them to parts of the *same* magnitude, that is, to a *common denominator*. $\frac{3}{4}$ are 3 parts. If each of these parts be divided into 2 equal parts, that is, if we multiply both terms of the fraction $\frac{3}{4}$ by 2, (¶ 46,) it will be changed to $\frac{6}{8}$; then $\frac{6}{8}$ and $\frac{1}{8}$ are $\frac{7}{8}$. *Ans.* $\frac{7}{8}$ of an orange.

7. A man had $\frac{2}{3}$ of a hogshead of molasses in one cask, and $\frac{3}{5}$ of a hogshead in another; how much more in one cask than in the other?

Here, 3ds cannot be so divided as to become 5ths, nor can 5ths be so divided as to become 3ds; but if the 3ds be each divided into 5 equal parts, and the 5ths each into 3 equal parts, they will all become 15ths. The $\frac{2}{3}$ will become $\frac{10}{15}$, and the $\frac{3}{5}$ will become $\frac{9}{15}$; then, $\frac{9}{15}$ taken from $\frac{10}{15}$ leaves $\frac{1}{15}$, *Ans.*

¶ **60.** From the very process of dividing each of the parts, that is, of increasing the denominators by *multiplying* them, it follows, that *each denominator* must be a *factor* of the *common denominator;* now, multiplying all the denominators together will evidently produce such a number.

Hence, *To reduce fractions of different denominators to equivalent fractions, having a common denominator*,—Rule: Multiply together all the denominators for a *common denominator*, and, as by this process each denominator is multiplied by all the others, so, to retain the value of each fraction, multiply each numerator by all the denominators, except its own, for a new numerator, and under it write the common denominator.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ to fractions of equal value, having a common denominator.

$3 \times 4 \times 5 = 60$, the common denominator.
$2 \times 4 \times 5 = 40$, the new numerator for the first fraction
$3 \times 3 \times 5 = 45$, the new numerator for the second fraction.
$3 \times 4 \times 4 = 48$, the new numerator for the third fraction.

The new fractions, therefore, are $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$. By an inspection of the operation, the pupil will perceive, that the numerator and denominator of each fraction have been multiplied by the same numbers; consequently, (¶ 46,) that their value has not been altered.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$ and $\frac{4}{5}$ to equivalent fractions, having a common denominator. *Ans.* $\frac{120}{240}$, $\frac{160}{240}$, $\frac{210}{240}$, $\frac{192}{240}$.

3. Reduce to equivalent fractions of a common denominator, and add together, $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{1}{4}$.

Ans. $\frac{20}{60} + \frac{36}{60} + \frac{15}{60} = \frac{71}{60} = 1\frac{11}{60}$, *Amount.*

4. Add together $\frac{3}{4}$ and $\frac{6}{7}$. *Amount*, $1\frac{17}{28}$.

5. What is the amount of $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{5}$?

Ans. $\frac{247}{210} = 1\frac{37}{210}$.

6. What are the fractions of a common denominator equivalent to $\frac{3}{4}$ and $\frac{5}{6}$? *Ans.* $\frac{18}{24}$ and $\frac{20}{24}$, or $\frac{9}{12}$ and $\frac{10}{12}$.

We have already seen, (¶ 59, ex. 7,) that the *common* denominator may be *any* number, of which each *given* denominator is a factor, that is, any number which may be divided by *each of them* without a remainder. Such a number is called a *common multiple* of all its common divisors, and the *least* number that will do this is called their *least* common multiple; therefore, the *least common denominator* of any fractions is the *least common multiple of all their denominators.* Though the rule already given will always find a *common* multiple of the given denominators, yet it will not always find their *least* common multiple. In the last example, 24 is evidently a common multiple of 4 and 6, for it will exactly measure both of them; but 12 will do the same, and as 12 is the *least* number that will do this, it is the *least* common multiple of 4 and 6. It will therefore be convenient to have a *rule* for finding this least common multiple. Let the numbers be 4 and 6.

It is evident, that one number is a multiple of another, when the former contains all the factors of the latter. The

factors of 4 are 2 and 2, $(2 \times 2 = 4.)$ The factors of 6 are 2 and 3, $(2 \times 3 = 6.)$ Consequently, $2 \times 2 \times 3 = 12$ contains the factors of 4, that is, 2×2; and also contains the factors of 6, that is, 2×3. 12, then, is a common multiple of 4 and 6, and it is the *least* common multiple, because it does not contain *any factor*, except those which make up the numbers 4 and 6; nor either of those repeated more than is necessary to produce 4 and 6. Hence it follows, that when any two numbers have a factor common to both, it may be once omitted; thus, 2 is a factor common both to 4 and 6, and is consequently once omitted.

¶ **61.** On this principle is founded the RULE *for finding the least common multiple of two or more numbers.* Write down the numbers in a line, and divide them by any number that will measure two or more of them; and write the quotients and undivided numbers in a line beneath. Divide this line as before, and so on, until there are no two numbers that can be measured by the same divisor; then the continual product of all the divisors and numbers in the last line will be the least common multiple required.

Let us apply the rule to find the least common multiple of 4 and 6.

$$\begin{array}{r|l} 2 & 4 \cdot 6 \\ \hline & 2 \cdot 3 \end{array}$$

4 and 6 may both be measured by 2; the quotients are 2 and 3. There is no number greater than 1, which will measure 2 and 3. Therefore, $2 \times 2 \times 3 = 12$ is the least common multiple of 4 and 6.

If the pupil examine the process, he will see that the divisor 2 is a factor common to 4 and 6, and that dividing 4 by this factor gives for a quotient its other factor, 2. In the same manner, dividing 6 gives its other factor, 3. Therefore the divisor and quotients make up all the factors of the two numbers, which, multiplied together, must give the common multiple.

7. Reduce $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{6}$ to equivalent fractions of the least common denominator.

OPERATION.

$$\begin{array}{r|l} 2 & 4 \cdot 2 \cdot 3 \cdot 6 \\ \hline 3 & 2 \cdot 1 \cdot 3 \cdot 3 \\ \hline & 2 \cdot 1 \cdot 1 \cdot 1 \end{array}$$

Then, $2 \times 3 \times 2 = 12$, least common denominator. It is evident we need not multiply by the 1s, as this would not alter the number.

To find the new numerators, that is, how many 12ths each fraction is, we may take $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{6}$ of 12. Thus:

$$\left.\begin{array}{l} \frac{3}{4} \text{ of } 12 = 9 \\ \frac{1}{2} \text{ of } 12 = 6 \\ \frac{2}{3} \text{ of } 12 = 8 \\ \frac{1}{6} \text{ of } 12 = 2 \end{array}\right\} \begin{array}{l}\text{New numerators, which,} \\ \text{written over the com-} \\ \text{mon denominators, give}\end{array} \left\{\begin{array}{l} \frac{9}{12} = \frac{3}{4} \\ \frac{6}{12} = \frac{1}{2} \\ \frac{8}{12} = \frac{2}{3} \\ \frac{2}{12} = \frac{1}{6} \end{array}\right.$$

Ans. $\frac{9}{12}$, $\frac{6}{12}$, $\frac{8}{12}$ and $\frac{2}{12}$.

8. Reduce $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{5}{6}$ to fractions having the least common denominator, and add them together.

Ans. $\frac{12}{24} + \frac{9}{24} + \frac{20}{24} = \frac{41}{24} = 1\frac{17}{24}$, amount.

9. Reduce $\frac{1}{6}$ and $\frac{1}{9}$ to fractions of the least common denominator, and subtract one from the other.

Ans. $\frac{3}{18} - \frac{2}{18} = \frac{1}{18}$, difference.

10. What is the least number that 3, 5, 8 and 10 will measure? *Ans.* 120.

11. There are 3 pieces of cloth, one containing $7\frac{3}{4}$ yards, another $13\frac{5}{6}$ yards, and the other $15\frac{7}{8}$ yards; how many yards in the 3 pieces.

Before adding, reduce the fractional parts to their least common denominator; this being done, we shall have,

$$\left.\begin{array}{r} 7\frac{3}{4} = 7\frac{18}{24} \\ 13\frac{5}{6} = 13\frac{20}{24} \\ 15\frac{7}{8} = 15\frac{21}{24} \\ \hline \textit{Ans. } 37\frac{11}{24}. \end{array}\right\}$$

Adding together all the 24ths, viz. $18 + 20 + 21$, we obtain 59, that is, $\frac{59}{24} = 2\frac{11}{24}$. We write down the fraction $\frac{11}{24}$ under the other fractions, and reserve the 2 integers to be carried to the amount of the other integers, making in the whole $37\frac{11}{24}$, *Ans.*

12. There was a piece of cloth containing $34\frac{3}{8}$ yards, from which were taken $12\frac{2}{3}$ yards; how much was there left?

$$\begin{array}{r} 34\frac{3}{8} = 34\frac{9}{24} \\ 12\frac{2}{3} = 12\frac{16}{24} \\ \hline \textit{Ans. } 21\frac{17}{24} \textit{ yds.} \end{array}$$

We cannot take 16 twenty-fourths ($\frac{16}{24}$) from 9 twenty-fourths, ($\frac{9}{24}$;) we must, therefore, borrow 1 integer, = 24 twenty-fourths, ($\frac{24}{24}$,) which, with $\frac{9}{24}$, makes $\frac{33}{24}$; we can now take $\frac{16}{24}$ from $\frac{33}{24}$, and there will remain $\frac{17}{24}$; but, as we borrowed, so also we must carry 1 to the 12, which makes it 13, and 13 from 34 leaves 21. *Ans.* $21\frac{17}{24}$.

13. What is the amount of $\frac{1}{2}$ of $\frac{3}{4}$ of a yard, $\frac{2}{3}$ of a yard, and $\frac{1}{5}$ of 2 yards?

Note. The compound fraction may be reduced to a *simple* fraction; thus, $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$; and $\frac{1}{5}$ of $2 = \frac{2}{5}$; then, $\frac{3}{8} + \frac{2}{3} + \frac{2}{5} = \frac{173}{120} = 1\frac{53}{120}$ yds., *Ans.*

¶ 62. From the foregoing examples we derive the following RULE:—*To add or subtract fractions*, add or subtract their *numerators*, when they have a common denominator; otherwise, they must first be reduced to a common denominator.

Note. Compound fractions must be reduced to simple fractions before adding or subtracting.

EXAMPLES FOR PRACTICE.

1. What is the amount of $\frac{6}{7}$, $4\frac{2}{3}$ and 12? *Ans.* $17\frac{11}{21}$.

2. A man bought a ticket, and sold $\frac{3}{4}$ of $\frac{1}{2}$ of it; what part of the ticket had he left? *Ans.* $\frac{5}{8}$.

3. Add together $\frac{1}{2}$, $\frac{5}{8}$, $\frac{1}{4}$, $\frac{7}{10}$, $\frac{1}{5}$ and $\frac{14}{20}$. *Amount*, $2\frac{39}{40}$.

4. What is the difference between $14\frac{8}{11}$ and $16\frac{7}{33}$? *Ans.* $1\frac{16}{33}$.

5. From $1\frac{1}{2}$ take $\frac{3}{4}$. *Remainder*, $\frac{3}{4}$.

6. From 3 take $\frac{1}{3}$. *Rem.* $2\frac{2}{3}$.

7. From $147\frac{1}{3}$ take $48\frac{4}{9}$. *Rem.* $98\frac{8}{9}$.

8. From $\frac{1}{4}$ of $\frac{4}{10}$ take $\frac{1}{2}$ of $\frac{2}{47}$. *Rem.* $\frac{37}{470}$.

9. Add together $112\frac{1}{2}$, $311\frac{2}{3}$, and $1000\frac{3}{4}$.

10. Add together 14, 11, $4\frac{2}{3}$, $\frac{1}{18}$ and $\frac{1}{2}$.

11. From $\frac{3}{4}$ take $\frac{1}{2}$. From $\frac{7}{8}$ take $\frac{3}{4}$.

12. What is the difference between $\frac{1}{2}$ and $\frac{1}{3}$? $\frac{2}{3}$ and $\frac{1}{2}$? $\frac{7}{8}$ and $\frac{2}{3}$? $\frac{4}{5}$ and $\frac{3}{4}$? $\frac{5}{6}$ and $\frac{4}{5}$? $\frac{5}{8}$ and $\frac{3}{4}$?

13. How much is $1-\frac{1}{4}$? $1-\frac{1}{2}$? $1-\frac{3}{8}$? $1-\frac{5}{8}$? $2-\frac{2}{3}$? $2-\frac{4}{7}$? $2\frac{1}{4}-\frac{2}{3}$? $3\frac{4}{5}-\frac{1}{10}$? $1000-\frac{1}{10}$?

REDUCTION OF FRACTIONS.

¶ 63. We have seen, (¶ 33,) that integers of one denomination may be reduced to integers of another denomination. It is evident, that *fractions* of one denomination, after the same manner, and by the same *rules*, may be reduced to *fractions* of another denomination; that is, *fractions*, like integers, may be brought into lower denominations by *multiplication*, and into higher denominations by *division*.

To reduce higher into LOWER *denominations.*

(RULE. See ¶ 34.)

1. Reduce $\frac{1}{280}$ of a pound to pence, or the fraction of a penny.

Note. Let it be recollected, that a fraction is *multiplied* either by *dividing* its *denominator*, or by *multiplying* its *numerator*.

$\frac{1}{280}$ £ $\times$ 20 $= \frac{1}{14}$ s. $\times$ 12 $= \frac{6}{7}$ d. *Ans.*

Or thus: $\frac{1}{280}$ of $\frac{20}{1}$ of $\frac{12}{1} = \frac{240}{280} = \frac{6}{7}$ of a penny, *Ans.*

3. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing.

$\frac{1}{1280}$ £. $\times$ 20 $= \frac{20}{1280}$ s. $\times$ 12 $= \frac{240}{1280}$ d. $\times$ 4 $= \frac{960}{1280} = \frac{3}{4}$ q.

Or thus:

Num. 1
20 s. in 1 £.
—
20
12 d. in 1 s.
—
240
4 q. in 1 d.
—
960

Then, $\frac{960}{1280} = \frac{3}{4}$ q. *Ans.*

5. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny.

7. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound.

Consult ¶ 34, ex. 11.

9. Reduce $\frac{3}{4}$ of a moidore, at 36 s. to the fraction of a guinea.

11. Reduce $\frac{2}{27}$ of a pound, Troy, to the fraction of an ounce.

To reduce lower into HIGHER *denominations.*

(RULE. See ¶ 34.)

2. Reduce $\frac{6}{7}$ of a penny to the fraction of a pound.

Note. Division is performed either by *dividing* the *numerator*, or by *multiplying* the *denominator*.

$\frac{6}{7}$ d. $\div$ 12 $= \frac{1}{14}$ s. $\div$ 20 $= \frac{1}{280}$ £. *Ans.*

Or thus: $\frac{6}{7}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{6}{1680} = \frac{1}{280}$ £. *Ans.*

4. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound.

$\frac{3}{4}$ q. $\div$ 4 $= \frac{3}{16}$ d. $\div$ 12 $= \frac{3}{192}$ s. $\div$ 20 $= \frac{3}{3840} = \frac{1}{1280}$ £.

Or thus:

Denom. 4
4 q. in 1 d.
—
16
12 d. in 1 s.
—
192
20 s. in 1 £.
—
3840

Then, $\frac{3}{3840} = \frac{1}{1280}$ £. *Ans.*

6. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea.

8. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea.

10. Reduce $\frac{27}{28}$ of a guinea to the fraction of a moidore.

12. Reduce $\frac{8}{9}$ of an ounce to the fraction of a pound Troy.

13. Reduce $\frac{1}{28}$ of a pound, avoirdupois, to the fraction of an ounce.

14. Reduce $\frac{4}{7}$ of an ounce to the fraction of a pound avoirdupois.

15. A man has $\frac{1}{728}$ of a hogshead of wine; what part is that of a pint?

16. A man has $\frac{9}{13}$ of a pint of wine; what part is that of a hogshead?

17. A cucumber grew to the length of $\frac{1}{3960}$ of a mile; what part is that of a foot?

18. A cucumber grew to the length of 1 foot 4 inches $=\frac{16}{12}=\frac{4}{3}$ of a foot; what part is that of a mile?

19. Reduce $\frac{2}{3}$ of $\frac{1}{6}$ of a pound to the fraction of a shilling.

20. $\frac{20}{27}$ of a shilling is $\frac{2}{3}$ of what fraction of a pound?

21. Reduce $\frac{1}{8}$ of $\frac{2}{11}$ of 3 pounds to the fraction of a penny.

22. $\frac{180}{11}$ of a penny is $\frac{1}{8}$ of what fraction of 3 pounds? $\frac{180}{11}$ of a penny is $\frac{2}{11}$ of what part of 3 pounds? $\frac{180}{11}$ of a penny is $\frac{1}{8}$ of $\frac{2}{11}$ of how many pounds?

¶ **64.** It will frequently be required *to find the value of a fraction*, that is, *to reduce a fraction to integers of less denominations.*

1. What is the value of $\frac{2}{3}$ of a pound? In other words, Reduce $\frac{2}{3}$ of a pound to shillings and pence.

$\frac{2}{3}$ of a pound is $\frac{40}{3}=13\frac{1}{3}$ shillings; it is evident from $\frac{1}{3}$ of a shilling may be obtained some pence; $\frac{1}{3}$ of a shilling is $\frac{12}{3}=4$ d. That is,—*Multiply the numerator by that number which will reduce it to the next less denomination, and divide the product by the denominator; if there be a remainder, multiply and divide as before, and so on; the several quotients, placed one after another, in their order, will be the answer.*

It will frequently be required to reduce integers to the fraction of a greater denomination.

2. Reduce 13 s. 4 d. to the fraction of a pound.

13 s. 4 d. is 160 pence; there are 240 pence in a pound; therefore, 13 s. 4 d. is $\frac{160}{240}=\frac{2}{3}$ of a pound. That is,—*Reduce the given sum or quantity to the least denomination mentioned in it, for a numerator; then reduce an integer of that greater denomination* (to a fraction of which it is required to reduce the given sum or quantity) *to the same denomination, for a denominator, and they will form the fraction required.*

EXAMPLES FOR PRACTICE.

3. What is the value of $\frac{3}{8}$ of a shilling?

OPERATION.

```
Numer.    3
         12
         --
Denom. 8)36(4 d. 2 q.   Ans.
         32
         --
          4
          4
         --
         16(2 q.
         16
         --
```

EXAMPLES FOR PRACTICE.

4. Reduce 4 d. 2 q. to the fraction of a shilling.

OPERATION.

```
4 d. 2 q.        1 s.
4               12
--              --
18 Numer.       12
                 4
                --
                48 Denom.
```

$\frac{18}{48} = \frac{3}{8}$ *Ans.*

5. What is the value of $\frac{3}{5}$ of a pound Troy?

6. Reduce 7 oz. 4 pwt. to the fraction of a pound Troy.

7. What is the value of $\frac{5}{9}$ of a pound avoirdupois?

8. Reduce 8 oz. $14\frac{2}{9}$ dr. to the fraction of a pound avoirdupois.

Note. Both the numerator and the denominator must be reduced to 9ths of a dr.

9. $\frac{4}{5}$ of a month is how many days, hours, and minutes?

10. 3 weeks, 1 d. 9 h. 36 m. is what fraction of a month?

11. Reduce $\frac{4}{7}$ of a mile to its proper quantity.

12. Reduce 4 fur. 125 yds. 2 ft. 1 in. $2\frac{1}{7}$ bar. to the fraction of a mile.

13. Reduce $\frac{7}{16}$ of an acre to its proper quantity.

14. Reduce 1 rood 30 poles to the fraction of an acre.

15. What is the value of $\frac{15}{16}$ of a dollar in shillings, pence, &c.?

16. Reduce 5 s. $7\frac{1}{2}$ d. to the fraction of a dollar.

17. What is the value of $\frac{9}{10}$ of a yard?

18. Reduce 2 ft. 8 in. $1\frac{1}{5}$ b. to the fraction of a yard.

19. What is the value of $\frac{3}{13}$ of a ton?

20. Reduce 4 cwt. 2 qr. 12 lb. 14 oz. $12\frac{4}{13}$ dr. to the fraction of a ton.

Note. Let the pupil be required to reverse and prove the following examples:

21. What is the value of $\frac{3}{14}$ of a guinea?

22. Reduce 3 roods $17\frac{1}{2}$ poles to the fraction of an acre.

23. A man bought 27 gal. 3 qts. 1 pt. of molasses; what part is that of a hogshead?

24. A man purchased $\frac{5}{13}$ of 7 cwt. of sugar; how much sugar did he purchase?

25. 13 h. 42 m. $51\frac{3}{7}$ s. is what part or fraction of a day?

SUPPLEMENT TO FRACTIONS.

QUESTIONS.

1. What are *fractions*? 2. Whence is it that the parts into which any thing or any number may be divided, take their name? 3. How are fractions *represented* by figures? 4. What is the number *above* the line called?—Why is it so called? 5. What is the number *below* the line called? —Why is it so called?—What does it show? 6. What is it which determines the *magnitude* of the parts?—Why? 7. What is a *simple* or *proper* fraction? —— an *improper* fraction? —— a *mixed* number? 8. How is an improper fraction reduced to a whole or mixed number? 9. How is a mixed number reduced to an improper fraction? —— a whole number? 10. What is understood by *the terms of the fraction*? 11. How is a fraction reduced to its most *simple* or *lowest* terms? 12. What is understood by a common divisor? —— by the greatest common divisor? 13. How is it found? 14. How many ways are there to multiply a fraction by a whole number? 15. How does it appear, that *dividing the denominator multiplies the fraction*? 16. How is a *mixed* number multiplied? 17. What is implied in multiplying by a fraction? 18. Of how many operations does it consist?—What are they? 19. When the multiplier is *less* than a unit, what is the product compared with the multiplicand? 20. How do you multiply a whole number by a fraction? 21. How do you multiply one fraction by another? 22. How do you multiply a mixed number by a mixed number? 23. How does it appear, that in multiplying both terms of the fraction by the same number the value of the fraction is not altered? 24. How many ways are there to divide a fraction by a whole number?—What are they? 25. How does it appear that a *fraction is divided* by *multiplying its denominator*? 26. How does *dividing* by a

fraction differ from *multiplying* by a fraction? 27. When the *divisor* is *less* than a unit, what is the quotient compared with the dividend? 28. What is understood by a *common* denominator? —— the *least* common denominator? 29. How does it appear, that each *given* denominator must be a factor of the *common* denominator? 30. How is the common denominator to two or more fractions found? 31. What is understood by a *multiple*? —— by a *common multiple*? —— by the *least* common multiple?—What is the process of finding it? 32. How are fractions added and subtracted? 33. How is a fraction of a greater denomination reduced to one of a less? —— of a less to a greater? 34. How are fractions of a greater denomination reduced to integers of a less? —— integers of a less denomination to the fraction of a greater?

EXERCISES.

1. What is the amount of $\frac{5}{6}$ and $\frac{3}{8}$? —— of $\frac{1}{2}$ and $\frac{2}{3}$? —— of $12\frac{1}{2}$, $3\frac{2}{3}$ and $4\frac{3}{4}$? *Ans. to the last*, $20\frac{11}{12}$.

2. To $\frac{7}{8}$ of a pound add $\frac{3}{4}$ of a shilling. *Amount*, $18\frac{1}{4}$ s.
Note. First reduce both to the same denomination.

3. $\frac{5}{6}$ of a day added to $\frac{3}{4}$ of an hour make how many hours? —— what part of a day? *Ans. to the last*, $\frac{83}{96}$ d.

4. Add $\frac{1}{2}$ lb. Troy to $\frac{7}{12}$ of an oz.
Amount, 6 oz. 11 pwt. 16 gr.

5. How much is $\frac{1}{4}$ less $\frac{1}{8}$? $\frac{3}{10} - \frac{1}{5}$? $\frac{3}{14} - \frac{8}{40}$? $14\frac{1}{2} - 4\frac{1}{7}$? $6 - 4\frac{3}{8}$? $\frac{109}{110} - \frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$?
Ans. to the last, $\frac{163}{220}$.

6. From $\frac{1}{2}$ shilling take $\frac{3}{4}$ of a penny. *Rem.* $5\frac{1}{4}$ d.

7. From $\frac{3}{5}$ of an ounce take $\frac{7}{8}$ of a pwt.
Rem. 11 pwt. 3 grs.

8. From 4 days $7\frac{1}{2}$ hours take 1 d. $9\frac{3}{18}$ h.
Rem. 2 d. 22 h. 20 m.

9. At \$ $\frac{5}{8}$ per yard, what costs $\frac{3}{4}$ of a yard of cloth?

¶ 65. The *price* of unity, or 1, being given, to find the *cost* of any quantity, either *less* or *more* than unity, *multiply the price by the quantity.* On the other hand, the *cost* of any quantity, either *less* or *more* than unity, being given, to find the price of unity, or 1, *divide the cost by the quantity.*

Ans. \$ $\frac{15}{32}$.

1. If $\frac{11}{13}$ lb. of sugar cost $\frac{7}{15}$ of a shilling, what will $\frac{32}{43}$ of a pound cost?*

This example will require two operations: first, as above, to find the price of 1 lb.; secondly, having found the price of 1 lb., to find the cost of $\frac{32}{43}$ of a pound. $\frac{7}{15}$ s. $\div$ $\frac{11}{13}$ ($\frac{13}{11}$ of $\frac{7}{15}$ s. ¶ 57) $=$ $\frac{91}{165}$ s. the price of 1 lb. Then, $\frac{91}{165}$ s. $\times$ $\frac{32}{43}$ ($\frac{32}{43}$ of $\frac{91}{165}$ s. ¶ 53) $=$ $\frac{2912}{7095}$ s. $=$ 4 d. $3\frac{4971}{7095}$ q., *the Answer.*

Or we may reason thus: first to find the price of 1 lb.: $\frac{11}{13}$ lb. costs $\frac{7}{15}$ s. If we knew what $\frac{1}{13}$ lb. would cost, we might repeat this 13 times, and the result would be the price of 1 lb. $\frac{11}{13}$ is 11 parts. If $\frac{11}{13}$ lb. costs $\frac{7}{15}$ s., it is evident $\frac{1}{13}$ lb. will cost $\frac{1}{11}$ of $\frac{7}{15}$ $=$ $\frac{7}{165}$ s., and $\frac{13}{13}$ lb. will cost 13 times as much, that is, $\frac{91}{165}$ s. $=$ the price of 1 lb. Then, $\frac{32}{43}$ of $\frac{91}{165}$ s. $=$ $\frac{2912}{7095}$ s., the cost of $\frac{32}{43}$ of a pound. $\frac{2912}{7095}$ s. $=$ 4 d. $3\frac{4971}{7095}$ q., as before. This process is called *solving the question by analysis.*

After the same manner let the pupil solve the following questions:

2. If 7 lb. of sugar cost $\frac{3}{4}$ of a dollar, what is that a pound? $\frac{1}{7}$ of $\frac{3}{4}$ $=$ how much? What is it for 4 lb.? $\frac{4}{7}$ of $\frac{3}{4}$ $=$ how much? What for 12 pounds? $\frac{12}{7}$ of $\frac{3}{4}$ $=$ how much? *Ans. to the last,* \$ $1\frac{2}{7}$.

3. If $6\frac{1}{2}$ yds. of cloth cost \$ 3, what cost $9\frac{1}{4}$ yards? *Ans.* \$ 4'269.

4. If 2 oz. of silver cost \$ 2'24, what costs $\frac{3}{4}$ oz.? *Ans.* \$ '84.

5. If $\frac{5}{7}$ oz. costs \$ $\frac{11}{12}$, what costs 1 oz.? *Ans.* \$ 1'283.

6. If $\frac{1}{3}$ lb. less by $\frac{1}{6}$ costs $13\frac{1}{5}$ d., what costs 14 lb. less by $\frac{1}{5}$ of 2 lb.? *Ans.* 4 £. 9 s. $9\frac{3}{25}$ d.

7. If $\frac{3}{5}$ yd. cost \$ $\frac{7}{8}$, what will $40\frac{1}{2}$ yds. cost? *Ans.* \$ 59'062 +.

8. If $\frac{7}{16}$ of a ship costs \$ 251, what is $\frac{3}{32}$ of her worth? *Ans.* \$ 53'785 +.

9. At $3\frac{5}{8}$ £. per cwt., what will $9\frac{2}{3}$ lb. cost? *Ans.* 6 s. $3\frac{5}{56}$ d.

10. A merchant, owning $\frac{4}{5}$ of a vessel, sold $\frac{2}{3}$ of his share for \$ 957; what was the vessel worth? *Ans.* \$ 1794'375.

11. If $\frac{5}{8}$ yds. cost $\frac{5}{7}$ £., what will $\frac{9}{15}$ of an ell Eng. cost? *Ans.* 17 s. 1 d. $2\frac{6}{7}$ q.

* This and the following are examples usually referred to the rule *Proportion,* or *Rule of Three.* See ¶ 95 ex. 35.

12. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of \$7 for 5 yards, and sold them out at the rate of \$11 for 7 yards, and gained \$200 by the bargain; how many bales were there?

First find for what he sold 5 yards; then what he gained on 5 yards—what he gained on 1 yard. Then, as many times as the sum gained on 1 yd. is contained in \$200, so many yards there must have been. Having found the number of yards, reduce them to bales. *Ans.* 9 bales.

13. If a staff, $5\frac{2}{3}$ ft. in length, cast a shadow of 6 feet, how high is that steeple whose shadow measures 153 feet?

Ans. $144\frac{1}{2}$ feet.

14. If 16 men finish a piece of work in $28\frac{1}{3}$ days, how long will it take 12 men to do the same work?

First find how long it would take 1 man to do it; then 12 men will do it in $\frac{1}{12}$ of that time. *Ans.* $37\frac{7}{9}$ days.

15. How many pieces of merchandise, at $20\frac{1}{8}$ s. apiece, must be given for 240 pieces, at $12\frac{1}{2}$ s. apiece? *Ans.* $149\frac{11}{161}$.

16. How many yards of bocking that is $1\frac{1}{4}$ yd. wide will be sufficient to line 20 yds. of camlet that is $\frac{3}{4}$ of a yard wide?

First find the contents of the camlet in square measure; then it will be easy to find how many yards in length of bocking that is $1\frac{1}{4}$ yd. wide it will take to make the same quantity. *Ans.* 12 yards of camlet.

17. If $1\frac{1}{4}$ yd. in breadth require $20\frac{1}{2}$ yds. in length to make a cloak, what in length that is $\frac{3}{4}$ yd. wide will be required to make the same? *Ans.* $34\frac{1}{6}$ yds.

18. If 7 horses consume $2\frac{3}{4}$ tons of hay in 6 weeks, how many tons will 12 horses consume in 8 weeks?

If we knew how much 1 horse consumed in 1 week, it would be easy to find how much 12 horses would consume in 8 weeks.

$2\frac{3}{4} = \frac{11}{4}$ tons. If 7 horses consume $\frac{11}{4}$ tons in 6 weeks, 1 horse will consume $\frac{1}{7}$ of $\frac{11}{4} = \frac{11}{28}$ of a ton in 6 weeks; and if a horse consume $\frac{11}{28}$ of a ton in 6 weeks, he will consume $\frac{1}{6}$ of $\frac{11}{28} = \frac{11}{168}$ of a ton in 1 week. 12 horses will consume 12 times $\frac{11}{168} = \frac{132}{168}$ in 1 week, and in 8 weeks they will consume 8 times $\frac{132}{168} = \frac{132}{21} = 6\frac{2}{7}$ tons, *Ans.*

19. A man with his family, which in all were 5 persons, did usually drink $7\frac{4}{5}$ gallons of cider in 1 week; how much will they drink in $22\frac{1}{2}$ weeks when 3 persons more are added to the family? *Ans.* $280\frac{4}{5}$ gallons.

20. If 9 students spend $10\frac{7}{9}$£. in 18 days, how much will 20 students spend in 30 days? *Ans.* 39£. 18 s. $4\frac{20}{81}$ d.

DECIMAL FRACTIONS.

¶ **66.** We have seen, that an individual thing or number may be divided into any number of equal parts, and that these parts will be called halves, thirds, fourths, fifths, sixths, &c., according to the number of parts into which the thing or number may be divided; and that each of these parts may be again divided into any other number of equal parts, and so on. Such are called *common*, or *vulgar fractions*. Their denominators are not uniform, but vary with every varying division of a unit. It is this circumstance which occasions the chief difficulty in the operations to be performed on them; for when numbers are divided into different kinds or parts, they cannot be so easily compared. This difficulty led to the invention of *decimal* fractions, in which an individual thing, or number, is supposed to be divided first into *ten* equal parts, which will be *tenths;* and each of these parts to be again divided into ten *other* equal parts, which will be *hundredths;* and each of these parts to be still further divided into ten other equal parts, which will be *thousandths;* and so on. Such are called *decimal fractions*, (from the Latin word *decem*, which signifies *ten*,) because they increase and decrease, in a *tenfold* proportion, in the same manner as whole numbers.

¶ **67.** In this way of dividing a unit, it is evident, that the denominator to a decimal fraction will always be 10, 100, 1000, or 1 with a number of ciphers annexed; consequently, the denominator to a decimal fraction need not be expressed, for the numerator only, written with a point before it (') called the *separatrix*, is sufficient of itself to express the true value. Thus,

$\frac{6}{10}$ are written '6.
$\frac{27}{100}$ .... '27.
$\frac{685}{1000}$ .... '685.

The denominator to a decimal fraction, although not expressed, is always understood, and is 1 with as many ciphers annexed as there are places in the numerator. Thus, '3765 is a decimal consisting of four places; consequently, 1 with four ciphers annexed (10000) is its proper denominator. Any decimal may be expressed in the form of a com-

mon fraction by writing under it its proper denominator. Thus, '3765 expressed in the form of a common fraction, is $\frac{3765}{10000}$.

When whole numbers and decimals are expressed together, in the same number, it is called *a mixed number*. Thus, 25'63 is a mixed number, 25', or all the figures on the left hand of the decimal point, being whole numbers, and '63, or all the figures on the right hand of the decimal point, being decimals.

The names of the places to ten-millionths, and, generally, how to read or write decimal fractions, may be seen from the following

TABLE.

		634 =	$25\frac{63}{100}$ =	$7\frac{9}{1000000}$ =	$\frac{86002}{100000}$ =	$\frac{6853}{10000}$ =	$\frac{50}{1000}$ =	$\frac{5}{100}$ =	$\frac{5}{10}$ =	
Whole Numbers.	3d place.	6								Hundreds.
	2d place.	3	2							Tens.
	1st place.	4	5	7	0					Units.
		'	'	'	'	'	'	'	'	'
Decimal Parts.	1st place.	0	6	0	8	6	0	0	5	Tenths.
	2d place.	0	3	0	6	8	5	5		Hundredths.
	3d place.	0		0	0	5	0			Thousandths.
	4th place.	0		0	0	3				Ten-Thousandths.
	5th place.	0		0	2					Hundred-Thousandths.
	6th place.	0		9						Millionths.
	7th place.	0								Ten-Millionths.
		..... 634.	..... 25 and 63 Hundredths.	..... 7 and 9 Millionths.	..... 86002 Hundred-Thousandths.	..... 6853 Ten-Thousandths.	..... 50 Thousandths.	..... 5 Hundredths.	read 5 Tenths.	

From the table it appears, that the first figure on the right hand of the decimal point signifies so many *tenth* parts of a unit; the second figure, so many *hundredth* parts of a unit; the third figure, so many *thousandth* parts of a unit, &c. It takes 10 thousandths to make 1 hundredth, 10 hundredths to make 1 tenth, and 10 tenths to make 1 unit, in the same manner as it takes 10 units to make 1 ten, 10 tens to make 1 hundred, &c. Consequently, we may regard unity as a starting point, from whence whole numbers proceed, continually *increasing* in a tenfold proportion towards the left hand, and decimals continually *decreasing*, in the same proportion, towards the right hand. But as decimals decrease towards the right hand, it follows of course, that they increase towards the left hand, in the same manner as whole numbers.

¶ **68.** The value of every figure is determined by its place from *units*. Consequently, ciphers placed at the *right hand* of decimals do *not* alter their value, since every significant figure continues to possess the same place from unity. Thus, '5, '50, '500 are all of the same value, each being equal to $\frac{5}{10}$, or $\frac{1}{2}$.

But every cipher, placed at the *left* hand of decimal fractions, *diminishes* them tenfold, by removing the significant figures further from unity, and consequently making each part ten times as small. Thus, '5, '05, '005, are of different value, '5 being equal to $\frac{5}{10}$, or $\frac{1}{2}$; '05 being equal to $\frac{5}{100}$, or $\frac{1}{20}$; and '005 being equal to $\frac{5}{1000}$, or $\frac{1}{200}$.

Decimal fractions, having *different denominators*, are readily reduced to a *common denominator*, by annexing ciphers until they are equal in number of places. Thus, '5, '06, '234 may be reduced to '500, '060, '234, each of which has 1000 for a common denominator,

¶ **69.** Decimals are read in the same manner as whole numbers, giving the name of the lowest denomination, or right hand figure, to the whole. Thus, '6853 (the lowest denomination, or right hand figure, being ten-thousandths) is read, 6853 ten-thousandths.

Any whole number may evidently be reduced to decimal parts, that is, to tenths, hundredths, thousandths, &c. by annexing ciphers. Thus, 25 is 250 tenths, 2500 hundredths, 25000 thousandths, &c. Consequently, any mixed number

may be read together, giving it the name of the lowest denomination or right hand figure. Thus, 25'63 may be read 2563 hundredths, and the whole may be expressed in the form of a common fraction, thus, $\frac{2563}{100}$.

The denominations in federal money are made to correspond to the decimal divisions of a unit now described, dollars being units or whole numbers, dimes tenths, cents hundredths, and mills thousandths of a dollar; consequently the expression of any sum in dollars, cents, and mills, is simply the expression of a mixed number in decimal fractions.

Forty-six and seven tenths $= 46\frac{7}{10} =$ 46'7.

Write the following numbers in the same manner:

Eighteen and thirty-four hundredths.
Fifty-two and six hundredths.
Nineteen and four hundred eighty-seven thousandths.
Twenty and forty-two thousandths.
One and five thousandths.
135 and 3784 ten-thousandths.
9000 and 342 ten-thousandths.
10000 and 15 ten-thousandths.
974 and 102 millionths.
320 and 3 tenths, 4 hundredths and 2 thousandths.
500 and 5 hundred-thousandths.
47 millionths.
Four hundred and twenty-three thousandths.

ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS.

¶ **70.** As the value of the parts in decimal fractions increases in the same proportion as units, tens, hundreds, &c., and may be read *together*, in the same manner as whole numbers, so, it is evident that all the operations on decimal fractions may be performed in the same manner as on whole numbers. The only difficulty, if any, that can arise, must be in finding *where to place the decimal point* in the result. This, in addition and subtraction, is determined by the same rule; consequently, they may be exhibited together.

1. A man bought a barrel of flour for $8, a firkin of but-

ter for $3'50, 7 pounds of sugar for 83½ cents, an ounce of pepper for 6 cents; what did he give for the whole?

OPERATION.

$8'	=	8000 mills, or 1000ths of a dollar.
3'50	=	3500 mills, or 1000ths.
'835	=	835 mills, or 1000ths.
'06	=	60 mills, or 1000ths.
Ans. $12'395	=	12395 mills, or 1000ths.

As the denominations of federal money correspond with the parts of decimal fractions, so the rules for adding and subtracting decimals are exactly the same as for the same operations in federal money. (See ¶ 28.)

2. A man, owing $375, paid $175'75; how much did he then owe?

OPERATION.

$375'	=	37500 cents, or 100ths of a dollar.
175'75	=	17575 cents, or 100ths of a dollar.
$199'25	=	19925 cents, or 100ths.

The operation is evidently the same as in subtraction of *federal money.* Wherefore,—In the addition and subtraction of decimal fractions,—RULE: Write the numbers under each other, tenths under tenths, hundredths under hundredths, according to the value of their places, and point off in the results as many places for decimals as are equal to the greatest number of decimal places in any of the given numbers.

EXAMPLES FOR PRACTICE.

3. A man sold wheat at several times as follows, viz. 13'25 bushels; 8'4 bushels; 23'051 bushels, 6 bushels, and '75 of a bushel; how much did he sell in the whole?

Ans. 51'451 bushels.

4. What is the amount of 429, $21\frac{37}{100}$, $355\frac{3}{1000}$, $1\frac{7}{100}$ and $1\frac{7}{10}$? *Ans.* $808\frac{143}{1000}$, or 808'143.

5. What is the amount of 2 tenths, 80 hundredths, 89 thousandths, 6 thousandths, 9 tenths, and 5 thousandths?

Ans. 2.

6. What is the amount of three hundred twenty-nine, and seven tenths; thirty-seven and one hundred sixty-two thousandths, and sixteen hundredths?

7. A man, owing \$4316, paid \$376'865; how much did he then owe? *Ans.* \$3939'135.

8. From thirty-five thousand take thirty-five thousandths. *Ans.* 34999'965.

9. From 5'83 take 4'2793. *Ans.* 1'5507.

10. From 480 take 245'0075. *Ans.* 234'9925.

11. What is the difference between 1793'13 and 817'05693? *Ans.* 976'07307.

12. From $4\frac{8}{100}$ take $2\frac{1}{10}$. *Remainder*, $1\frac{98}{100}$, or 1'98.

13. What is the amount of $29\frac{3}{10}$, $374\frac{9}{1000000}$, $97\frac{253}{1000}$, $315\frac{4}{1000}$, 27, and $100\frac{4}{10}$? *Ans.* 942'957009.

MULTIPLICATION OF DECIMAL FRACTIONS.

¶ 71. 1. How much hay in 7 loads, each containing 23'571 cwt.?

OPERATION.

	23'571 cwt. =	23571	1000ths of a cwt.
	7	7	
Ans.	164'997 cwt. =	164997	1000ths of a cwt.

We may here (¶ 69) consider the multiplicand so many *thousandths* of a cwt., and then the product will evidently be *thousandths*, and will be reduced to a mixed or whole number by pointing off 3 figures, that is, the same number as are in the multiplicand; and as either factor may be made the multiplier, so, if the decimals had been in the *multiplier*, the same number of places must have been pointed off for decimals. Hence it follows, *we must always point off in the product as many places for decimals as there are decimal places in both factors.*

2. Multiply '75 by '25.

OPERATION.

```
 '75
 '25
 ---
 375
150
----
'1875 Product.
```

In this example, we have 4 decimal places in both factors; we must therefore point off 4 places for decimals in the product. The reason of pointing off this number may appear still more plain, if we consider the two factors as

common or vulgar fractions. Thus, ‘75 is $\frac{75}{100}$, and ‘25 is $\frac{25}{100}$: now, $\frac{75}{100} \times \frac{25}{100} = \frac{1875}{10000}$ = ‘1875, *Ans.* same as before.

3. Multiply ‘125 by ‘03.

OPERATION.

```
  ‘125
   ‘03
 ------
‘00375 Prod.
```

Here, as the number of significant figures in the product is not equal to the number of decimals in both factors, the deficiency must be supplied by prefixing ciphers, that is, placing them at the left hand. The correctness of the rule may appear from the following process: ‘125 is $\frac{125}{1000}$, and ‘03 is $\frac{3}{100}$: now, $\frac{125}{1000} \times \frac{3}{100} = \frac{375}{100000}$ = ‘00375, the same as before.

These examples will be sufficient to establish the following

RULE.

In the multiplication of decimal fractions, multiply as in whole numbers, and from the product point off so many figures for decimals as there are decimal places in the multiplicand and multiplier counted together, and, if there are not so many figures in the product, supply the deficiency by prefixing ciphers.

EXAMPLES FOR PRACTICE.

4. At $ 5‘47 per yard, what cost 8‘3 yards of cloth? *Ans.* $ 45‘401.
5. At $ ‘07 per pound, what cost 26‘5 pounds of rice? *Ans.* $ 1‘855.
6. If a barrel contain 1‘75 cwt. of flour, what will be the weight of ‘63 of a barrel? *Ans.* 1‘1025 cwt.
7. If a melon be worth $ ‘09, what is ‘7 of a melon worth? *Ans.* $6\frac{3}{10}$ cents.
8. Multiply five hundredths by seven thousandths. *Product,* ‘00035.
9. What is ‘3 of 116? *Ans.* 34‘8.
10. What is ‘85 of 3672? *Ans.* 3121‘2.
11. What is ‘37 of ‘0563? *Ans.* ‘020831.
12. Multiply 572 by ‘58. *Product,* 331‘76.
13. Multiply eighty-six by four hundredths. *Product,* 3‘44.
14. Multiply ‘0062 by ‘0008.
15. Multiply forty-seven tenths by one thousand eighty-six hundredths.

16. Multiply two hundredths by eleven thousandths.

17. What will be the cost of thirteen hundredths of a ton of hay, at $ 11 a ton?

18. What will be the cost of three hundred seventy-five thousandths of a cord of wood, at $ 2 a cord?

19. If a man's wages be seventy-five hundredths of a dollar a day, how much will he earn in 4 weeks, Sundays excepted?

DIVISION OF DECIMAL FRACTIONS.

¶ **72.** Multiplication is proved by division. We have seen, in multiplication, that the decimal places in the product must always be equal to the number of decimal places in the multiplicand and multiplier counted together. The multiplicand and multiplier, in proving multiplication, become the divisor and quotient in division. It follows of course, in division, that *the number of decimal places in the divisor and quotient, counted together, must always be equal to the number of decimal places in the dividend.* This will still further appear from the examples and illustrations which follow:

1. If 6 barrels of flour cost $ 44'718, what is that a barrel?

By taking away the decimal point, $ 44'718 = 44718 mills, or 1000ths, which, divided by 6, the quotient is 7453 mills, = $ 7'453, the *Answer.*

Or, retaining the decimal point, divide as in whole numbers.

OPERATION.

6)44'718

Ans. 7'453

As the decimal places in the divisor and quotient, counted together, must be equal to the number of decimal places in the dividend, there being *no* decimals in the *divisor*,—therefore point off *three* figures for decimals in the *quotient*, equal to the number of decimals in the dividend, which brings us to the same result as before.

2. At $ 4'75 a barrel for cider, how many barrels may be bought for $ 31?

In this example, there are decimals in the divisor, and none in the dividend. $ 4'75 = 475 cents, and $ 31, by annexing two ciphers, = 3100 cents; that is, reduce the di

vidend to parts of the same denomination as the divisor. Then, it is plain, as many times as 475 cents are contained in 3100 cents, so many barrels may be bought.

```
475)3100(6 250/475
    2850
    ----
     250
```

barrels, the *Answer;* that is, 6 barrels and $\frac{250}{475}$ of another barrel.

But the remainder, 250, instead of being expressed in the form of a common fraction, may be reduced to 10ths by annexing a cipher, which, in effect, is multiplying it by 10, and the division continued, placing the decimal point after the 6, or whole ones already obtained, to distinguish it from the decimals which are to follow. The points may be withdrawn or not from the divisor and dividend.

OPERATION.

```
4‘75)31‘00(6‘526 +
     2850
     ----
      2500
      2375
      ----
       1250
        950
       ----
        3000
        2850
        ----
         150
```

barrels, the *Answer;* that is, 6 barrels and 526 thousandths of another barrel.

By annexing a cipher to the first remainder, thereby reducing it to 10ths, and continuing the division, we obtain from it ‘5, and a still further remainder of 125, which, by annexing another cipher, is reduced to 100ths, and so on.

The last remainder, 150, is $\frac{150}{475}$ of a thousandth part of a barrel, which is of so trifling a value, as not to merit notice.

If now we count the decimals in the dividend, (for every cipher annexed to the remainder is evidently to be counted a decimal of the dividend,) we shall find them to be *five,* which corresponds with the number of decimal places in the divisor and quotient counted together.

3. Under ¶ 71, ex. 3, it was required to multiply ‘125 by ‘03; the product was ‘00375. Taking this product for a dividend, let it be required to divide ‘00375 by ‘125. One operation will prove the other. Knowing that the number of decimal places in the quotient and divisor, counted together, will be equal to the decimal places in the dividend, we may divide as in whole numbers, being careful to retain the decimal points in their proper places. Thus,

OPERATION.
‘125)‘00375(‘03
375
———
000

The divisor, 125, in 375 goes 3 times, and no remainder. We have only to place the decimal point in the quotient, and the work is done. There are five decimal places in the dividend; consequently there must be five in the divisor and quotient counted together; and, as there are *three* in the divisor, there must be *two* in the quotient; and, since we have but one figure in the quotient, the *deficiency* must be supplied by prefixing a cipher.

The operation by vulgar fractions will bring us to the same result. Thus, ‘125 is $\frac{125}{1000}$, and ‘00375 is $\frac{375}{100000}$: now, $\frac{375}{100000} \div \frac{125}{1000} = \frac{375000}{12500000} = \frac{3}{100} =$ ‘03, the same as before.

¶ 73. The foregoing examples and remarks are sufficient to establish the following

RULE.

In the division of decimal fractions, divide as in whole numbers, and from the right hand of the quotient point off as many figures for decimals, as the decimal figures in the dividend exceed those in the divisor, and if there are not so many figures in the quotient, supply the deficiency by prefixing ciphers.

If at any time there is a remainder, or if the decimal figures in the divisor exceed those in the dividend, ciphers may be annexed to the dividend or the remainder, and the quotient carried to any necessary degree of exactness; but the ciphers annexed must be counted so many decimals of the dividend.

EXAMPLES FOR PRACTICE.

4. If $ 472‘875 be divided equally between 13 men, how much will each one receive? *Ans.* $ 36‘375.
5. At $ ‘75 per bushel, how many bushels of rye can be bought for $ 141? *Ans.* 188 bushels.
6. At 12½ cents per lb., how many pounds of butter may be bought for $ 37? *Ans.* 296 lb.
7. At 6¼ cents apiece, how many oranges may be bought for $ 8? *Ans.* 128 oranges.
8. If ‘6 of a barrel of flour cost $ 5, what is that per barrel? *Ans.* $ 8‘333 +.
9. Divide 2 by 53‘1. *Quot.* ‘037 +.

10. Divide ‘012 by ‘005. *Quot.* 2‘4.
11. Divide three thousandths by four hundredths. *Quot.* ‘075.
12. Divide eighty-six tenths by ninety four thousandths.
13. How many times is ‘17 contained in 8?

REDUCTION OF COMMON OR VULGAR FRACTIONS TO DECIMALS.

¶ **74.** 1. A man has $\frac{4}{5}$ of a barrel of flour; what is that expressed in decimal parts?

As many times as the denominator of a fraction is contained in the numerator, so many whole ones are contained in the fraction. We can obtain no whole ones in $\frac{4}{5}$, because the denominator is not contained in the numerator. We may, however, reduce the numerator to *tenths*, (¶ 72, ex. 2,) by annexing a cipher to it, (which, in effect, is multiplying it by 10,) making 40 tenths, or 4‘0. Then, as many times as the denominator, 5, is contained in 40, so many *tenths* are contained in the fraction. 5 into 40 goes 8 times, and no remainder. *Ans.* ‘8 of a bushel.

2. Express $\frac{3}{4}$ of a dollar in decimal parts.

The numerator, 3, reduced to tenths, is $\frac{30}{10}$, 3‘0, which, divided by the denominator, 4, the quotient is 7 tenths, and a remainder of 2. This remainder must now be reduced to *hundredths* by annexing another cipher, making 20 hundredths. Then, as many times as the denominator, 4, is contained in 20, so many *hundredths* also may be obtained. 4 into 20 goes 5 times, and no remainder. $\frac{3}{4}$ of a dollar, therefore, reduced to decimals, is 7 tenths and 5 hundredths, that is, ‘75 of a dollar.

The operation may be presented in form as follows:—

```
          Num.
Denom. 4 ) 3‘0 (‘75 of a dollar, the Answer.
           28
           --
            20
            20
            --
```

3. Reduce $\frac{4}{66}$ to a decimal fraction.

The numerator must be reduced to *hundredths*, by annexing two ciphers, before the division can begin.

66) 4‘00 (‘0606 +, the *Answer*.
     396
     ———
       400   As there can be no *tenths*, a cipher must
       396   be placed in the quotient, in tenth's place.
       ———
          4

Note. $\frac{4}{66}$ cannot be reduced *exactly;* for, however long the division be continued, there will still be a remainder.* It is sufficiently exact for most purposes, if the decimal be extended to three or four places.

From the foregoing examples we may deduce the following general Rule :—*To reduce a common to a decimal frac-*

---

* Decimal figures, which *continually repeat*, like ‘06, in this example, are called *Repetends*, or *Circulating* Decimals. If only *one figure* repeats, as ‘3333 or ‘7777, &c., it is called a *single repetend*. If *two* or *more figures* circulate alternately, as ‘060606, ‘234234234, &c., it is called a *compound repetend*. If other figures arise *before* those which circulate, as ‘743333, ‘143010101, &c., the decimal is called a *mixed repetend*.

A single repetend is denoted by writing only the *circulating figure* with a point over it: thus, ‘$\dot{3}$, signifies that the 3 is to be continually repeated, forming an *infinite* or *never-ending series* of 3's.

A compound repetend is denoted by a point over the *first* and *last* repeating *figure:* thus, ‘$\dot{2}3\dot{4}$ signifies that 234 is to be continually repeated.

It may not be amiss, here to show how the *value* of any *repetend* may be found, or, in other words, how it may be *reduced to its equivalent vulgar fraction.*

If we attempt to reduce $\frac{1}{9}$ to a *decimal*, we obtain a continual repetition of the figure 1: thus, ‘11111, that is, the *repetend* ‘$\dot{1}$. The value of the repetend ‘$\dot{1}$, then, is $\frac{1}{9}$; the value of ‘222, &c., the repetend ‘$\dot{2}$, will evidently be *twice* as much, that is, $\frac{2}{9}$. In the same manner, $\dot{3} = \frac{3}{9}$, and ‘$\dot{4} = \frac{4}{9}$, and ‘$\dot{5} = \frac{5}{9}$, and so on to $\dot{9}$, which $= \frac{9}{9} = 1$.

1. What is the value of ‘$\dot{8}$? *Ans.* $\frac{8}{9}$.

2. What is the value of ‘$\dot{6}$? *Ans.* $\frac{6}{9} = \frac{2}{3}$. What is the value of ‘$\dot{3}$? —— of ‘$\dot{7}$? —— of ‘$\dot{4}$? —— of ‘$\dot{5}$? —— of ‘$\dot{9}$? —— of ‘$\dot{1}$?

If $\frac{1}{99}$ be reduced to a decimal, it produces ‘010101, or the repetend $\dot{0}\dot{1}$. The repetend ‘$\dot{0}\dot{2}$, being 2 times as much, must be $\frac{2}{99}$ and ‘$\dot{0}\dot{3} = \frac{3}{99}$, and ‘$\dot{4}\dot{8}$, being 48 times as much, must be $\frac{48}{99}$. and ‘$\dot{7}\dot{4} = \frac{74}{99}$, &c.

tion,—Annex one or more ciphers, as may be necessary, to the numerator, and divide it by the denominator. If then there be a remainder, annex another cipher, and divide as before, and so continue to do so long as there shall continue to be a remainder, or until the fraction shall be reduced to any necessary degree of exactness. The quotient will be the decimal required, which must consist of as many decimal places as there are ciphers annexed to the numerator; and, if there are not so many figures in the quotient, the deficiency must be supplied by prefixing ciphers.

EXAMPLES FOR PRACTICE.

4. Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{12}{480}$, and $\frac{9}{1129}$ to decimals.
Ans. '5; '25; '025; '00797 +.

5. Reduce $\frac{27}{39}$, $\frac{3}{1000}$, $\frac{5}{1785}$, and $\frac{11}{60000}$ to decimals.
Ans. '692 +; '003; '0028 +; '000183 +.

6. Reduce $\frac{478}{962}$, $\frac{10}{367}$, $\frac{16}{8600}$ to decimals.

7. Reduce $\frac{4}{9}$, $\frac{5}{99}$, $\frac{8}{999}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{11}$, $\frac{4}{11}$, $\frac{1}{909}$ to decimals.

8. Reduce $\frac{1}{8}$, $\frac{2}{8}$, $\frac{5}{8}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{3}{75}$ to decimals.

If $\frac{1}{999}$ be reduced to a decimal, it produces '$\dot{0}0\dot{1}$; consequently, '$\dot{0}0\dot{2} = \frac{2}{999}$, and '$\dot{0}3\dot{7} = \frac{37}{999}$, and $\dot{4}2\dot{5} = \frac{425}{999}$, &c. As this principle will apply to any number of places, we have this *general* RULE *for reducing a circulating decimal to a vulgar fraction,*—Make the *given repetend* the *numerator*, and the *denominator* will be as many 9s as there are *repeating figures*.

3. What is the vulgar fraction equivalent to '$\dot{7}0\dot{4}$? *Ans.* $\frac{704}{999}$.

4. What is the value of '$\dot{0}0\dot{3}$? —— '$\dot{0}1\dot{4}$? —— '$\dot{3}2\dot{4}$? —— '$\dot{0}102\dot{1}$? '$\dot{2}46\dot{3}$? —— '$\dot{0}0210\dot{3}$? *Ans. to last,* $\frac{701}{333333}$.

5. What is the value of '$4\dot{3}$?

In this fraction, the repetend begins in the second place, or place of hundredths. The first figure, 4, is $\frac{4}{10}$, and the *repetend*, 3, is $\frac{3}{9}$ of $\frac{1}{10}$, that is, $\frac{3}{90}$; these two parts must be added together. $\frac{4}{10} + \frac{3}{90} = \frac{39}{90} = \frac{13}{30}$, *Ans.* Hence, to find the value of a *mixed repetend,*—Find the value of the two parts, *separately*, and add them together.

6. What is the value of '$15\dot{3}$? $\frac{15}{100} + \frac{3}{900} = \frac{138}{900} = \frac{23}{150}$, *Ans*

7. What is the value of '$004\dot{7}$? *Ans.* $\frac{43}{9000}$.

8. What is the value of '$13\dot{8}$? —— '$1\dot{6}$? —— '$4\dot{1}2\dot{3}$?

It is plain, that circulates may be added, subtracted, multiplied, and divided, by first reducing them to their equivalent *vulgar fractions*

REDUCTION OF DECIMAL FRACTIONS.

¶ **75.** Fractions, we have seen, (¶ 63,) like *integers*, are reduced from low to higher denominations by *division*, and from high to lower denominations by *multiplication*.

To reduce a compound number to a decimal of the highest denomination.

1. Reduce 7 s. 6 d. to the decimal of a pound.

6 d. reduced to the decimal of a shilling, that is, divided by 12, is ‘5 s., which annexed to the 7 s. making 7‘5 s., and divided by 20, is ‘375 £. the *Ans.*

The process may be presented in form of a *rule*, thus:—Divide the *lowest* denomination given, annexing to it one or more ciphers, as may be necessary, by that number which it takes of the same to make *one* of the next higher denomination, and annex the quotient, as a decimal to that higher denomination; so continue to do, until the whole shall be reduced to the decimal required.

EXAMPLES FOR PRACTICE.

3. Reduce 1 oz. 10 pwt. to the fraction of a pound.

OPERATION.

```
20)10‘0  pwt.
   ------
12)1‘5   oz.
   ------
    ‘125 lb. Ans.
```

To reduce the decimal of a higher denomination to integers of lower denominations.

2. Reduce ‘375 £. to integers of lower denominations.

‘375 £. reduced to shillings, that is, multiplied by 20, is 7‘50 s.; then the *fractional* part, ‘50 s., reduced to pence, that is, multiplied by 12, is 6 d. *Ans.* 7 s. 6 d.

That is,—Multiply the given decimal by that number which it takes of the next *lower* denomination to make *one* of this higher, and from the right hand of the product point off as many figures for decimals as there are figures in the given decimal, and so continue to do through all the denominations; the several numbers at the left hand of the decimal points will be the value of the fraction in the proper denominations.

EXAMPLES FOR PRACTICE.

4. Reduce ‘125 lbs. Troy to integers of lower denominations.

OPERATION.

```
lb.  ‘125
       12
     ----
oz. 1‘500
       20
     ----
pwt. 10‘000.  Ans. 1 oz. 10 pwt.
```

5. Reduce 4 cwt. $2\frac{3}{5}$ qrs. to the decimal of a ton.

Note. $2\frac{3}{5} = 2‘6$.

6. What is the value of ‘2325 of a ton?

7. Reduce 38 gals. 3‘52 qts. of beer, to the decimal of a hhd.

8. What is the value of ‘72 hhd. of beer?

9. Reduce 1 qr. 2 n. to the decimal of a yard.

10. What is the value of ‘375 of a yard?

11. Reduce 17 h. 6 m. 43 sec. to the decimal of a day.

12. What is the value of ‘713 of a day?

13. Reduce 21 s. $10\frac{1}{2}$ d. to the decimal of a guinea.

14. What is the value of ‘78125 of a guinea?

15. Reduce 3 cwt. 0 qr. 7 lbs. 8 oz. to the decimal of a ton.

16. What is the value of ‘15334821 of a ton?

Let the pupil be required to reverse and prove the following examples:

17. Reduce 4 rods to the decimal of an acre.
18. What is the value of ‘7 of a lb. of silver?
19. Reduce 18 hours, 15 m. 50‘4 sec. to the decimal of a day.
20. What is the value of ‘67 of a league?
21. Reduce 10 s. $9\frac{1}{4}$ d. to the fraction of a pound.

¶ **76.** There is a method of reducing shillings, pence and farthings to the decimal of a pound, by *inspection*, more simple and concise than the foregoing. The reasoning in relation to it is as follows:

$\frac{1}{10}$ of 20 s. is 2 s.; therefore every 2 s. is $\frac{1}{10}$, or ‘1 £. Every shilling is $\frac{1}{20} = \frac{5}{100}$, or ‘05 £. Pence are readily reduced to farthings. Every farthing is $\frac{1}{960}$ £. Had it so happened, that 1000 farthings, instead of 960, had made a pound, then every farthing would have been $\frac{1}{1000}$, or ‘001 £. But 960 increased by $\frac{1}{24}$ part of itself is 1000; consequently, 24 farthings are exactly $\frac{25}{1000}$, or ‘025 £., and 48 farthings are exactly $\frac{50}{1000}$, or ‘050 £. Wherefore, if the number of farthings, in the given pence and farthings, be *more* than 12, $\frac{1}{24}$ part will be more than $\frac{1}{2}$; therefore add 1 to them: if they be more than 36, $\frac{1}{24}$ part will be more than $1\frac{1}{2}$; therefore add 2 to them: then call them so many thousandths, and the result will be correct within less than $\frac{1}{2}$ of $\frac{1}{1000}$ of a pound. Thus, 17 s. $5\frac{3}{4}$ d. is reduced to the

decimal of a pound as follows: 16 s. = '8 £. and 1 s. = '05 £. Then, 5¾ d. = 23 farthings, which, increased by 1, (the number being more than 12, but not exceeding 36,) is '024 £., and the whole is '874 £. the *Ans.*

Wherefore, *to reduce shillings, pence and farthings to the decimal of a pound by inspection,—Call every two shillings one tenth of a pound; every odd shilling, five hundredths; and the number of farthings, in the given pence and farthings, so many thousandths, adding one, if the number be more than twelve and not exceeding thirty-six, and two, if the number be more than thirty-six.*

¶ 77. Reasoning as above, the result, or the three first figures in any decimal of a pound, may readily be reduced back to shillings, pence and farthings, by *inspection.* Double the *first* figure, or *tenths,* for shillings, and, if the second figure, or hundredths, be *five,* or *more* than five, reckon *another* shilling; then, after the five is deducted, call the figures in the second and third place so many farthings, abating *one* when they are above twelve, and *two* when above thirty-six, and the result will be the answer, sufficiently exact for all practical purposes. Thus, to find the value of '876 £. by inspection:—

'8	tenths of a pound - - -	=	16 shillings.
'05	hundredths of a pound - -	=	1 shilling.
'026	thousandths, abating 1, = 25 farthings	=	0 s. 6¼ d.
'876	of a pound - - - - -	=	17 s. 6¼ d. *Ans.*

EXAMPLES FOR PRACTICE.

1. Find, by inspection, the decimal expressions of 9 s. 7 d., and 12 s. 0¾ d. *Ans.* '479 £., and '603 £.

2. Find, by inspection, the value of '523 £., and '694 £. *Ans.* 10 s. 5½ d., and 13 s. 10½ d.

3. Reduce to decimals, by inspection, the following sums, and find their amount, viz.: 15 s. 3 d.; 8 s. 11½ d.; 10 s. 6¼ d.; 1 s. 8½ d.; ½ d., and 2¼ d. *Amount,* £1'833.

4. Find the value of '47 £.

Note. When the decimal has but *two* figures, after taking out the shillings, the remainder, to be reduced to *thousandths,* will require a cipher to be annexed to the right hand, or supposed to be so. *Ans.* 9 s. 4¾ d.

5. Value the following decimals, by inspection, and find their amount, viz.: '785 £.; '357 £.; '916 £.; '74 £.; '5 £.; '25 £.; '09 £.; and '008 £. *Ans.* 3 £. 12 s. 11 d.

SUPPLEMENT TO DECIMAL FRACTIONS.

QUESTIONS.

1. What are decimal fractions? 2. Whence is the term derived? 3. How do decimal differ from common fractions? 4. How are decimal fractions written? 5. How can the proper denominator to a decimal fraction be known, if it be not expressed? 6. How is the value of every figure determined? 7. What does the first figure on the right hand of the decimal point signify? —— the second figure? —— third figure? —— fourth figure? 8. How do ciphers, placed at the *right* hand of decimals, affect their value? 9. Placed at the *left* hand, how do they affect their value? 10. How are decimals read? 11. How are decimal fractions, having different denominators, reduced to a *common* denominator? 12. What is a mixed number? 13. How may any *whole* number be reduced to decimal parts? 14. How can any mixed number be read together, and the whole expressed in the form of a common fraction? 15. What is observed respecting the denominations in federal money? 16. What is the rule for addition and subtraction of decimals, particularly as respects placing the decimal point in the results? —— multiplication? —— division? 17. How is a common or vulgar fraction reduced to a decimal? 18. What is the rule for reducing a compound number to a decimal of the highest denomination contained in it? 19. What is the rule for finding the value of any given decimal of a higher denomination in terms of a lower? 20. What is the rule for reducing shillings, pence and farthings to the decimal of a pound, by *inspection*? 21. What is the reasoning in relation to this rule? 22. How may the three first figures of any decimal of a pound be reduced to shillings, pence and farthings, by *inspection*?

EXERCISES.

1. A merchant had several remnants of cloth, measuring as follows, viz.:

$7\frac{3}{8}$ yds.	How many yards in the whole, and what would the whole come to at $ 3'67 per yard?
$6\frac{5}{8}$	
$1\frac{4}{5}$	*Note.* Reduce the common fractions to decimals. Do the same wherever they occur in the examples which follow.
$9\frac{2}{5}$	
$8\frac{1}{4}$	
$3\frac{1}{40}$	*Ans.* 36'475 yards. $ 133'863 +, cost.

2. From a piece of cloth, containing $36\frac{5}{8}$ yds., a merchant sold, at one time, $7\frac{3}{10}$ yds., and, at another time, $12\frac{5}{8}$ yds.; how much of the cloth had he left? *Ans.* 16'7 yds.

3. A farmer bought 7 yards of broadcloth for $8\frac{6}{13}$ £., a barrel of flour for $2\frac{4}{15}$ £., a cask of lime for $1\frac{5}{8}$ £., and 7 lbs. of rice for $\frac{5}{24}$ £.; he paid 1 ton of hay at $3\frac{7}{16}$ £., 1 cow at $6\frac{2}{3}$ £., and the balance in pork at $\frac{1}{40}$ £. per lb.; how many were the pounds of pork?

Note. In reducing the common fractions in this example, it will be sufficiently exact if the decimal be extended to three places. *Ans.* $108\frac{4}{5}$ lb.

4. At $12\frac{1}{2}$ cents per lb., what will $37\frac{3}{4}$ lbs. of butter cost? *Ans.* $ $4'718\frac{3}{4}$.

5. At $ 17'37 per ton for hay, what will $11\frac{5}{8}$ tons cost? *Ans.* $ $201'92\frac{5}{8}$.

6. *The above example reversed.* At $ $201'92\frac{5}{8}$ for $11\frac{5}{8}$ tons of hay, what is that per ton? *Ans.* $ 17'37.

7. If '45 of a ton of hay cost $ 9, what is that per ton? *Consult* ¶ 65. *Ans.* $ 20.

8. At '4 of a dollar a gallon, what will '25 of a gallon of molasses cost? *Ans.* $ '1.

9. At $ 9 per cwt., what will 7 cwt. 3 qrs. 16 lbs. of sugar cost?

Note. Reduce the 3 qrs. 16 lbs. to the decimal of a cwt., extending the decimal in this, and the examples which follow, to *four* places. *Ans.* 71'035+.

10. At $ 69'875 for 5 cwt. 1 qr. 14 lbs. of raisins, what is that per cwt.? *Ans.* $ 13.

11. What will 2300 lbs. of hay come to at 7 mills per lb.? *Ans.* $ 16'10.

12. What will $765\frac{1}{2}$ lbs. of coffee come to, at 18 cents per lb.? *Ans.* $ 137'79.

N*

13. What will 12 gals. 3 qts. 1 pt. of gin cost, at 28 cents per quart?

Note. Reduce the whole quantity to quarts and the decimal of a quart. *Ans.* $ 14'42.

14. Bought 16 yds. 2 qrs. 3 na. of broadcloth for $ 100'125; what was that per yard? *Ans.* $ 6.

15. At $ 1'92 per bushel, how much wheat may be bought for $ '72? *Ans.* 1 peck 4 quarts.

16. At $ 92'72 per ton, how much iron may be purchased for $ 60'268? *Ans.* 13 cwt.

17. Bought a load of hay for $ 9'17, paying at the rate of $ 16 per ton; what was the weight of the hay?
Ans. 11 cwt. 1 qr. 23 lbs.

18. At $ 302'4 per tun, what will 1 hhd. 15 gals. 3 qts. of wine cost? *Ans.* $ 94'50.

19. *The above reversed.* At $ 94'50 for 1 hhd. 15 gals. 3 qts. of wine, what is that per tun? *Ans.* $ 302'4.

Note. The following examples reciprocally prove each other, excepting when there are some fractional losses, as explained above, and even then the results will be sufficiently exact for all practical purposes. If, however, *greater* exactness be required, the decimals must be extended to a greater number of places.

20. At $ 1'80 for 3½ qts. of wine, what is that per gal.?

21. At $ 2'215 per gal., what cost 3½ qts.?

22. If ⅝ of a ton of pot-ashes cost $ 60'45, what is that per ton?

23. At $ 96'72 per ton for pot-ashes, what will ⅝ of a ton cost?

24. If '8 of a yard of cloth cost $ 2, what is that per yard?

25. If a yard of cloth cost $ 2'5, what will '8 of a yard cost?

26. At $ 2'5 per yard, how much cloth may be purchased for $ 2?

27. If 14 cwt. of pot-ashes cost 19 £. 5 s., what is that per ton?

28. If a ton of pot-ashes cost 27 £. 10 s., what will 14 cwt. cost?

29. At 27 £. 10 s. a ton for pot-ashes, what quantity may be bought for 19 £. 5 s.?

Note. After the same manner let the pupil reverse and prove the following examples:

30. At $ 18'50 per ton, how much hay may be bou.. for $ 12'025?

31. What will 3 qrs. 2 na. of broadcloth cost, at $ 6 per yard?

32. At $ 22'10 for transportation of 65 cwt. 46 miles, what is that per ton?

33. Bought a silver cup, weighing 9 oz. 4 pwt. 16 grs. for 3 £. 2 s. 3 d. 3⅗ q.; what was that per ounce?

34. Bought 9 chests of tea, each weighing 3 cwt. 2 qrs. 21 lbs. at 4 £. 9 s. per cwt.; what came they to?

35. If 5 acres 1 rood produce 26 quarters 2 bushels of wheat, how many acres will be required to produce 47 quarters 4 bushels? A quarter is 8 bushels.

Note. The above example will require two operations, for which consult ¶ 65, ex. 1.

36. A lady purchased a gold ring, giving at the rate of $ 20 per ounce; she paid for the ring $ 1'25; how much did it weigh?

REDUCTION OF CURRENCIES.

¶ **78.** Previous to the act of Congress in 1786 establishing federal money, all calculations in money, throughout the United States, were made in pounds, shillings, pence and farthings, the same as in England. But these denominations, although *the same in name*, were different in value in different countries.

Thus, 1 dollar is reckoned in

England,	4 s. 6 d., called *English*, or *sterling* money.
Canada and Nova Scotia,	5 s. called *Canada* currency.
The New England States, Virginia, Kentucky, and Tennessee,	6 s., called *New England* currency.
New York, Ohio, and N. Carolina,	8 s., called *New York* currency.

1 dollar is reckoned in

New Jersey, Pennsylvania, Delaware, and Maryland,	7 s. 6 d., called *Pennsylvania* currency
S. Carolina and Georgia,	4 s. 8 d., called *Georgia* currency.

1. Reduce 6£. 11 s. $6\frac{1}{4}$ d. to federal money.

Note. To reduce pounds, shillings, pence and farthings, in either of the above-named currencies, to federal money,— First, reduce the shillings, pence and farthings (if any be contained in the given sum) *to the decimal of a pound by inspection*, as already taught, ¶ 76.

6£. 11 s. $6\frac{1}{4}$ d. = £6'576.

English money.—Now, supposing the above sum to be English money,—1£. is 20 s. = 240 pence, in all the above currencies. 1 dollar, in English money, is reckoned 4 s. 6 d. = 54 pence, that is, $\frac{54}{240} = \frac{9}{40}$ of 1 pound. Now, as many times as $\frac{9}{40}$, the fraction which 1 dollar is of 1 pound, English money, is contained in £6'576, so many dollars, it is evident, there must be; that is,—*To reduce English to federal money*,—Divide the given sum by $\frac{9}{40}$, the quotient will be federal money.

```
 £6'576  English money.
     40
 ------
9)263'040
 ------
  29'226⅔ federal money, Answer.
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Note. It will be recollected, to divide by a fraction, we multiply by the denominator, and divide the product by the numerator.

Canada currency.—Supposing the above sum to be Canada currency,—1 dollar, in this currency, is 5 s. = 60 pence, that is, $\frac{60}{240} = \frac{1}{4}$ of 1 pound. Therefore,—*To reduce Canada currency to federal money*,—Divide the given sum by $\frac{1}{4}$, and the quotient will be federal money; or, which is the same thing,—Multiply the given sum by 4.

```
 £6'576  Canada currency.
      4
 ------
$27'304  federal money, Answer.
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NEW ENGLAND CURRENCY.—1 dollar, in this currency, is 6 s. = 72 pence, that is, $\frac{72}{240} = \frac{3}{10}$, or '3 of a pound. Therefore,—*To reduce New England currency to federal money*,—Divide the given sum by '3.

'3) £. 6'576 New England currency.

$ 21'92 federal money, *Answer.*

NEW YORK CURRENCY.—1 dollar, in this currency, is 8 s. = 96 pence, that is, $\frac{96}{240} = \frac{4}{10}$, or '4 of a pound. Therefore, —*To reduce New York currency to federal money*,—Divide the given sum by '4.

'4) £. 6'576 New York currency.

$ 16'44 federal money, *Answer.*

PENNSYLVANIA CURRENCY.—1 dollar, in this currency, is 7 s. 6 d. = 90 pence, that is, $\frac{90}{240} = \frac{3}{8}$ of a pound. Therefore,—*To reduce Pennsylvania currency to federal money*,—Divide by $\frac{3}{8}$, that is, multiply the given sum by 8, and divide the product by 3.

£. 6'576 Pennsylvania currency.
8

3)52'608

$ 17'536 federal money, *Answer.*

GEORGIA CURRENCY.—1 dollar, Georgia currency, is 4 s. 8 d. = 56 pence, that is, $\frac{56}{240} = \frac{7}{30}$ of a pound. Therefore,—*To reduce Georgia currency to federal money*,—Divide by $\frac{7}{30}$, that is, multiply the given sum by 30, and divide the product by 7.

£. 6'576 Georgia currency.
30

7)197'280

$ 28'182$\frac{6}{7}$ federal money, *Answer.*

From the foregoing examples, we derive the following general RULE:—*To reduce English money, and the currencies of Canada and the several States, to federal money*,—First, reduce the shillings, &c., if any in the given sum, to the decimal of a pound; this being done, divide the given sum by such fractional part as 1 dollar, in the given currency, is a fractional part of 1 pound.

EXAMPLES FOR PRACTICE.

2. Reduce 125£., in each of the before named currencies, to federal money.

Answers.

125£. English money,	is	$555'555$\frac{5}{9}$.
125£. Canada currency,	...	$500.
125£. New England currency,	...	$416'666$\frac{2}{3}$.
125£. New York	....	$312'50.
125£. Pennsylvania	....	$333'333$\frac{1}{3}$.
125£. Georgia	....	$535'714$\frac{2}{7}$.

3. Reduce 1 s. 6 d., in the several currencies, to federal money.

Answers. 1 s. 6 d. = '075£. English money, is $'333$\frac{1}{3}$; Canada currency, it is $'30; New England currency, it is $'25; New York currency, it is $'187$\frac{1}{2}$; Pennsylvania currency, it is $'20; Georgia currency, it is $'321$\frac{3}{7}$.

4. Reduce 75£. 15 s., in the several currencies, to federal money.

5. Reduce 18 £. 0 s. 8$\frac{3}{4}$ d., in the several currencies, to federal money.

6. Reduce 4$\frac{1}{2}$ d., in the several currencies, to federal money.

7. Reduce 36£. 3 s. 7$\frac{1}{4}$ d., in the several currencies, to federal money.

¶ **79.** *To reduce federal money to any of the before named currencies*, reverse the process in the foregoing operations; that is,—*Multiply* the given sum in federal money by such fractional part as 1 dollar, in that currency to which you would reduce it, is of 1 pound. The product will be the answer in pounds and decimals of a pound, which must be reduced to shillings, pence and farthings, *by inspection, as already taught*, ¶ 77.

EXAMPLES FOR PRACTICE.

1. Reduce $118'25 to the several before named currencies.

Answer. $118'25, changed to

		£.	s.	d.
English money,	is	26	12	1$\frac{1}{2}$.
Canada currency,	...	29	11	3.
N. England currency,	...	35	9	6.
N. York	....	47	6	0.
Pennsylvania	....	44	6	10$\frac{1}{2}$.
Georgia	....	27	11	9$\frac{3}{4}$.

2. Change $250 to the several currencies.
3. Change 56 cents to the several currencies.
4. Change $45'12½ to the several currencies.

¶ **80.** It may sometimes be required to reduce one currency to the par, or equality of another currency.

1. Reduce 35£. 6 s. 8 d., English money, to N. England currency.

$1 is 4 s. 6 d. = 54 d. *English money.* $1 is 6 s. = 72 d. *N. England currency;* that is, the value of any number of pounds, shillings, pence, &c., English money, is $\frac{72}{54} = \frac{4}{3}$ of the same in N. England currency; consequently,—*To reduce English money to N. England currency,*—Multiply by $\frac{4}{3}$, or, which is the same, increase it by $\frac{1}{3}$ part of itself. Thus,

	£.	s.	d.	q.	
3)	35	6	8		English money, is
	11	15	6	2	
	47	2	2	2	New England currency, *Answer.*

Hence we have this general RULE for finding a *multiplier* to reduce any currency to the par of another:—

Make $1 in *pence*, of the currency *to be reduced*, the *denominator* of a fraction, over which write $1 in *pence*, of the currency to which it is to be reduced, for a *numerator*. This fraction may then be reduced to its lowest terms before multiplying.

On the same principles, let the pupil form for himself *multipliers*, by which

To reduce English money to Canada, N. York, Pennsylvania, and Georgia currencies.
.... Canada currency to English, N. England, N. York, Pennsylvania, and Georgia currencies.
.... N. England currency to Canada, N. York, Pennsylvania, and Georgia currencies.
.... N. York currency to English, Canada, N. England, Pennsylvania, and Georgia currencies.
.... Pennsylvania currency to English, Canada, N. England, N. York, and Georgia currencies.
.... Georgia currency to English, Canada, N. England, N. York, and Pennsylvania currencies.

Rates at which the following foreign coins are estimated at the Custom Houses of the United States.

Livre of France,	$ '18½.
Franc do.	$ '18¾.
Silver Rouble of Russia,	$ '75.
Florin or Guilder of the United Netherlands,	$ '40.
Mark Banco of Hamburg,	$ '33⅓.
Real of Plate of Spain,	$ '10.
Real of Vellon of do.	$ '05.
Milrea of Portugal,	$ 1'24.
Tale of China,	$ 1'48.
Pagoda of India,	$ 1'84.
Rupee of Bengal,	$ '50.

2. Reduce 8764 livres to federal money.
3. Reduce 10,000 francs to federal money.
4. Reduce 250,000 florins to federal money.
5. In $ 1000, how many francs?

INTEREST.

¶ **81.** Interest is an allowance made by a debtor to a creditor for the use of money. It is computed at a certain number of dollars for the use of each hundred dollars, or so many pounds for each hundred pounds, &c. one year, and in the same proportion for a greater or less sum, or for a longer or shorter time.

The number of dollars so paid for the use of a hundred dollars, one year, is called the *rate per cent.* or *per centum;* the words *per cent.* or *per centum* signifying *by the hundred.*

The highest rate allowed by law in the New England States, is 6 *per cent.*,* that is, 6 dollars for a 100 dollars, 6 cents for a 100 cents, 6 pounds for a 100, &c.; in other words, $\frac{6}{100}$ *of the sum lent or due is paid for the use of it one year.* This is called *legal interest*, and will here be understood when no other rate is mentioned.

---

* In the State of New York, 7 per cent. is the legal interest; in England the legal interest is 5 per cent.

Let us suppose the sum lent, or due, to be $ 1. The 100th part of $ 1, or $\frac{1}{100}$ of a dollar, is 1 cent, and $\frac{6}{100}$ of a dollar, the legal interest, is 6 cents, which, written as a decimal fraction, is expressed thus, - - - - - - - '06.

So of any other rate per cent.

1 per cent., expressed as a common fraction, is $\frac{1}{100}$; decimally, - - - - - - - - - - - - - - '01.

$\frac{1}{2}$ per cent. is a half of 1 per cent., that is, - - '005.

$\frac{1}{4}$ per cent., is a fourth of 1 per cent., that is, - - '0025.

$\frac{3}{4}$ per cent. is 3 times $\frac{1}{4}$ per cent., that is, - - - '0075.

Note. The rate per cent. is a decimal carried to *two places*, that is, to *hundredths;* all decimal expressions *lower* than hundredths are parts of 1 per cent. $\frac{5}{8}$ per cent., for instance, is '625 of 1 per cent., that is, '00625.

Write $2\frac{1}{2}$ per cent. as a decimal fraction.

2 per cent. is '02, and $\frac{1}{2}$ per cent. is '005. *Ans.* '025.

Write 4 per cent. as a decimal fraction. —— $4\frac{1}{2}$ per cent. —— $4\frac{3}{4}$ per cent. —— 5 per cent. —— $7\frac{1}{4}$ per cent. —— 8 per cent. —— $8\frac{3}{4}$ per cent. —— 9 per cent. —— $9\frac{1}{2}$ per cent. —— 10 per cent. (10 per cent. is $\frac{10}{100}$; decimally, '10.) —— $10\frac{1}{2}$ per cent. —— 11 per cent. —— $12\frac{1}{2}$ per cent. —— 15 per cent.

1. If the interest on $ 1, for 1 year, be 6 cents, what will be the interest on $ 17 for the same time?

It will be 17 times 6 cents, or 6 times 17, which is the same thing:—

$ 17
'06
———
1'02 *Answer;* that is, 1 dollar and 2 cents.

To find the interest on any sum for 1 year, it is evident we need only to multiply it by the *rate per cent.* written as a *decimal* fraction. The product, observing to place the point as directed in multiplication of decimal fractions, will be the interest required.

Note. PRINCIPAL is the money *due*, for which interest is paid. AMOUNT is the principal and interest added together.

2. What will be the interest of \$32'15, 1 year, at $4\frac{1}{2}$ per cent.?

	\$32'15 *principal.*
	'045 *rate per cent.*
	16075
	12860
Ans.	\$1'44675

There being five decimal places in the multiplicand and multiplier, five figures must be pointed off for decimals from the product, which gives the answer,—1 dollar, 44 cents, 6 mills, and $\frac{75}{100}$ of a mill. *Parts* of a mill are not generally regarded; hence, \$1'446 is sufficiently exact for the answer.

3. What will be the interest of \$11'04 for 1 year, at 3 per cent.? —— at $5\frac{1}{2}$ per cent.? —— at 6 per cent.? —— at $7\frac{1}{4}$ per cent.? —— at $8\frac{1}{2}$ per cent.? —— at $9\frac{3}{4}$ per cent.? —— at 10 per cent.? —— at $10\frac{1}{4}$ per cent.? —— at 11 per cent.? —— at $11\frac{3}{4}$ per cent.? —— at 12 per cent.? —— at $12\frac{1}{2}$ per cent.?

4. A tax on a certain town is \$1627'18, on which the collector is to receive $2\frac{1}{2}$ per cent. for collecting; what will he receive for collecting the whole tax at that rate?

Ans. \$40'679.

Note. In the same way are calculated commission, insurance, buying and selling stocks, loss and gain, or any thing else rated at so much per cent. *without respect to time.*

5. What must a man, paying \$0'$37\frac{1}{2}$ on a dollar, pay on a debt of \$132'25? *Ans.* \$49'593.

6. A merchant, having purchased goods to the amount of \$580, sold them so as to gain $12\frac{1}{2}$ per cent., that is, $12\frac{1}{2}$ cents on each 100 cents, and in the same proportion for a greater or less sum; what was his whole gain, and what was the whole amount for which he sold the goods?

Ans. His whole gain was \$72'50; whole amount \$652'50.

7. A merchant bought a quantity of goods for \$763'$37\frac{1}{2}$; how much must he sell them for to gain 15 per cent.?

Ans. \$877'881.

¶ 82. COMMISSION is an allowance of so much per cent. to a person called a *correspondent*, *factor*, or *broker*, for assisting merchants and others in purchasing and selling goods.

8. My correspondent sends me word that he has purchased goods to the value of $1286, on my account; what will his commission come to at 2½ per cent.? *Ans.* $32‘15.

9. What must I allow my correspondent for selling goods to the amount of $2317‘46, at a commission of 3¼ per cent.? *Ans.* $75‘317.

Insurance is an exemption from hazard, obtained by the payment of a certain sum, which is generally so much *per cent.* on the estimated value of the property insured.

Premium is the sum paid by the insured for the insurance.

Policy is the name given to the instrument or writing, by which the contract of indemnity is effected between the insurer and insured.

10. What will be the premium for insuring a ship and cargo from Boston to Amsterdam, valued at $37800, at 4½ per cent.? *Ans.* $1701.

11. What will be the annual premium for insurance on a house against loss from fire, valued at $3500, at ¾ per cent.?

By removing the separatrix 2 figures towards the left, it is evident, the sum itself may be made to express the premium at 1 per cent., of which the given rate parts may be taken; thus, 1 per cent. on $3500 is $35‘00, and ¾ of $35‘00 is $26‘25, *Answer.*

12. What will be the premium for insurance on a ship and cargo valued at $25156‘86, at ½ per cent.? —— at ⅔ per cent.? —— at ¾ per cent.? —— at ⅛ per cent? —— at ⅝ per cent.? *Ans.* At ⅝ per cent. the premium is $157‘23.

Stock is a general name for the capital of any trading company or corporation, or of a fund established by government.

The value of stock is variable. When 100 dollars of stock sells for 100 dollars in *money*, the stock is said to be at *par*, which is a Latin word signifying *equal;* when for *more*, it is said to be *above* par; when for *less*, it is said to be *below* par.

13. What is the value of $7564 of stock, at 112½ per cent.? that is, when 1 dollar of stock sells for 1 dollar 12½

cents in *money*, which is 12½ per cent. above par, or 12½ per cent. *advance*, as it is sometimes called. *Ans.* $ 8509'50.

14. What is the value of $ 3700 of bank stock, at 95½ per cent., that is, 4½ per cent. *below* par? *Ans.* $ 3533'50.

15. What is the value of $ 120 of stock, at 92½ per cent.? —— at 86¼ per cent.? —— at 67¾ per cent.? —— at 104½ per cent.? —— at 108¼ per cent.? —— at 115 per cent.? —— at 37½ per cent. *advance*?

Loss and Gain. 16. Bought a hogshead of molasses for $ 60; for how much must I sell it to gain 20 per cent.? *Ans.* $ 72.

17. Bought broadcloth at $ 2'50 per yard; but, it being damaged, I am willing to sell it so as to lose 12 per cent.; how much will it be per yard? *Ans.* $ 2'20.

18. Bought calico at 20 cents per yard; how must I sell it to gain 5 per cent.? —— 10 per cent.? —— 15 per cent.? —— to lose 20 per cent.? *Ans. to the last*, 16 cents per yard.

¶ **83.** We have seen how interest is cast on any sum of money, when the time is *one year;* but it is frequently necessary to cast interest for months and days.

Now, the interest on $ 1 for 1 year, at 6 per cent., being '06, is

'01 cent for 2 months,

'005 mills (or ½ a cent) for 1 month of 30 days, (for so we reckon a month in casting interest,) and

'001 mill for every 6 days; 6 being contained 5 times in 30.

Hence, it is very easy to find by *inspection*, that is, to cast in the mind, the interest on 1 dollar, at 6 per cent. for *any given time.* The *cents*, it is evident, will be equal to *half* the greatest even number of the months; the *mills* will be 5 for the odd month, if there be one, and 1 for every time 6 is contained in the given number of the days.

Suppose the interest of $ 1, at 6 per cent., be required for 9 months, and 18 days. The greatest even number of the months is 8 half of which will be the cents, '04; the mills, reckoning 5 for the odd month, and 3 for the 18 (3 times 6 = 18) days, will be '008, which, united with the cents, ('048,) give 4 cents 8 mills for the interest of $ 1 for 9 months and 18 days.

1. What will be the interest on $1 for 5 months 6 days? —— 6 months 12 days? —— 7 months? —— 8 months 24 days? —— 9 months 12 days? —— 10 months? —— 11 months 6 days? —— 12 months 18 days? —— 15 months 6 days? —— 16 months?

ODD DAYS. 2. What is the interest of $1 for 13 months 16 days?

The cents will be 6, and the mills 5, for the odd month, and 2 for 2 times 6 = 12 days, and there is a remainder of 4 days, the interest for which will be such part of 1 mill as 4 days is part of 6 days, that is, $\frac{4}{6} = \frac{2}{3}$ of a mill. *Ans.* '067$\frac{2}{3}$.

3. What will be the interest of $1 for 1 month 8 days? —— 2 months 7 days? —— 3 months 15 days? —— 4 months 22 days? —— 5 months 11 days? —— 6 months 17 days? —— 7 months 3 days? —— 8 months 11 days? —— 9 months 2 days? —— 10 months 15 days? —— 11 months 4 days? —— 12 months 3 days?

Note. If there is no odd month, and the number of days be less than 6, *so that there are no mills*, it is evident, a *cipher* must be put in the place of mills; thus, in the last example,—12 months 3 days,—the cents will be '06, the mills 0, the 3 days $\frac{1}{2}$ a mill. *Ans.* '060$\frac{1}{2}$.

4. What will be the interest of $1 for 2 months 1 day? —— 4 months 2 days? —— 6 months 3 days? —— 8 months 4 days? —— 10 months 5 days? —— for 3 days? —— for 1 day? —— for 2 days? —— for 4 days? —— for 5 days?

5. What is the interest of $56'13 for 8 months 5 days? The interest of $1, for the given time, is '040$\frac{5}{6}$; therefore,

$\frac{1}{2}$) and $\frac{1}{3}$) $56'13 principal.
'040$\frac{5}{6}$ interest of $1 for the given time.

224520 interest for 8 months.
2806 interest for 3 days.
1871 interest for 2 days.

2'29197, *Ans.* $2'291.

5 days = 3 days + 2 days. As the multiplicand is taken *once* for every 6 days, for 3 days take $\frac{1}{2}$, for 2 days take $\frac{1}{3}$,

of the multiplicand. $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$. So also, if the odd days be $4 = 2$ days $+ 2$ days, take $\frac{1}{3}$ of the multiplicand *twice*; for 1 day, take $\frac{1}{6}$.

Note. If the sum on which interest is to be cast be *less* than $ 10, the interest, for any number of days *less* than 6, will be *less* than 1 cent; consequently, *in business*, if the sum be less than $ 10, such days need not be regarded.

From the illustrations now given, it is evident,—*To find the interest of any sum in federal money, at* 6 *per cent.*, it is only necessary to multiply the principal by the interest of $ 1 for the given time, found as above directed, and written as a decimal fraction, remembering to point off as many places for decimals in the product as there are decimal places in both the factors counted together.

EXAMPLES FOR PRACTICE.

6. What is the interest of $ 87‘19 for 1 year 3 months?

Ans. $ 6‘539.

7.	Interest of $ 116,08 for 11 mo. 19 days?	$ 6‘751.
8.	.... of $ 200 for 8 mo. 4 days?	$ 8‘132.
9.	.... of $ 0‘85 for 19 mo.?	$ ‘08.
10.	.... of $ 8‘50 for 1 year 9 mo. 12 days?	$ ‘909.
11.	.... of $ 675 for 1 mo. 21 days?	$ 5‘737.
12.	.... of $ 8673 for 10 days?	$ 14‘455.
13.	.... of $ 0‘73 for 10 mo.?	$ ‘036.
14.	.... of $ 96 for 3 days?	
15.	.... of $ 73‘50 for 2 days?	
16.	.... of $ 180‘75 for 5 days?	
17.	.... of $ 15000 for 1 day?	

Note. The interest of $ 1 for 6 days being 1 mill, the dollars *themselves* express the interest in *mills for six days*, of which we may take parts.

Thus, 6) 15000 mills,

2‘500, that is, $ 2‘50, *Ans.* to the last.

When the interest is required for a large number of years, it will be more convenient to find the interest for one year, and multiply it by the number of years; after which find the interest for the months and days, if any, as usual.

18. What is the interest of $ 1000 for 120 years?

Ans. $ 7200

19. What is the interest of $ 520‘04 for 30 years and 6 months? *Ans.* $ 951‘673.

20. What is the interest on $ 400 for 10 years 3 months and 6 days? *Ans.* $ 246'40.

21. What is the interest of $ 220 for 5 years? —— for 12 years? —— 50 years? *Ans. to last,* $ 660.

22. What is the amount of $ 86, at interest 7 years? *Ans.* $ 122'12.

23. What is the interest of 36 £. 9 s. 6½ d. for 1 year? Reduce the shillings, pence, &c. to the decimal of a pound, by inspection, (¶ 76;) then proceed in all respects as in federal money. Having found the interest, reverse the operation, and reduce the three first decimals to shillings, &c., by inspection. (¶ 77.) *Ans.* 2 £. 3 s. 9 d.

24. Interest of 36 £. 10 s. for 18 mo. 20 days? *Ans.* 3 £. 8 s. 1½ d. Interest of 95 £. for 9 mo.? *Ans.* 4 £. 5 s. 6 d.

25. What is the amount of 18 £. 12 s. at interest 10 months 3 days? *Ans.* 19 £. 10 s. 9¼ d.

26. What is the amount of 100 £. for 8 years? *Ans.* 148 £.

27. What is the amount of 400 £. 10 s. for 18 months? *Ans.* 436 £. 10 s. 10 d. 3 q.

28. What is the amount of 640 £. 8 s. at interest for 1 year? —— for 2 years 6 months? —— for 10 years? *Ans. to last,* 1024 £. 12 s. 9½ d.

¶ **84.** 1. What is the interest of 36 dollars for 8 months, at 4½ per cent.?

Note. When the rate is any other than *six per cent.*, first find the interest at 6 per cent., then divide the interest so found by such part as the interest, at the rate *required*, exceeds or falls short of the interest at 6 per cent., and the quotient added to, or subtracted from the interest at 6 per cent., as the case may be, will give the interest at the rate required.

```
   $ 36
    '04
   ----
¼)1'44
   '36
   ----
  1'08 Ans.
```

4½ per cent. is ¾ of 6 per cent.; therefore, from the interest at 6 per cent. subtract ¼; the remainder will be the interest at 4½ per cent.

2. Interest of $ 54'81 for 18 mo., at 5 per ct.? *Ans.* $ 4'11.
3. .... of $ 500 for 9 mo. 9 days, at 8 per ct.? $ 31'00
4. .... of $ 62'12 for 1 mo. 20 days, at 4 per ct.? $ '345.

5. Interest of $ 85 for 10 mo. 15 days, at 12½ per cent.? *Ans.* $ 9‘295.

6. What is the amount of $ 53 at 10 per ct. for 7 mo.? *Ans.* $ 56‘091.

The time, rate per cent. and amount given, to find the principal.

¶ **85.** 1. What sum of money, put at interest at 6 per cent., will amount to $ 61‘02, in 1 year 4 months?

The amount of $ 1, at the given rate and time, is $ 1‘08; hence, $ 61‘02 ÷ $ 1‘08 = 56‘50, the principal required; that is,—*Find the amount of $ 1 at the given rate and time, by which divide the given amount; the quotient will be the principal required.* *Ans.* $ 56‘50.

2. What principal, at 8 per cent., in 1 year 6 months, will amount to $ 85‘12? *Ans.* $ 76.

3. What principal, at 6 per cent., in 11 months 9 days, will amount to $ 99‘311?

Note. The interest of $ 1, for the given time, is ‘056½; but, in these cases, when there are odd days, instead of writing the parts of a mill as a common fraction, it will be more convenient to write them as a *decimal*, thus, ‘0565; that is, extend the decimal to four places. *Ans.* $ 94.

4. A factor receives $ 988 to lay out after deducting his commission of 4 per cent.; how much will remain to be laid out?

It is evident, he ought not to receive commission on his *own* money. This question, therefore, in principle, does not differ from the preceding.

Note. In questions like this, where no respect is had to *time*, (¶ 81, ex. 4, note,) add the *rate* to $ 1. *Ans.* $ 950.

5. A factor receives $ 1008 to lay out after deducting his commission of 5 per cent.; what does his commission amount to? *Ans.* $ 48.

Discount. 6. Suppose I owe a man $ 397‘50, to be paid in 1 year, without interest, and I wish to pay him now; how much ought I to pay him when the usual rate is 6 per cent.?

I ought to pay him such a sum as, put at interest, would, in 1 year, amount to $ 397‘50. The question, therefore, does not differ from the preceding. *Ans.* $ 375.

Note. An allowance made for the payment of any sum

of money before it becomes due, as in the last example, is called *Discount.*

The sum which, put at interest, would, in the time and at the rate per cent. for which discount is to be made, amount to the given sum, or debt, is called the *present worth.*

7. What is the present worth of $ 834, payable in 1 year 7 months and 6 days, discounting at the rate of 7 per cent.? *Ans.* $ 750.

8. What is the discount on $ 321'63, due 4 years hence, discounting at the rate of 6 per cent.? *Ans.* $ 62'26.

9. How much ready money must be paid for a note of $ 18, due 15 months hence, discounting at the rate of 6 per cent.? *Ans.* $ 16'744.

10. Sold goods for $ 650, payable one half in 4 months, and the other half in 8 months; what must be discounted for present payment? *Ans.* $ 18.

11. What is the present worth of $ 56'20, payable in 1 year 8 months, discounting at 6 per cent.? —— at $4\frac{1}{2}$ per cent.? —— at 5 per cent.? —— at 7 per cent.? —— at $7\frac{1}{2}$ per cent.? —— at 9 per cent.?

Ans. to the last, $ 48'869.

The time, rate per cent., and interest being given, to find the principal.

¶ **86.** 1. What sum of money, put at interest 16 months, will gain $ 10'50, at 6 per cent.?

$ 1, at the given rate and time, will gain '08; hence, $ 10'50 ÷ $ '08 = $ 131'25, the principal required; that is,—*Find the interest of $ 1, at the given rate and time, by which divide the given gain, or interest; the quotient will be the principal required.* *Ans.* $ 131'25.

2. A man paid $ 4'52 interest, at the rate of 6 per cent. at the end of 1 year 4 months; what was the principal? *Ans.* $ 56'50.

3. A man received, for interest on a certain note, at the end of 1 year, $ 20; what was the principal, allowing the rate to have been 6 per cent.? *Ans.* $ 333'333$\frac{1}{3}$.

The principal, interest, and time being given, to find the rate per cent.

¶ **87.** 1. If I pay $3'78 interest, for the use of $36 for 1 year and 6 months, what is that per cent.?

The interest on $36, at *one per cent.* the given time, is $'54; hence, $3'78 ÷ $'54 = '07, the rate required; that is,— *Find the interest on the given sum, at* 1 *per cent. for the given time, by which divide the given interest; the quotient will be the rate at which interest was paid.* *Ans.* 7 per cent.

2. If I pay $2'34 for the use of $468, 1 month, what is the rate per cent.? *Ans.* 6 per cent.

3. At $46'80 for the use of $520, 2 years, what is that per cent.? *Ans.* $4\frac{1}{2}$ per cent.

The prices at which goods are bought and sold being given, to find the rate per cent. of GAIN *or* LOSS.

¶ **88.** 1. If I purchase wheat at $1'10 per bushel, and sell it at $1'$37\frac{1}{2}$ per bushel, what do I gain per cent.?

This question does not differ essentially from those in the foregoing paragraph. Subtracting the cost from the price at sale, it is evident I gain $27\frac{1}{2}$ cents on a bushel, that is, $\frac{'275}{1'10}$ of the first cost. $\frac{'275}{1'10}$ = '25 per cent., the *Answer.* That is, —*Make a common fraction, writing the gain or loss for the numerator, and the price at which the article was bought for the denominator; then reduce it to a decimal.*

2. A merchant purchases goods to the amount of $550; what per cent. profit must he make to gain $66? *Ans.* 12 per cent.

3. —— What per cent. profit must he make on the same purchase to gain $38'50? —— to gain $24'75? —— to gain $2'75?

Note. The last gain gives for a quotient '005, which is $\frac{1}{2}$ per cent. The rate per cent., it must be recollected, (¶ 81, note,) is a decimal carried to *two* places, or *hundredths;* all decimal expressions *lower* than hundredths are parts of 1 per cent.

4. Bought a hogshead of rum, containing 114 gallons, at 96 cents per gallon, and sold it again at $1'0032 per gallon; what was the whole gain, and what was the gain per cent.?

Ans. { $4'924, whole gain, $4\frac{1}{2}$ gain per cent. }

5. A merchant bought a quantity of tea for $ 365, which, proving to have been damaged, he sold for $ 332'15; what did he lose per cent.? *Ans.* 9 per cent.

6. If I buy cloth at $ 2 per yard, and sell it for $ 2'50 per yard, what should I gain in laying out $ 100?

Ans. $ 25.

7. Bought indigo at $ 1'20 per lb., and sold the same at 90 cents per lb.; what was lost per cent.? *Ans.* 25 per cent.

8. Bought 30 hogsheads of molasses, at $ 600; paid in duties $ 20'66; for freight, $ 40'78; for porterage, $ 6'05, and for insurance, $ 30'84: if I sell them at $ 26 per hogshead, how much shall I gain per cent.? *Ans.* 11'695 per cent.

The principal, rate per cent., and interest being given, to find the time.

¶ **89.** 1. The interest on a note of $ 36, at 7 per cent., was $ 3'78; what was the time?

The interest on $ 36, 1 year, at 7 per cent., is $ 2'52; hence, $ 3'78 ÷ $ 2'52 = 1'5 years, the time required; that is,—*Find the interest for 1 year on the principal given, at the given rate, by which divide the given interest; the quotient will be the time required, in years and decimal parts of a year;* the latter may then be reduced to months and days.

Ans. 1 year 6 months.

2. If $ 31'71 interest be paid on a note of $ 226'50, what was the time, the rate being 6 per cent.?

Ans. 2'33$\frac{1}{3}$ = 2 years 4 months.

3. On a note of $ 600, paid interest $ 20, at 8 per cent.; what was the time?

Ans. '416 + = 5 months so nearly as to be called 5, and would be exactly 5 but for the fraction lost.

4. The interest on a note of $ 217'25, at 4 per cent., was $ 28'242; what was the time? *Ans.* 3 years 3 months.

Note. When the rate is 6 per cent., we may divide the interest by $\frac{1}{2}$ the principal, removing the separatrix *two* places to the left, and the *quotient* will be the answer in *months.*

To find the interest due on notes, &c. when partial payments have been made.

¶ **90.** In Massachusetts the law provides, that payments shall be applied to keep down the interest, and that neither *interest* nor *payment* shall ever *draw interest.* Hence, if the payment at any time *exceed* the interest computed to the *same* time, that excess is taken from the principal; but if the payment be *less* than the interest, the principal remains unaltered. Wherefore, we have this RULE :—Compute the interest to the first time when a payment was made, which, either alone, or together with the preceding payments, if any, exceeds the interest then due; add that interest to the principal, and from the sum subtract the payment, or the *sum* of the payments, made within the time for which the interest was computed, and the remainder will be a new principal, with which proceed as with the first.

1. *For value received, I promise to pay* JAMES CONANT, *or order, one hundred sixteen dollars sixty-six cents and six mills, with interest.* *May* 1, 1822.

$ 116,666. SAMUEL ROOD.

On this note were the following endorsements:

Dec. 25, 1822, received	$ 16'666	*Note.* In finding the *times* for computing the interest, consult ¶ 40.
July 10, 1823, ...	$ 1'666	
Sept. 1, 1824, ...	$ 5'000	
June 14, 1825, ...	$ 33'333	
April 15, 1826, ...	$ 62'000	

What was due August 3, 1827? *Ans.* $ 23'775.

The first principal on interest from May 1, 1822,	$ 116'666
Interest to Dec. 25, 1822, time of the first payment, (7 months 24 days,) - - -	4'549
Amount,	$ 121'215
Payment, Dec. 25, exceeding interest then due,	16'666
Remainder for a new principal, - - - -	104'549
Interest from Dec. 25, 1822, to June 14, 1825, (29 months 19 days,) - - - - -	15'490
Amount carried forward,	$ 120'039

Amount brought forward,		$ 120'039
Payment, July 10, 1823, less than interest then due, - - - - -	$ 1'666	
Payment, Sept. 1, 1824, less than interest then due, - - - - -	5'000	
Payment, June 14, 1825, exceeding interest then due, - - -	33'333	
		$ 39'999
Remainder for a new principal, (June 14, 1825,)		80'040
Interest from June 14, 1825, to April 15, 1826, (10 months 1 day,) - - - -		4'015
Amount,		$ 84'055
Payment, April 15, 1825, exceeding interest then due, - - - - - - - -		62'000
Remainder for a new principal, (April 15, 1826,)		$ 22'055
Interest due Aug. 3, 1827, from April 15, 1826, (15 months 18 days,) - - - - -		1'720
Balance due Aug 3, 1827, - - -		$ 23'775

2. *For value received, I promise to pay* James Lowell, *or order, eight hundred sixty-seven dollars and thirty-three cents with interest.* *Jan.* 6. 1820.

$ 867'33. Hiram Simson.

On this note were the following endorsements, viz.

April 16, 1823, received $ 136'44.
April 16, 1825, received $ 319.
Jan. 1, 1826, received $ 518'68.

What remained due July 11, 1827? *Ans.* $ 215'103.

COMPOUND INTEREST.

¶ 91. A promises to pay B $ 256 in 3 years, with interest annually; but at the end of 1 year, not finding it convenient to pay the interest, he consents to pay interest on the interest from that time, the same as on the principal.

Note. *Simple interest* is that which is allowed for the *principal* only; *compound interest* is that which is allowed

for both *principal* and *interest*, when the latter is not paid at the time it becomes due.

Compound interest is calculated by adding the interest to the principal at the end of each year, and making the *amount* the principal for the next succeeding year.

1. What is the compound interest of $ 256 for 3 years, at 6 per cent.?

$ 256	given sum, or first principal.	
‘06		
15‘36	interest,	to be added together.
256‘00	principal,	
271‘36	amount, or principal for 2d year.	
‘06		
16‘2816	compound interest, 2d year,	added together.
271‘36	principal, do.	
287‘6416	amount, or principal for 3d year.	
‘06		
17‘25846	compound interest, 3d year,	added together.
287‘641	principal, do.	
304‘899	amount.	
256	first principal subtracted.	
Ans. $ 48‘899	compound interest for 3 years.	

2. At 6 per cent., what will be the compound interest, and what the amount, of $ 1 for 2 years? —— what the amount for 3 years? —— for 4 years? —— for 5 years? —— for 6 years? —— for 7 years? —— for 8 years?

Ans. to the last, $ 1‘593+.

It is plain that the amount of $ 2, for any given time, will be 2 times as much as the amount of $ 1; the amount of $ 3 will be 3 times as much, &c.

Hence, we may form the amounts of $ 1, for several years, into a table of *multipliers* for finding the amount of *any sum* for the same time.

TABLE,

Showing the amount of $1, or 1£., &c. for any number of years, not exceeding 24, at the rates of 5 and 6 per cent. compound interest.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1‘05	1‘06	13	1‘88564 +	2‘13292 +
2	1‘1025	1‘1236	14	1‘97993 +	2‘26090 +
3	1‘15762 +	1‘19101 +	15	2‘07892 +	2‘39655 +
4	1‘21550 +	1‘26247 +	16	2‘18287 +	2‘54035 +
5	1‘27628 +	1‘33822 +	17	2‘29201 +	2‘69277 +
6	1‘34009 +	1‘41851 +	18	2‘40661 +	2‘85433 +
7	1‘40710 +	1‘50363 +	19	2‘52695	3‘02559 +
8	1‘47745 +	1‘59384 +	20	2‘65329 +	3‘20713 +
9	1‘55132 +	1‘68947 +	21	2‘78596 +	3‘39956 +
10	1‘62889 +	1‘79084 +	22	2‘92526 +	3‘60353 +
11	1‘71033 +	1‘89829 +	23	3‘07152 +	3‘81974 +
12	1‘79585 +	2‘01219 +	24	3‘22509 +	4‘04893 +

Note 1. Four decimals in the above numbers will be sufficiently accurate for most operations.

Note 2. When there are months and days, you may first find the amount for the *years*, and on that amount cast the interest for the months and days; this, added to the amount, will give the answer.

3. What is the amount of $600‘50 for 20 years, at 5 per cent. compound interest? —— at 6 per cent.?

$1 at 5 per cent., by the table, is $2‘65329; therefore, 2‘65329 × 600‘50 = $1593‘30 + *Ans.* at 5 per cent.; and 3‘20713 × 600‘50 = $1925‘881 + *Ans.* at 6 per cent.

4. What is the amount of $40‘20 at 6 per cent. compound interest, for 4 years? —— for 10 years? —— for 18 years? —— for 12 years? —— for 3 years and 4 months? —— for 24 years, 6 months, and 18 days?

Ans. to last, $168‘137.

Note. Any sum at compound interest will double itself in 11 years, 10 months, and 22 days.

From what has now been advanced we deduce the following general

RULE.

I. *To find the interest when the time is* 1 *year, or, to find the rate per cent. on any sum of money, without respect to time, as*

the premium for insurance, commission, &c.,—Multiply the principal, or given sum, by the rate per cent., written as a decimal fraction; the product, remembering to point off as many places for decimals as there are decimals in both the factors, will be the interest, &c. required.

II. *When there are months and days in the given time, to find the interest on any sum of money at* 6 *per cent.*,—Multiply the principal by the interest on $ 1 for the given time, found by inspection, and the product, as before, will be the interest required.

III. *To find the interest on* $ 1 *at* 6 *per cent., for any given time, by inspection,*—It is only to consider, that the *cents* will be equal to half the greatest even number of the months; and the *mills* will be 5 for the odd month, (if there be one,) and 1 for every 6 days.

IV. *If the sum given be in pounds, shillings, pence and farthings,*—Reduce the shillings, &c. to the decimal of a pound, by inspection, (¶ 76;) then proceed in all respects as in federal money. Having found the interest, the decimal part, by reversing the operation, may be reduced back to shillings, pence and farthings.

V. *If the interest required be at any other rate than* 6 *per cent.*, (*if there be months, or months and days, in the given time,*) —First find the interest at 6 per cent.; then divide the interest so found by such part or parts, as the interest, at the rate required, exceeds or falls short of the interest at 6 per cent., and the quotient, or quotients, added to or subtracted from the interest at 6 per cent., as the case may require, will give the interest at the rate required.

Note. The interest on any number of dollars, for 6 days, at 6 per cent., is readily found by cutting off the unit or right hand figure; those at the *left* hand will show the interest in *cents* for 6 days.

EXAMPLES FOR PRACTICE.

1. What is the interest of $ 1600 for 1 year and 3 months? *Ans.* $ 120.

2. What is the interest of $ 5'811, for 1 year 11 months? *Ans.* $ '668.

3. What is the interest of $ 2'29, for 1 month 19 days, at 3 per cent.? *Ans.* $ '009.

4. What is the interest of $ 18, for 2 years 14 days, at 7 per cent.? *Ans.* $ 2'569.

5. What is the interest of $17'68, for 11 months 28 days? *Ans.* $1'054.

6. What is the interest of $200 for 1 day? —— 2 days? —— 3 days? —— 4 days? —— 5 days? *Ans.* for 5 days, $0'166.

7. What is the interest of half a mill for 567 years? *Ans.* $0'017.

8. What is the interest of $81, for 2 years 14 days, at $\frac{1}{2}$ per cent.? —— $\frac{3}{4}$ per cent.? —— $\frac{5}{8}$ per cent.? —— 2 per cent.? —— 3 per cent.? —— $4\frac{1}{2}$ per cent.? —— 5 per cent.? —— 6 per cent.? —— 7 per cent.? —— $7\frac{1}{2}$ per cent.? —— 8 per cent.? —— 9 per cent.? —— 10 per cent.? —— 12 per cent.? —— $12\frac{1}{2}$ per cent.? *Ans. to last,* $20'643.

9. What is the interest of 9 cents for 45 years, 7 months, 11 days? *Ans.* $0'245.

10. A's note of $175 was given Dec. 6, 1798, on which was endorsed one year's interest; what was there due Jan. 1, 1803?

Note. Consult ex. 16, Supplement to Subtraction of Compound Numbers. *Ans.* $207'22.

11. B's note of $56'75 was given June 6, 1801, on interest after 90 days; what was there due Feb. 9, 1802? *Ans.* $58'19.

12. C's note of $365'37 was given Dec. 3, 1797; June 7, 1800, he paid $97'16; what was there due Sept. 11, 1800? *Ans.* $328'32.

13. Supposing a note of $317'92, dated July 5, 1797, on which were endorsed the following payments, viz. Sept. 13, 1799, $208'04; March 10, 1800, $76; what was there due Jan. 1, 1801? *Ans.* $83'991.

SUPPLEMENT TO INTEREST.

QUESTIONS.

1. What is interest? 2. How is it computed? 3. What is understood by rate per cent.? 4. —— by principal? 5. —— by amount? 6. —— by legal interest? 7. —— by commission? 8. —— insurance? 9. —— premium? 10. —— policy? 11. —— stock? 12. What is understood by stock being at *par*? 13. —— above par? 14.

—— below par? 15. The rate per cent. is a decimal carried to how many places? 16. What are decimal expressions *lower* than hundredths? 17. How is interest, (when the time is 1 year,) commission, insurance, or any thing else rated at so much per cent. without respect to time, found? 18. When the rate is 1 per cent., or less, how may the operation be contracted? 19. How is the interest on $ 1, at 6 per cent. for any given time, found by inspection? 20. How is interest cast, at 6 per cent., when there are months and days in the given time? 21. When the given time is *less* than 6 days, how is the interest most readily found? 22. If the sum given be in pounds, shillings, &c., how is interest cast? 23. When the rate is any other than 6 per cent, if there be months and days in the given time, how is the interest found? 24. What is the rule for casting interest on notes, &c. when partial payments have been made, and what is the principle on which the rule is founded? 25. How may the principal be found, the time, rate per cent., and amount being given? 26. What is understood by *discount*? 27. —— by *present worth*? 28. How is the principal found, the time, rate per cent., and interest being given? 29. How is the rate per cent. of gain or loss found, the prices at which goods are bought and sold being given? 30. How is the rate per cent. found, the principal, interest, and time being given? 31. How is the time found, the principal, rate per cent., and interest being given? 32. What is simple interest? 33. —— compound interest? 34. How is compound interest computed?

EXERCISES.

1. What is the interest of $ 273‘51 for 1 year 10 days, at 7 per cent.? *Ans.* $ 19‘677.
2. What is the interest of $ 486 for 1 year, 3 months, 19 days, at 8 per cent.? *Ans.* $ 50‘652.
3. D's note of $ 203‘17 was given Oct. 5, 1808, on interest after three months; Jan. 5, 1809, he paid $ 50; what was there due May 2, 1811? *Ans.* $ 174‘53.
4. E's note of $ 870‘05 was given Nov. 17, 1800, on interest after 90 days; Feb. 11, 1805, he paid $ 186‘06; what was there due Dec. 23, 1807? *Ans.* $ 1041‘58.
5. What will be the annual insurance, at $\frac{5}{8}$ per cent., on a house valued at $ 1600? *Ans.* $ 10.

6. What will be the insurance of a ship and cargo, valued at $ 5643, at $1\frac{1}{2}$ per cent.? —— at $\frac{1}{4}$ per cent.? —— at $\frac{7}{16}$ per cent.? —— at $1\frac{1}{2}$ per cent.? —— at $\frac{3}{4}$ per cent.?

Note. Consult ¶ 82, ex. 11.

Ans. at $\frac{3}{4}$ per cent. $ 42'322.

7. A man having compromised with his creditors at $62\frac{1}{2}$ cents on a dollar, what must he pay on a debt of $ 137'46?

Ans. $ 85'912.

8. What is the value of $ 800 United States Bank stock, at $112\frac{1}{2}$ per cent.? *Ans.* $ 900.

9. What is the value of $ 560'75 of stock, at 93 per cent.?

Ans. $ 521'497

10. What principal at 7 per cent. will, in 9 months 18 days, amount to $ 422'40? *Ans.* $ 400.

11. What is the present worth of $ 426, payable in 4 years and 12 days, discounting at the rate of 5 per cent.?

In large sums, to bring out the cents correctly, it will sometimes be necessary to extend the decimal in the divisor to five places. *Ans.* $ 354'506.

12. A merchant purchased goods for $ 250 ready money, and sold them again for $ 300, payable in 9 months; what did he gain, discounting at 6 per cent.? *Ans.* $ 37'081.

13. Sold goods for $ 3120, to be paid, one half in 3 months, and the other half in 6 months; what must be discounted for present payment? *Ans.* 68'492.

14. The interest on a certain note, for 1 year 9 months, was $ 49'875; what was the principal? *Ans.* $ 475.

15. What principal, at 5 per cent., in 16 months 24 days, will gain $ 35? *Ans.* $ 500.

16. If I pay $ 15'50 interest for the use of $ 500, 9 months and 9 days, what is the rate per cent.?

17. If I buy candles at $ '167 per lb., and sell them at 20 cents, what shall I gain in laying out $ 100?

Ans. $ 19'76.

18. Bought hats at 4 s. apiece, and sold them again at 4 s. 9 d.; what is the profit in laying out 100 £.?

Ans. 18 £. 15 s.

19. Bought 37 gallons of brandy, at $ 1'10 per gallon, and sold it for $ 40; what was gained or lost per cent.?

20. At 4 s. 6 d. profit on 1 £., how much is gained in laying out 100 £., that is, how much per cent.? *Ans.* 22 £. 10 s.

21. Bought cloth at $ 4'48 per yard; how must I sell it to gain $12\frac{1}{2}$ per cent.? *Ans.* $ 5'04

22. Bought a barrel of powder for 4 £.; for how much must it be sold to lose 10 per cent.? *Ans.* 3 £. 12 s.

23. Bought cloth at 15 s. per yard, which not proving so good as I expected, I am content to lose '17½ per cent.; how must I sell it per yard? *Ans.* 12 s. 4½ d.

24. Bought 50 gallons of brandy, at 92 cents per gallon, but by accident 10 gallons leaked out; at what rate must I sell the remainder per gallon to gain upon the whole cost at the rate of 10 per cent.? *Ans.* $ 1'265 per gallon.

25. A merchant bought 10 tons of iron for $ 950; the freight and duties came to $ 145, and his own charges to $ 25; how must he sell it per lb. to gain 20 per cent. by it? *Ans.* 6 cents per lb.

EQUATION OF PAYMENTS.

¶ **92.** Equation of payments is the method of finding the mean time for the payment of several debts, due at different times.

1. In how many months will $ 1 gain as much as 5 dollars will gain in 6 months?

2. In how many months will $ 1 gain as much as $ 40 will gain in 15 months? *Ans.* 600.

3. In how many months will the use of $ 5 be worth as much as the use of $ 1 for 40 months?

4. Borrowed of a friend $ 1 for 20 months; afterwards lent my friend $ 4; how long ought he to keep it to become indemnified for the use of the $ 1?

5. I have three notes against a man; one of $ 12, due in 3 months; one of $ 9, due in 5 months; and the other of $ 6, due in 10 months; the man wishes to pay the whole at once; in what time ought he to pay it?

$ 12	for 3 months is the same as	$ 1	for 36	months, and
$ 9	for 5 months is the same as	$ 1	for 45	months, and
$ 6	for 10 months is the same as	$ 1	for 60	months.
27			141	

He might, therefore, have $ 1 141 months, and he may keep 27 dollars $\frac{1}{27}$ part as long; that is, $\frac{141}{27} = 5$ months 6 + days, *Answer.*

Hence, *To find the mean time for several payments*,—RULE: —Multiply each sum by its *time* of payment, and divide the sum of the *products* by the sum of the *payments*, and the quotient will be the answer.

Note. This rule is founded on the supposition, that what is gained by keeping a debt a certain time *after* it is due, is the same as what is lost by paying it an equal time *before* it is due; but, in the first case, the *gain* is evidently equal to the *interest* on the debt for the given time, while, in the second case, the *loss* is only equal to the *discount* of the debt for that time, which is always *less* than the *interest;* therefore, the rule is not exactly true. The error, however, is so trifling, in most questions that occur in business, as scarce to merit notice.

6. A merchant has owing him $300, to be paid as follows: $50 in 2 months, $100 in 5 months, and the rest in 8 months; and it is agreed to make one payment of the whole: in what time ought that payment to be?

Ans. 6 months.

7. A owes B $136, to be paid in 10 months; $96, to be paid in 7 months; and $260, to be paid in 4 months: what is the equated time for the payment of the whole?

Ans. 6 months, 7 days +.

8. A owes B $600, of which $200 is to be paid at the present time, 200 in 4 months, and 200 in 8 months; what is the equated time for the payment of the whole?

Ans. 4 months.

9. A owes B $300, to be paid as follows: $\frac{1}{3}$ in 3 months, $\frac{1}{3}$ in 4 months, and the rest in 6 months: what is the equated time? *Ans.* $4\frac{1}{3}$ months.

RATIO;

OR

THE RELATION OF NUMBERS.

¶ 93. 1. What part of 1 gallon is 3 quarts? 1 gallon is 4 quarts, and 3 quarts is $\frac{3}{4}$ of 4 quarts. *Ans.* $\frac{3}{4}$ of a gallon.

2. What part of 3 quarts is 1 gallon? 1 gallon, being 4 quarts, is $\frac{4}{3}$ of 3 quarts; that is, 4 quarts is 1 time 3 quarts and $\frac{1}{3}$ of another time. *Ans.* $\frac{4}{3} = 1\frac{1}{3}$.

3. What part of 5 bushels is 12 bushels?

Finding what part one number is of another is the same as finding what is called the *ratio*, or *relation* of one number to another; thus, the question, What part of 5 bushels is 12 bushels? is the same as What is the ratio of 5 bushels to 12 bushels? The *Answer* is $\frac{12}{5} = 2\frac{2}{5}$.

Ratio, therefore, may be defined, the number of times one number is contained in another; or, the number of times one quantity is contained in another quantity of the same kind.

4. What part of 8 yards is 13 yards? or, What is the ratio of 8 yards to 13 yards?

13 yards is $\frac{13}{8}$ of 8 yards, expressing the division *fractionally*. If now we perform the division, we have for the ratio $1\frac{5}{8}$; that is, 13 yards is 1 time 8 yards, and $\frac{5}{8}$ of another time.

We have seen, (¶ 15, *sign*,) that division may be expressed *fractionally*. So also the *ratio* of one number to another, or the part one number is of another, may be expressed fractionally, to do which, make the number which is called the *part*, whether it be the larger or the smaller number, the *numerator* of a fraction, under which write the other number for a denominator. When the question is, What is the ratio, &c.? the number *last* named is the *part*; consequently it must be made the *numerator* of the fraction, and the number *first* named the denominator.

5. What part of 12 dollars is 11 dollars? or, 11 dollars is what part of 12 dollars? 11 is the number which expresses the *part*. To put this question in the other form, viz. What is the *ratio*, &c.? let that number, which expresses the *part*, be the number *last* named; thus, What is the ratio of 12 dollars to 11 dollars? *Ans.* $\frac{11}{12}$.

6. What part of 1 £. is 2 s. 6 d.? or, What is the ratio of 1 £. to 2 s. 6 d.?

1 £. = 240 pence, and 2 s. 6 d. = 30 pence; hence, $\frac{30}{240} = \frac{1}{8}$, is the *Answer*.

7. What part of 13 s. 6 d. is 1 £. 10 s.? or, What is the ratio of 13 s. 6 d. to 1 £. 10 s.? *Ans.* $\frac{20}{9}$.

8. What is the ratio of 3 to 5? —— of 5 to 3? —— of 7 to 19? —— of 19 to 7? —— of 15 to 90? —— of 90 to 15? —— of 84 to 160? —— of 160 to 84? —— of 615 to 1107? —— of 1107 to 615? *Ans. to the last,* $\frac{9}{5}$.

PROPORTION;

OR

THE RULE OF THREE.

¶ **94.** 1. If a piece of cloth, 4 yards long, cost 12 dollars, what will be the cost of a piece of the same cloth 7 yards long?

Had this piece contained twice the number of yards of the first piece, it is evident the price would have been twice as much; had it contained 3 times the number of yards, the price would have been 3 times as much; or had it contained only half the number of yards, the price would have been only half as much; that is, the cost of 7 yards will be such part of 12 dollars as 7 yards is part of 4 yards. 7 yards is $\frac{7}{4}$ of 4 yards; consequently, the price of 7 yards must be $\frac{7}{4}$ of the price of 4 yards, or $\frac{7}{4}$ of 12 dollars. $\frac{7}{4}$ of 12 dollars, that is, $12 \times \frac{7}{4} = \frac{84}{4} = 21$ dollars, *Answer.*

2. If a horse travel 30 miles in 6 hours, how many miles will he travel in 11 hours, at that rate?

11 hours is $\frac{11}{6}$ of 6 hours, that is, 11 hours is 1 time 6 hours, and $\frac{5}{6}$ of another time; consequently, he will travel, in 11 hours, 1 time 30 miles, and $\frac{5}{6}$ of another time, that is, the ratio between the distances will be equal to the ratio between the times.

$\frac{11}{6}$ of 30 miles, that is, $30 \times \frac{11}{6} = \frac{330}{6} = 55$ miles. If, then, no error has been committed, 55 miles must be $\frac{11}{6}$ of 30 miles. This is actually the case; for $\frac{55}{30} = \frac{11}{6}$.

Ans. 55 miles.

Quantities which have the same ratio between them are said to be *proportional.* Thus, these four quantities,

hours.	hours.	miles.	miles.
6,	11,	30,	55,

written in this order, being such, that the second contains the first as many times as the fourth contains the third, that is, the ratio between the third and fourth being equal to the ratio between the first and second, form what is called a proportion.

It follows, therefore, that *proportion is a combination of two equal ratios. Ratio* exists between *two* numbers; but *proportion* requires at least *three.*

To denote that there is a proportion between the numbers 6, 11, 30, and 55, they are written thus :—

$$6 : 11 :: 30 : 55$$

which is read, 6 is to 11 as 30 is to 55; that is, 6 is the same part of 11, that 30 is of 55; or, 6 is contained in 11 as many times as 30 is contained in 55; or, lastly, the ratio or relation of 11 to 6 is the same as that of 55 to 30.

¶ **95.** The first term of a ratio, or relation, is called the *antecedent*, and the second the *consequent*. In a proportion there are two antecedents, and two consequents, viz. the antecedent of the first ratio, and that of the second; the consequent of the first ratio, and that of the second. In the proportion 6 : 11 :: 30 : 55, the antecedents are 6, 30; the consequents, 11, 55.

The consequent, as we have already seen, is taken for the numerator, and the antecedent for the denominator of the fraction, which expresses the ratio or relation. Thus, the first ratio is $\frac{11}{6}$, the second $\frac{55}{30} = \frac{11}{6}$; and that these two ratios are equal, we know, because the fractions are equal.

The two fractions $\frac{11}{6}$ and $\frac{55}{30}$ being equal, it follows that, by reducing them to a common denominator, the numerator of the one will become equal to the numerator of the other, and, consequently, that 11 multiplied by 30 will give the same product as 55 multiplied by 6. This is actually the case; for $11 \times 30 = 330$, and $55 \times 6 = 330$. Hence it follows,—*If four numbers be in proportion, the product of the first and last, or of the two extremes, is equal to the product of the second and third, or of the two means.*

Hence it will be easy, having three terms in a proportion given, to find the fourth. Take the last example. Knowing that the distances travelled are in proportion to the times or hours occupied in travelling, we write the proportion thus :—

hours.		hours.		miles.		miles.
6	:	11	::	30		

Now, since the product of the extremes is equal to the product of the means, we multiply together the two means, 11 and 30, which makes 330, and, dividing this product by the *known* extreme, 6, we obtain for the result 55, that is, 55 miles, which is the other extreme, or term, sought.

3. At $54 for 9 barrels of flour, how many barrels may be purchased for $186 ?

In this question, the unknown quantity is the number of barrels bought for $186, which ought to contain the 9 barrels as many times as $186 contains $54; we thus get the following proportion:

```
dollars. dollars.  barrels. barrels.
 54  :  186  : :  9  :  ......
          9
       ----
 54 ) 1674 ( 31 barrels, the Answer.
      162
      ----
        54
        54
        --
```

The product, 1674, of the two means, divided by 54, the known extreme, gives 31 barrels for the other extreme, which is the term sought, or *Answer*.

Any three terms of a proportion being given, the operation by which we find the fourth is called the *Rule of Three.* A just solution of the question will sometimes require, that the order of the terms of a proportion be changed. This may be done, provided the terms be so placed, that the product of the extremes shall be equal to that of the means.

4. If 3 men perform a certain piece of work in 10 days, how long will it take 6 men to do the same ?

The number of days in which 6 men will do the work being the term sought, the known term of the same kind, viz. 10 days, is made the third term. The two remaining terms are 3 men and 6 men, the ratio of which is $\frac{6}{3}$. But the *more** men there are employed in the work, the *less* time will be required to do it; consequently, the days will be *less* in

* The rule of three has sometimes been divided into *direct* and *inverse*, a distinction which is totally useless. It may not however be amiss to explain, in this place, in what this distinction consists.

The *Rule of Three Direct* is when *more* requires *more*, or *less* requires *less*, as in this example :—If 3 men dig a trench 48 feet long in a certain time, how many feet will 12 men dig in the same time ? Here it is obvious, that the *more* men there are employed, the *more* work will be done; and therefore, in this instance, more requires more. Again :—If 6 men dig 48 feet in a given time, how much will 3 men dig in the same time ? Here *less* requires *less*, for the *less* men there are employed, the *less* work will be done.

The *Rule of Three Inverse* is when *more* requires *less*, or *less* requires *more*, as in this example :—If 6 men dig a certain quantity of trench in 14 hours, how many hours will it require 12 men to dig the same quantity ? Here *more* requires *less*; that is, 12 men being *more* than 6, will require *less* time. Again :—If 6 men perform a piece of work in 7 days, how long will 3 men be in performing the same work ? Here *less* requires *more*; for the number of men, being *less*, will require *more* time.

proportion as the number of men is *greater*. There is still a proportion in this case, but the order of the terms is inverted; for the number of men in the second set, being two times that in the first, will require only one half the time. The first number of days, therefore, ought to contain the second as many times as the second number of men contains the first. This order of the terms being the reverse of that assigned to them in announcing the question, we say, that the number of men is in the *inverse ratio* of the number of days. With a view, therefore, to the just solution of the question, we reverse the order of the two first terms, (in doing which we invert the ratio,) and, instead of writing the proportion, 3 men : 6 men, ($\frac{6}{3}$,) we write it, 6 men : 3 men, ($\frac{3}{6}$,) that is,

men.		men.		days.		days.
6	:	3	::	10	.	…..,

Note. We invert the ratio when we reverse the order of the terms in the proportion, because then the antecedent takes the place of the consequent, and the consequent that of the antecedent; consequently, the terms of the fraction which express the ratio are inverted; hence the ratio is inverted. Thus, the ratio expressed by $\frac{6}{3} = 2$, being inverted, is $\frac{3}{6} = \frac{1}{2}$.

Having stated the proportion as above, we divide the product of the means, ($10 \times 3 = 30$,) by the known extreme, 6, which gives 5, that is, 5 days, for the other extreme, or term sought. *Ans.* 5 days.

From the examples and illustrations now given we deduce the following general

RULE.

Of the three given numbers, make that the third term which is of the same kind with the answer sought. Then consider, from the nature of the question, whether the answer will be greater or less than this term. If the answer is to be greater, place the greater of the two remaining numbers for the second term, and the less number for the first term; but if it is to be less, place the less of the two remaining numbers for the second term, and the greater for the first; and, in either case, multiply the second and third terms together, and divide the product by the first for the answer, which will always be of the same denomination as the third term.

Note 1. If the first and second terms contain *different* denominations, they must both be reduced to the *same* denomination; and if the third term be a *compound* number, it either must be reduced to *integers of the lowest denomination*, or the low denominations must be reduced to a *fraction of the highest denomination* contained in it.

Note 2. The same rule is applicable, whether the given quantities be integral, fractional, or decimal.

EXAMPLES FOR PRACTICE.

5. If 6 horses consume 21 bushels of oats in 3 weeks, how many bushels will serve 20 horses the same time?
Ans. 70 bushels.

6. *The above question reversed.* If 20 horses consume 70 bushels of oats in 3 weeks, how many bushels will serve 6 horses the same time? *Ans.* 21 bushels.

7. If 365 men consume 75 barrels of provisions in 9 months, how much will 500 men consume in the same time?
Ans. $102\frac{54}{73}$ barrels.

8. If 500 men consume $102\frac{54}{73}$ barrels of provisions in 9 months, how much will 365 men consume in the same time? *Ans.* 75 barrels.

9. A goldsmith sold a tankard for 10 £. 12 s., at the rate of 5 s. 4 d. per ounce; I demand the weight of it.
Ans. 39 oz. 15 pwt.

10. If the moon move 13° 10′ 35″ in 1 day, in what time does it perform one revolution? *Ans.* 27 days, 7 h. 43 m.

11. If a person, whose rent is $145, pay $12‘63 parish taxes, how much should a person pay whose rent is $378?
Ans. $32‘925.

12. If I buy 7 lbs. of sugar for 75 cents, how many pounds can I buy for $6? *Ans.* 56 lbs.

13. If 2 lbs. of sugar cost 25 cents, what will 100 lbs. of coffee cost, if 8 lbs. of sugar are worth 5 lbs. of coffee?
Ans. $20.

14. If I give $6 for the use of $100 for 12 months, what must I give for the use of $357‘82 the same time?
Ans. $21‘469.

15. There is a cistern which has 4 pipes; the first will fill it in 10 minutes, the second in 20 minutes, the third in

40 minutes, and the fourth in 80 minutes; in what time will all four, running together, fill it?

$\frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \frac{1}{80} = \frac{15}{80}$ cistern in 1 minute.

Ans. $5\frac{1}{3}$ minutes.

16. If a family of 10 persons spend 3 bushels of malt in a month, how many bushels will serve them when there are 30 in the family? *Ans.* 9 bushels.

Note. The rule of proportion, although of frequent use, is not of indispensable necessity; for all questions under it may be solved on general principles, without the formality of a proportion; that is, by *analysis*, as already shown, ¶ 65, ex. 1. Thus, in the above example,—If 10 persons spend 3 bushels, 1 person, in the same time, would spend $\frac{1}{10}$ of 3 bushels, that is, $\frac{3}{10}$ of a bushel; and 30 persons would spend 30 times as much, that is, $\frac{90}{10} = 9$ bushels, as before.

17. If a staff, 5 ft. 8 in. in length, cast a shadow of 6 feet, how high is that steeple whose shadow measures 153 feet?

Ans. $144\frac{1}{2}$ feet.

18. *The same by analysis.* If 6 ft. shadow require a staff of 5 ft. 8 in. $= 68$ in., 1 ft. shadow will require a staff of $\frac{1}{6}$ of 68 in. or $\frac{68}{6}$ in.; then, 153 ft. shadow will require 153 times as much; that is, $\frac{68}{6} \times 153 = \frac{10404}{6} = 1734$ in. $=$ $144\frac{1}{2}$ ft., as before.

19. If 3 £. sterling be equal to 4 £. Massachusetts, how much Massachusetts is equal to 1000 £. sterling?

Ans. 1333 £. 6 s. 8 d.

20. If 1333 £. 6 s. 8 d. Massachusetts, be equal to 1000 £. sterling, how much sterling is equal to 4 £. Massachusetts?

Ans. 3 £.

21. If 1000 £. sterling be equal to 1333 £. 6 s. 8 d. Massachusetts, how much Massachusetts is equal to 3 £. sterling? *Ans.* 4 £.

22. If 3 £. sterling be equal to 4 £. Massachusetts, how much sterling is equal to 1333 £. 6 s. 8 d. Massachusetts?

Ans. 1000 £.

23. Suppose 2000 soldiers had been supplied with bread sufficient to last them 12 weeks, allowing each man 14 ounces a day; but, on examination, they find 105 barrels, containing 200 lbs. each, wholly spoiled; what must the allowance be to each man, that the remainder may last them the same time? *Ans.* 12 oz. a day.

24. Suppose 2000 soldiers were put to an allowance of 12 oz. of bread per day for 12 weeks, having a seventh part of their bread spoiled; what was the whole weight of their bread, good and bad, and how much was spoiled?

Ans. { The whole weight, 147000 lbs.
Spoiled, - - 21000 lbs.

25. —— 2000 soldiers, having lost 105 barrels of bread, weighing 200 lbs. each, were obliged to subsist on 12 oz. a day for 12 weeks; had none been lost, they might have had 14 oz. a day; what was the whole weight, including what was lost, and how much had they to subsist on?

Ans. { Whole weight, 147000 lbs.
Left, to subsist on, 126000 lbs.

26. —— 2000 soldiers, after losing one seventh part of their bread, had each 12 oz. a day for 12 weeks; what was the whole weight of their bread, including that lost, and how much might they have had per day, each man, if none had been lost?

Ans. { Whole weight, 147000 lbs.
Loss, - - 21000 lbs.
14 oz. per day, had none been lost.

27. There was a certain building raised in 8 months by 120 workmen; but, the same being demolished, it is required to be built in 2 months; I demand how many men must be employed about it. *Ans.* 480 men.

28. There is a cistern having a pipe which will empty it in 10 hours; how many pipes of the same capacity will empty it in 24 minutes? *Ans.* 25 pipes.

29. A garrison of 1200 men has provisions for 9 months, at the rate of 14 oz. per day; how long will the provisions last, at the same allowance, if the garrison be reinforced by 400 men? *Ans.* $6\frac{3}{4}$ months.

30. If a piece of land, 40 rods in length and 4 in breadth, make an acre, how wide must it be when it is but 25 rods long? *Ans.* $6\frac{2}{5}$ rods.

31. If a man perform a journey in 15 days when the days are 12 hours long, in how many will he do it when the days are but 10 hours long? *Ans.* 18 days.

32. If a field will feed 6 cows 91 days, how long will it feed 21 cows? *Ans.* 26 days.

33. Lent a friend 292 dollars for 6 months; some time after, he lent me 806 dollars; how long may I keep it to balance the favour? *Ans.* 2 months 5 + days.

Q *

34. If 30 men can perform a piece of work in 11 days, how many men will accomplish another piece of work, 4 times as big, in a fifth part of the time? *Ans.* 600 men.

35. If $\frac{11}{13}$ lb. of sugar cost $\frac{7}{15}$ of a shilling, what will $\frac{32}{43}$ of a lb. cost? *Ans.* 4 d. $3\frac{4971}{7095}$ q.

Note. See ¶ 65, ex. 1, where the above question is solved by analysis. The eleven following are the next succeeding examples in the same ¶.

36. If 7 lbs. of sugar cost $\frac{3}{4}$ of a dollar, what cost 12 lbs.? *Ans.* \$ $1\frac{2}{7}$.

37. If $6\frac{1}{2}$ yds. of cloth cost \$ 3, what cost $9\frac{1}{4}$ yds.? *Ans.* \$ 4‘269.

38. If 2 oz. of silver cost \$ 2‘24, what costs $\frac{3}{4}$ oz.? *Ans.* \$ 0‘84.

39. If $\frac{5}{7}$ oz. cost \$ $\frac{11}{12}$, what costs 1 oz.? *Ans.* \$ 1‘283.

40. If $\frac{1}{3}$ lb. less by $\frac{1}{6}$ lb. cost $13\frac{1}{5}$ d., what cost 14 lbs. less by $\frac{1}{5}$ of 2 lbs.? *Ans.* 4 £. 9 s. $9\frac{3}{25}$ d.

41. If $\frac{3}{5}$ yd. cost \$ $\frac{7}{8}$, what will $40\frac{1}{2}$ yds. cost? *Ans.* \$ 59‘062.

42. If $\frac{7}{16}$ of a ship cost \$ 251, what is $\frac{3}{32}$ of her worth? *Ans.* \$ 53‘785.

43. At $3\frac{5}{8}$ £. per cwt., what will $9\frac{2}{3}$ lbs. cost? *Ans.* 6 s. $3\frac{5}{56}$ d.

44. A merchant, owning $\frac{4}{5}$ of a vessel, sold $\frac{2}{3}$ of his share for \$ 957; what was the vessel worth? *Ans.* \$ 1794‘375.

45. If $\frac{5}{8}$ yd. cost $\frac{4}{7}$ £., what will $\frac{9}{15}$ of an ell English cost? *Ans.* 17 s. 1 d. $2\frac{6}{7}$ q.

46. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yds., at the rate of \$ 7 for 5 yds., and sold them out at the rate of \$ 11 for 7 yds., and gained \$ 200 by the bargain; how many bales were there? *Ans.* 9 bales.

47. At \$ 33 for 6 barrels of flour, what must be paid for 178 barrels? *Ans.* \$ 979.

48. At \$ 2‘25 for 3‘17 cwt. of hay, how much is that per ton? *Ans.* \$ 14‘195.

49. If 2‘5 lbs. of tobacco cost 75 cents, how much will 185 lbs. cost? *Ans.* \$ 5‘55.

50. What is the value of ‘15 of a hogshead of lime, at \$ 2‘39 per hhd.? *Ans.* \$ 0‘3585.

51. If ‘15 of a hhd. of lime cost \$ 0‘3585, what is it per hhd.? *Ans.* \$ 2‘39.

COMPOUND PROPORTION.

¶ 96. It frequently happens, that the relation of the quantity required, to the given quantity of the same kind, depends upon several circumstances combined together; it is then called *Compound Proportion*, or *Double Rule of Three.*

1. If a man travel 273 miles in 13 days, travelling only 7 hours in a day, how many miles will he travel in 12 days, if he travel 10 hours in a day?

This question may be solved several ways. First, by *analysis:*—

If we knew how many miles the man travelled in 1 hour, it is plain, we might take this number 10 times, which would be the number of miles he would travel in 10 hours, or in 1 of these long days, and this again, taken 12 times, would be the number of miles he would travel in 12 days, travelling 10 hours each day.

If he travel 273 miles in 13 days, he will travel $\frac{1}{13}$ of 273 miles; that is, $\frac{273}{13}$ miles in 1 day of 7 hours; and $\frac{1}{7}$ of $\frac{273}{13}$ miles is $\frac{273}{91}$ miles, the distance he travels in 1 hour: then, 10 times $\frac{273}{91} = \frac{2730}{91}$ miles, the distance he travels in 10 hours; and 12 times $\frac{2730}{91} = \frac{32760}{91} = 360$ miles, the distance he travels in 12 days, travelling 10 hours each day.

Ans. 360 miles.

But the object is to show how the question may be solved by *proportion:*—

First; it is to be regarded, that the number of miles travelled over depends upon two circumstances, viz. the number of *days* the man travels, and the number of *hours* he travels each day.

We will not at first consider this *latter* circumstance, but suppose the number of hours to be the same in each case: the question then will be,—*If a man travel* 273 *miles in* 13 *days, how many miles will he travel in* 12 *days?* This will furnish the following proportion:—

13 days : 12 days : : 273 miles : .... miles

which gives for the fourth term, or answer, 252 miles.

Now, taking into consideration the *other* circumstance, or that of the *hours*, we must say,—*If a man, travelling* 7 *hours a day for a certain number of days, travels* 252 *miles, how far*

will he travel in the same time, if he travel 10 *hours in a day?* This will lead to the following proportion :—

7 hours : 10 hours : : 252 miles : .... miles.

This gives for the fourth term, or answer, 360 miles.

We see, then, that 273 miles has to the fourth term, or answer, the same proportion that 13 days has to 12 days, and that 7 hours has to 10 hours. Stating this in the form of a proportion, we have

$$\left.\begin{array}{rcl} 13 \text{ days} & : & 12 \text{ days} \\ 7 \text{ hours} & : & 10 \text{ hours} \end{array}\right\} : : 273 \text{ miles} : \ldots\ldots \text{ miles}$$

by which it appears, that 273 is to be multiplied by both 12 and 10; that is, 273 is to be multiplied by the product of 12×10, and divided by the product of 13×7, which, being done, gives 360 miles for the fourth term, or answer, as before.

In the same manner, any question relating to compound proportion, however complicated, may be stated and solved.

2. If 248 men, in 5 days, of 11 hours each, can dig a trench 230 yards long, 3 wide, and 2 deep, in how many days, of 9 hours each, will 24 men dig a trench 420 yards long, 5 wide, and 3 deep?

Here the number of days, in which the proposed work can be done, depends on *five circumstances*, viz. the number of men employed, the number of hours they work each day, the length, breadth, and depth of the trench. We will consider the question in relation to each of these circumstances, in the order in which they have been named :—

1st. *The number of men employed.* Were all the circumstances in the two cases alike, except the number of men and the number of days, the question would consist only in finding in how many days 24 men would perform the work which 248 men had done in 5 days; we should then have

24 men : 248 men : : 5 days : .... days.

2d. *Hours in a day.* But the first labourers worked 11 hours in a day, whereas the others worked only 9; *less* hours will require *more* days, which will give

9 hours : 11 hours : : 5 days : .... days.

3d. *Length of the ditches.* The ditches being of unequal

length, as many more days will be necessary as the second is longer than the first; hence we shall have

230 length : 420 length : : 5 days : .... days.

4th. *Widths.* Taking into consideration the widths, which are different, we have

3 wide : 5 wide : : 5 days : .... days.

5th. *Depths.* Lastly, the depths being different, we have

2 deep : 3 deep : : 5 days : .... days.

It would seem, therefore, that 5 days has to the fourth term, or answer, the same proportion

that 24 men has to 248 men, whose ratio is $\frac{248}{24}$,
that 9 hours has to 11 hours, the ratio of which is $\frac{11}{9}$,
that 230 length has to 420 length, ... $\frac{420}{230}$,
that 3 width has to 5 width, ... $\frac{5}{3}$,
that 2 depth has to 3 depth, ... $\frac{3}{2}$;
all which stated in form of a proportion, we have

$$\left.\begin{array}{lrcr} \text{Men,} & 24 & : & 248 \\ \text{Hours,} & 9 & : & 11 \\ \text{Length,} & 230 & : & 420 \\ \text{Width,} & 3 & : & 5 \\ \text{Depth,} & 2 & : & 3 \end{array}\right\} :: \overset{\text{common term.}}{5 \text{ days}} : \ldots\ldots \text{ days.}$$

¶ 97. The continued product of all the second terms $248 \times 11 \times 420 \times 5 \times 3$, multiplied by the third term, 5 days, and this product divided by the continued product of the first terms, $24 \times 9 \times 230 \times 3 \times 2$, gives $288\frac{84960}{298080}$ days for the fourth term, or answer. $288\frac{59}{207}$.

But the first and second terms are the fractions $\frac{248}{24}$, $\frac{11}{9}$, $\frac{420}{230}$, $\frac{5}{3}$ and $\frac{3}{2}$, which express the ratios of the men, and of the hours, of the lengths, widths and depths of the two ditches. Hence it follows, that the ratio of the number of days given to the number of days sought, is equal to the product of all the ratios, which result from a comparison of the terms relating to each circumstance of the question.

The product of all the ratios is found by multiplying together the fractions which express them, thus, $\frac{248 \times 11 \times 420 \times 5 \times 3}{24 \times 9 \times 230 \times 3 \times 2} = \frac{17186400}{298080}$, and this fraction, $\frac{17186400}{298080}$, represents the

ratio of the quantity required to the given quantity of the same kind. A ratio resulting in this manner, from the multiplication of several ratios, is called a *compound ratio*.

From the examples and illustrations now given we deduce the following general

RULE

for solving questions in compound proportion, or double rule of three, viz.—Make that number which is of the same kind with the required answer, the third term; and, of the remaining numbers, take away two that are of the *same kind*, and arrange them according to the directions given in simple proportion; then, any *other* two of the same kind, and so on till all are used.

Lastly, multiply the third term by the continued product of the second terms, and divide the result by the continued product of the first terms, and the quotient will be the fourth term, or answer required.

EXAMPLES FOR PRACTICE.

1. If 6 men build a wall 20 ft. long, 6 ft. high, and 4 ft. thick, in 16 days, in what time will 24 men build one 200 ft. long, 8 ft. high, and 6 ft. thick? *Ans.* 80 days.

2. If the freight of 9 hhds. of sugar, each weighing 12 cwt., 20 leagues, cost 16 £., what must be paid for the freight of 50 tierces, each weighing $2\frac{1}{2}$ cwt., 100 leagues?
Ans. 92 £. 11 s. $10\frac{2}{3}$ d.

3. If 56 lbs. of bread be sufficient for 7 men 14 days, how much bread will serve 21 men 3 days? *Ans.* 36 lbs.

The same by analysis. If 7 men consume 56 lbs. of bread, 1 man, in the same time, would consume $\frac{1}{7}$ of 56 lbs. $= \frac{56}{7}$ lbs.; and if he consume $\frac{56}{7}$ lbs. in 14 days, he would consume $\frac{1}{14}$ of $\frac{56}{7} = \frac{56}{98}$ lb. in 1 day. 21 men would consume 21 times so much as 1 man; that is, 21 times $\frac{56}{98} = \frac{1176}{98}$ lbs. in 1 day, and in 3 days they would consume 3 times as much; that is, $\frac{3528}{98} = 36$ lbs., as before.
Ans. 36 lbs.

Note. Having wrought the following examples by the rule of proportion, let the pupil be required to do the same by *analysis*.

4. If 4 reapers receive \$11'04 for 3 days' work, how many men may be hired 16 days for \$103'04?
Ans. 7 men.

5. If 7 oz. 5 pwt. of bread be bought for $4\frac{3}{4}$ d. when corn is 4 s. 2 d. per bushel, what weight of it may be bought for 1 s. 2 d. when the price per bushel is 5 s. 6 d.?

Ans. 1 lb. 4 oz. $3\frac{478}{627}$ pwts.

6. If \$100 gain \$6 in 1 year, what will \$400 gain in 9 months?

Note. This and the three following examples reciprocally prove each other.

7. If \$100 gain \$6 in 1 year, in what time will \$400 gain \$18?

8. If \$400 gain \$18 in 9 months, what is the rate per cent. per annum?

9. What principal, at 6 per cent. per. ann., will gain \$18 in 9 months?

10. A usurer put out \$75 at interest, and, at the end of 8 months, received, for principal and interest, \$79; I demand at what rate per cent. he received interest.

Ans. 8 per cent.

11. If 3 men receive $8\frac{9}{10}$ £. for $19\frac{1}{2}$ days' work, how much must 20 men receive for $100\frac{1}{4}$ days'?

Ans. 305 £. 0 s. 8 d.

SUPPLEMENT TO THE SINGLE RULE OF THREE.

QUESTIONS.

1. What is proportion? 2. How many numbers are required to form a ratio? 3. How many to form a proportion? 4. What is the first term of a ratio called? 5. —— the second term? 6. Which is taken for the numerator, and which for the denominator of the fraction expressing the ratio? 7. How may it be known when four numbers are in proportion? 8. Having three terms in a proportion given, how may the fourth term be found? 9. What is the operation, by which the fourth term is found, called? 10. How does a ratio become inverted? 11. What is the rule in proportion? 12. In what denomination will the fourth term, or answer, be found? 13. If the first and second terms contain different denominations, what is to be done? 14. What is compound proportion, or double rule of three? 15. Rule?

EXERCISES.

1. If I buy 76 yds. of cloth for $113'17, what does it cost per ell English? *Ans.* $1'861.

2. Bought 4 pieces of Holland, each containing 24 ells English, for $96; how much was that per yard? *Ans.* $0'80.

3. A garrison had provision for 8 months, at the rate of 15 ounces to each person per day; how much must be allowed per day, in order that the provision may last $9\frac{1}{2}$ months? *Ans.* $12\frac{12}{19}$ oz.

4. How much land, at $2'50 per acre, must be given in exchange for 360 acres, at $3'75 per acre? *Ans.* 540 acres.

5. Borrowed 185 quarters of corn when the price was 19 s.; how much must I pay when the price is 17 s. 4 d.? *Ans.* $202\frac{41}{52}$.

6. A person, owning $\frac{3}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for 171 £.; what is the whole mine worth? *Ans.* 380 £.

7. If $\frac{3}{8}$ of a gallon cost $\frac{3}{8}$ of a dollar, what costs $\frac{5}{8}$ of a tun? *Ans.* $140.

8. At $1\frac{1}{2}$ £. per cwt., what cost $3\frac{1}{3}$ lbs.? *Ans.* $10\frac{5}{7}$ d.

9. If $4\frac{1}{2}$ cwt. can be carried 36 miles for 35 shillings, how many pounds can be carried 20 miles for the same money? *Ans.* $907\frac{1}{5}$ lbs.

10. If the sun appears to move from east to west 360 degrees in 24 hours, how much is that in each hour? —— in each minute? —— in each second? *Ans. to last*, 15″ of a deg.

11. If a family of 9 persons spend $450 in 5 months, how much would be sufficient to maintain them 8 months if 5 persons more were added to the family? *Ans.* $1120.

Note. Exercises 14th, 15th, 16th, 17th, 18th, 19th, and 20th, "*Supplement to Fractions*," afford additional examples in single and double proportion, should more examples be thought necessary.

FELLOWSHIP.

¶ 98. 1. Two men own a ticket; the first owns $\frac{1}{4}$, and the second owns $\frac{3}{4}$ of it; the ticket draws a prize of 40 dollars; what is each man's share of the money?

2. Two men purchase a ticket for 4 dollars, of which one pays 1 dollar, and the other 3 dollars; the ticket draws 40 dollars; what is each man's share of the money?

3. A and B bought a quantity of cotton; A paid 100 dollars, and B 200 dollars; they sold it so as to gain 30 dollars; what were their respective shares of the gain?

The process of ascertaining the respective gains or losses of individuals, engaged in joint trade, is called the *Rule of Fellowship.*

The money, or value of the articles employed in trade, is called the *Capital*, or *Stock;* the gain or loss to be shared is called the *Dividend.*

It is plain, that *each man's* gain or loss ought to have the same relation to the *whole* gain or loss, as his *share of the stock* does to the *whole stock.*

Hence we have this RULE:—As the *whole* stock : to each man's *share* of the stock : : the *whole* gain or loss : his *share* of the gain or loss.

4. Two persons have a joint stock in trade; A put in \$250, and B \$350; they gain \$400; what is each man's share of the profit?

OPERATION.

A's stock,	\$250	Then,
B's stock,	\$350	600 : 250 : : 400 : 166'666$\frac{2}{3}$ dolls. A's gain.
Whole stock,	\$600	600 : 350 : : 400 : 233'333$\frac{1}{3}$ dolls. B's gain.

The pupil will perceive, that the process may be contracted by cutting off an equal number of ciphers from the *first* and *second*, or *first* and *third* terms; thus, 6 : 250 : : 4 : 166'666$\frac{2}{3}$, &c.

It is obvious, the correctness of the work may be ascertained by finding whether the sums of the *shares* of the gains are equal to the *whole* gain; thus, \$166'666$\frac{2}{3}$ + \$233'333$\frac{1}{3}$ = \$400, whole gain.

5. A, B and C trade in company; A's capital was \$175, B's \$200, and C's \$500; by misfortune they lose \$250; what loss must each sustain?

Ans. \$ 50', A's loss. \$ 57'142$\frac{6}{7}$, B's loss. \$ 142'857$\frac{1}{7}$, C's loss.

6. Divide \$600 among 3 persons, so that their shares may be to each other as 1, 2, 3, respectively.

Ans. \$100, \$200, and \$300.

7. Two merchants, A and B, loaded a ship with 500 hhds. of rum; A loaded 350 hhds., and B the rest; in a storm, the seamen were obliged to throw overboard 100 hhds.; how much must each sustain of the loss?

Ans. A 70, and B 30 hhds.

8. A and B companied; A put in $45, and took out $\frac{3}{5}$ of the gain; how much did B put in? *Ans.* $30.

Note. They took out in the same proportion as they put in; if 3 fifths of the stock is $45, how much is 2 fifths of it?

9. A and B companied, and trade with a joint capital of $400; A receives, for his share of the gain, $\frac{1}{2}$ as much as B; what was the stock of each?

Ans. { $133'333$\frac{1}{3}$, A's stock. $266'666$\frac{2}{3}$, B's stock. }

10. A bankrupt is indebted to A $780, to B $460, and to C $760; his estate is worth only $600; how must it be divided?

Note. The question evidently involves the principles of fellowship, and may be wrought by it.

Ans. A $234, B $138, and C $228.

11. A and B venture equal stocks in trade, and clear $164; by agreement, A was to have 5 per cent. of the profits, because he managed the concerns; B was to have but 2 per cent.; what was each one's gain? and how much did A receive for his trouble?

Ans. A's gain was $117'142$\frac{6}{7}$, and B's $46,857$\frac{1}{7}$, and A received $70'285$\frac{5}{7}$ for his trouble.

12. A cotton factory, valued at $12000, is divided into 100 shares; if the profits amount to 15 per cent. yearly, what will be the profit accruing to 1 share? —— to 2 shares? —— to 5 shares? —— to 25 shares?

Ans. to the last. $450.

13. In the above-mentioned factory, repairs are to be made which will cost $340; what will be the tax, on each share, necessary to raise the sum? —— on 2 shares? —— on 3 shares? —— on 10 shares? *Ans. to the last,* $34.

14. If a town raise a tax of $1850, and the whole town be valued at $37000, what will that be on $1? What will be the tax of a man whose property is valued at $1780?

Ans. $'05 on a dollar, and $89 on $1780.

¶ 99. In assessing taxes, it is necessary to have an inventory of the property, both real and personal, of the whole town, and also of the whole number of polls; and, as the polls are rated at so much each, we must first take out from the *whole* tax what the *polls* amount to, and the remainder is to be assessed on the *property*. We may then find the tax upon 1 dollar, and make a table containing the taxes on 1, 2, 3, &c., to 10 dollars; then on 20, 30, &c., to 100 dollars; and then on 100, 200, &c., to 1000 dollars. Then, knowing the inventory of any individual, it is easy to find the tax upon his property.

15. A certain town, valued at $64530, raises a tax of $2259'90; there are 540 polls, which are taxed $'60 each; what is the tax on a dollar, and what will be A's tax, whose *real estate* is valued at $1340, his personal property at $874, and who pays for 2 polls?

540 × '60 = $324, amount of the poll taxes, and $2259'90 — $324 = 1935'90, to be assessed on property. $64530 : $1935'90 :: $1 : '03; or, $\frac{1935'90}{64530}$ = '03, tax on $1.

TABLE.

	dolls.	dolls.		dolls.	dolls.		dolls.	dolls.
Tax on	1 is	'03	Tax on	10 is	'30	Tax on	100 is	3'
....	2 ..	'06	....	20 ..	'60	....	200 ..	6'
....	3 ..	'09	....	30 ..	'90	....	300 ..	9'
....	4 ..	'12	....	40 ..	1'20	....	400 ..	12'
....	5 ..	'15	....	50 ..	1'50	....	500 ..	15'
....	6 ..	'18	....	60 ..	1'80	....	600 ..	18'
....	7 ..	'21	....	70 ..	2'10	....	700 ..	21'
....	8 ..	'24	....	80 ..	2'40	....	800 ..	24'
....	9 ..	'27	....	90 ..	2'70	....	900 ..	27'
						....	1000 ..	30'

Now, to find A's tax, his real estate being $1340, I find, by the table, that

The tax on - - -	$1000	- - is - -	$30'
The tax on - - -	300	- - - - - -	9'
The tax on - - -	40	- - - - - -	1'20
Tax on his real estate - - - - - - - - - -			$40'20
In like manner I find the tax on his personal property to be - - - - - - - - -			26'22
2 polls at '60 each, are - - - - - - - -			1'20
		Amount,	$67'62

16. What will B's tax amount to, whose inventory is 874 dollars *real*, and 210 dollars *personal* property, and who pays for 3 polls? *Ans.* $ 34'32.

17. What will be the tax of a man, paying for 1 poll, whose property is valued at $ 3482? —— at $ 768? —— at $ 940? —— at $ 4657? *Ans. to the last*, $ 140'31.

18. Two men paid 10 dollars for the use of a pasture 1 month; A kept in 24 cows, and B 16 cows; how much should each pay?

19. Two men hired a pasture for $ 10; A put in 8 cows 3 months, and B put in 4 cows 4 months; how much should each pay?

¶ **100.** The pasturage of 8 cows for 3 months is the same as of 24 cows for 1 month, and the pasturage of 4 cows for 4 months is the same as of 16 cows for 1 month. The shares of A and B, therefore, are 24 to 16, as in the former question. Hence, when *time* is regarded in fellowship,— *Multiply each one's stock by the time he continues it in trade, and use the product for his share.* This is called *Double Fellowship.* *Ans.* A 6 dollars, and B 4 dollars.

20. A and B enter into partnership; A puts in $ 100 6 months, and then puts in $ 50 more; B puts in $ 200 4 months, and then takes out $ 80; at the close of the year, they find that they have gained $ 95; what is the profit of each? *Ans.* { $ 43'711, A's share. $ 51'288, B's share.

21. A, with a capital of $ 500, began trade Jan. 1, 1826, and, meeting with success, took in B as a partner, with a capital of $ 600, on the first of March following; four months after, they admit C as a partner, who brought $ 800 stock; at the close of the year, they find the gain to be $ 700; how must it be divided among the partners?

Ans. { $ 250, A's share. $ 250, B's share. $ 200, C's share.

QUESTIONS.

1. What is fellowship? 2. What is the rule for operating? 3. When *time* is regarded in fellowship, what is it called? 4. What is the method of operating in double fellowship? 5. How are taxes assessed? 6. How is fellowship proved?

ALLIGATION.

¶ **101.** Alligation is the method of mixing two or more simples, of different qualities, so that the composition may be of a mean, or middle quality.

When the *quantities* and *prices* of the simples are given, to find the *mean price* of the mixture, compounded of them, the process is called *Alligation Medial.*

1. A farmer mixed together 4 bushels of wheat, worth 150 cents per bushel, 3 bushels of rye, worth 70 cents per bushel, and 2 bushels of corn, worth 50 cents per bushel; what is a bushel of the mixture worth?

It is plain, that *the cost of the whole, divided by the number of bushels, will give the price of one bushel.*

4 bushels,	at 150 cents,	cost 600 cents.		
3	at 70	 210		
2	at 50	 100		$\frac{910}{9} = 101\frac{1}{9}$ cts. *Ans.*
9 bushels cost		910 cents.		

2. A grocer mixed 5 lbs. of sugar, worth 10 cents per lb., 8 lbs. worth 12 cents, 20 lbs. worth 14 cents; what is a pound of the mixture worth? *Ans.* $12\frac{10}{11}$.

3. A goldsmith melted together 3 ounces of gold 20 carats fine, and 5 ounces 22 carats fine; what is the fineness of the mixture? *Ans.* $21\frac{1}{4}$.

4. A grocer puts 6 gallons of water into a cask containing 40 gallons of rum, worth 42 cents per gallon; what is a gallon of the mixture worth? *Ans.* $36\frac{12}{23}$ cents.

5. On a certain day the mercury was observed to stand in the thermometer as follows: 5 hours of the day, it stood at 64 degrees; 4 hours, at 70 degrees; 2 hours, at 75 degrees, and 3 hours, at 73 degrees: what was the *mean* temperature for that day?

It is plain this question does not differ, in the mode of its operation, from the former. *Ans.* $69\frac{3}{14}$ degrees.

¶ **102.** When the *mean price* or rate, and the *prices* or rates of the *several simples* are given, to find the *proportions* or *quantities* of each simple, the process is called *Alligation Alternate:* alligation alternate is, therefore, the reverse of alligation medial, and may be proved by it.

1. A man has oats worth 40 cents per bushel, which he wishes to mix with corn worth 50 cents per bushel, so that the mixture may be worth 42 cents per bushel; what proportions, or quantities of each, must he take?

Had the price of the mixture required *exceeded* the price of the oats, by *just as much* as it *fell short* of the price of the corn, it is plain, he must have taken *equal quantities* of oats and corn; had the price of the mixture exceeded the price of the oats by only ½ as much as it fell short of the price of the corn, the compound would have required 2 times as much oats as corn; and in all cases, the *less* the difference between the price of the *mixture* and that of *one* of the simples, the *greater* must be the quantity of *that simple*, in proportion to the *other;* that is, the quantities of the simples must be *inversely* as the *differences* of their prices from the price of the mixture; therefore, if these differences be mutually *exchanged*, they will, *directly*, express the *relative quantities* of each simple necessary to form the compound required. In the above example, the price of the *mixture* is 42 cents, and the price of the *oats* is 40 cents; consequently, the difference of their prices is 2 cents: the price of the *corn* is 50 cents, which differs from the price of the mixture by 8 cents. Therefore, by exchanging these differences, we have 8 bushels of *oats* to 2 bushels of *corn*, for the proportion required.

Ans. 8 bushels of *oats* to 2 bushels of *corn*, or in *that proportion.*

The correctness of this result may now be ascertained by the last rule; thus, the cost of 8 bushels of oats, at 40 cents, is 320 cents; and 2 bushels of corn, at 50 cents, is 100 cents; then, $320 + 100 = 420$, and 420, divided by the number of bushels, $(8 + 2,) = 10$, gives 42 cents for the price of the *mixture.*

2. A merchant has several kinds of tea; some at 8 shillings, some at 9 shillings, some at 11 shillings, and some at 12 shillings per pound; what proportions of each must he mix, that he may sell the compound at 10 shillings per pound?

Here we have 4 simples; but it is plain, that what has just been proved of *two* will apply to any number of *pairs*, if in each pair the price of *one* simple is *greater*, and that of the *other less*, than the price of the mixture required. Hence we have this

RULE.

The mean rate and the several prices being reduced to the same denomination,—*connect with a continued line each price that is* LESS *than the mean rate with one or more that is* GREATER, *and each price* GREATER *than the mean rate with one or more that is* LESS.

Write the difference between the MEAN *rate, or price, and the price of* EACH SIMPLE *opposite the price with which it is connected;* (thus the difference of the two prices in each pair will be mutually exchanged;) *then the sum of the differences, standing against any price, will express the* RELATIVE QUANTITY *to be taken of that price.*

By attentively considering the rule, the pupil will perceive, that there may be as many different ways of mixing the simples, and consequently as many different answers, as there are different ways of linking the several prices.

We will now apply the rule to solve the last question:—

OPERATIONS.

		lbs.	
10s.	8s.	—2	Ans.
	9s.	—1	
	11s.	—1	
	12s.	—2	

Or,

10	8	—2+1=3	Ans.
	9	1 =1	
	11	—1+2=3	
	12	—2 =2	

Here we set down the prices of the simples, one directly under another, in order, from least to greatest, as this is most convenient, and write the mean rate, (10 s.) at the left hand. In the first way of linking, we find, that we may take in the proportion of 2 pounds of the teas at 8 and 12 s. to 1 pound at 9 and 11 s. In the second way, we find for the answer, 3 pounds at 8 and 11 s. to 1 pound at 9 and 12 s.

3. What proportions of sugar, at 8 cents, 10 cents, and 14 cents per pound, will compose a mixture worth 12 cents per pound?

Ans. In the proportion of 2 lbs. at 8 and 10 cents to 6 lbs. at 14 cents.

Note. As these quantities only express the *proportions* of each kind, it is plain, that a compound of the *same mean price* will be formed by taking 3 times, 4 times, one half, or any proportion, of each quantity. Hence,

When the quantity of one simple is given, after finding

the *proportional* quantities, by the above rule, we may say, *As the* PROPORTIONAL *quantity : is to the* GIVEN *quantity : : so is each of the other* PROPORTIONAL *quantities : to the* REQUIRED *quantities of each.*

4. If a man wishes to mix 1 gallon of brandy worth 16 s. with rum at 9 s. per gallon, so that the mixture may be worth 11 s. per gallon, how much rum must he use?

Taking the differences as above, we find the *proportions* to be 2 of brandy to 5 of rum; consequently, 1 gallon of brandy will require $2\frac{1}{2}$ gallons of rum. *Ans.* $2\frac{1}{2}$ gallons.

5. A grocer has sugars worth 7 cents, 9 cents, and 12 cents per pound, which he would mix so as to form a compound worth 10 cents per pound; what must be the *proportions* of each kind?

Ans. 2 lbs. of the first and second to 4 lbs. of the third kind.

6. If he use 1 lb. of the first kind, how much must he take of the others? —— if 4 lbs., what? —— if 6 lbs., what? —— if 10 lbs., what? —— if 20 lbs., what?

Ans. to the last, 20 lbs. of the second, and 40 of the third.

7. A merchant has spices at 16 d. 20 d. and 32 d. per pound; he would mix 5 pounds of the first sort with the others, so as to form a compound worth 24 d. per pound; how much of each sort must he use?

Ans. 5 lbs. of the second, and $7\frac{1}{2}$ lbs. of the third.

8. How many gallons of water, of no value, must be mixed with 60 gallons of rum, worth 80 cents per gallon, to reduce its value to 70 cents per gallon? *Ans.* $8\frac{4}{7}$ gallons.

9. A man would mix 4 bushels of wheat, at \$1'50 per bushel, rye at \$1'16, corn at \$'75, and barley at \$'50, so as to sell the mixture at \$'84 per bushel; how much of each may he use?

10. A goldsmith would mix gold 17 carats fine with some 19, 21, and 24 carats fine, so that the compound may be 22 carats fine; what proportions of each must he use?

Ans. 2 of the 3 first sorts to 9 of the last.

11. If he use 1 oz. of the first kind, how much must he use of the others? What would be the quantity of the compound? *Ans. to last,* $7\frac{1}{2}$ ounces.

12. If he would have the whole compound consist of 15 oz., how much must he use of each kind? —— if of 30 oz., how much of each kind? —— if of $37\frac{1}{2}$ oz., how much?

Ans. to the last, 5 oz. of the 3 first, and $22\frac{1}{2}$ oz. of the last.

Hence, when the *quantity* of the compound is given, we may say, *As the sum of the* PROPORTIONAL *quantities, found by the* ABOVE RULE, *is to the quantity* REQUIRED, *so is each* PROPORTIONAL *quantity, found by the rule, to the* REQUIRED *quantity of* EACH.

13. A man would mix 100 pounds of sugar, some at 8 cents, some at 10 cents, and some at 14 cents per pound, so that the compound may be worth 12 cents per pound; how much of each kind must he use?

We find the proportions to be, 2, 2, and 6. Then, 2 + 2 + 6 = 10, and

$$10 : 100 :: \left\{\begin{array}{l} 2 : 20 \text{ lbs. at } 8 \text{ cts.} \\ 2 : 20 \text{ lbs. at } 10 \text{ cts.} \\ 6 : 60 \text{ lbs. at } 14 \text{ cts.} \end{array}\right\} \textit{Ans.}$$

14. How many gallons of water, of no value, must be mixed with brandy at $1'20 per gallon, so as to fill a vessel of 75 gallons, which may be worth 92 cents per gallon?

Ans. 17½ gallons of water to 57½ gallons of brandy.

15. A grocer has currants at 4 d., 6 d., 9d. and 11 d. per lb.; and he would make a mixture of 240 lbs., so that the mixture may be sold at 8 d. per lb.; how many pounds of each sort may he take?

Ans. 72, 24, 48, and 96 lbs., or 48, 48, 72, 72, &c.

Note. This question may have five different answers.

QUESTIONS.

1. What is alligation? 2. —— medial? 3. —— the rule for operating? 4. What is alligation alternate? 5. When the price of the mixture, and the price of the several simples, are given, how do you find the *proportional quantities* of each simple? 6. When the quantity of one simple is given, how do you find the others? 7. When the quantity of the whole compound is given, how do you find the quantity of each simple?

DUODECIMALS.

¶ **103.** Duodecimals are fractions of a foot. The word is derived from the Latin word *duodecim*, which signifies *twelve*. A foot, instead of being divided *decimally* into *ten* equal parts, is divided *duodecimally* into *twelve* equal parts,

called *inches*, or *primes*, marked thus, ($'$). Again, each of these parts is conceived to be divided into twelve other equal parts, called *seconds*, ($''$). In like manner, each second is conceived to be divided into twelve equal parts, called *thirds*, ($'''$); each third into twelve equal parts, called *fourths*, ($''''$); and so on to any extent.

In this way of dividing a foot, it is obvious, that

$1'$ *inch*, or *prime*, is - - - - - - - $\frac{1}{12}$ of a foot.
$1''$ second is $\frac{1}{12}$ of $\frac{1}{12}$, - - - = $\frac{1}{144}$ of a foot.
$1'''$ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, - - = $\frac{1}{1728}$ of a foot.
$1''''$ fourth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, = $\frac{1}{20736}$ of a foot.
$1'''''$ fifth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, = $\frac{1}{248832}$ of a foot, &c.

Duodecimals are added and subtracted in the same manner as compound numbers, 12 of a *less* denomination making 1 of a *greater*, as in the following

TABLE.

$12''''$	fourths	make	$1'''$	third,
$12'''$	thirds	- - -	$1''$	second,
$12''$	seconds	- -	$1'$	inch or prime,
$12'$	inches, or primes,	1		foot.

Note. The marks, $'$, $''$, $'''$, $''''$, &c., which distinguish the different parts, are called the *indices* of the *parts* or denominations.

MULTIPLICATION OF DUODECIMALS.

Duodecimals are chiefly used in measuring *surfaces* and *solids*.

1. How many square feet in a board 16 feet 7 inches long, and 1 foot 3 inches wide?

Note. Length × breadth = superficial contents, (¶ 25.)

OPERATION.

	ft.		
Length,	16	$7'$	
Breadth,	1	$3'$	
	4	$1'$	$9''$
	16	$7'$	
Ans.	20	$8'$	$9''$

7 inches, or primes, = $\frac{7}{12}$ of a foot, and 3 inches = $\frac{3}{12}$ of a foot; consequently, the product of $7' \times 3' = \frac{21}{144}$ of a foot, that is, $21'' = 1'$ and $9''$; wherefore, we set down the $9''$, and reserve the $1'$ to be carried forward to its proper place. To multiply 16 feet by $3'$

is to take $\frac{3}{12}$ of $\frac{16}{1} = \frac{48}{12}$, that is, 48′; and the 1′ which we reserved makes 49′, = 4 feet 1′; we therefore set down the 1′, and carry forward the 4 feet to its proper place. Then, multiplying the multiplicand by the 1 foot in the multiplier, and adding the two products together, we obtain the *Answer*, 20 feet, 8′, and 9″.

The only difficulty that can arise in the multiplication of duodecimals is, in finding of what denomination is the product of any two denominations. This may be ascertained as above, and in all cases it will be found to hold true, that the *product of any two denominations will always be of the denomination denoted by the sum of their* INDICES. Thus, in the above example, the sum of the indices of 7′ × 3′ is ″; consequently, the product is 21″; and thus *primes* multiplied by *primes* will produce *seconds*; *primes* multiplied by *seconds* produce *thirds*; *fourths* multiplied by *fifths* produce *ninths*, &c.

It is generally most convenient, in practice, to multiply the multiplicand *first* by the *feet* of the multiplier, then by the inches, &c., thus:—

ft.		
16	7′	
1	3′	
16	7′	
4	1′	9″
20	8′	9″

16 ft. × 1 ft. = 16 ft., and 7′ × 1 ft. = 7′. Then, 16 ft. × 3′ = 48′ = 4 ft., and 7′ × 3′ = 21″ = 1′ 9″. The two products, added together, give for the *Answer*, 20 ft. 8′ 9″, as before.

2. How many solid feet in a block 15 ft. 8′ long, 1 ft. 5′ wide, and 1 ft. 4′ thick?

OPERATION.

	ft.			
Length,	15	8′		
Breadth,	1	5′		
	15	8′		
	6	6′	4″	
	22	2′	4″	
Thickness,	1	4′		
	22	2′	4″	
	7	4′	9″	4‴
Ans.	29	7′	1″	4‴

The length multiplied by the breadth, and that product by the thickness, gives the *solid contents*, (¶ 36.)

From these examples we derive the following RULE:— Write down the denominations as compound numbers, and in multiplying remember, that the product of any two denominations will always be of that denomination denoted by the *sum of their indices.*

EXAMPLES FOR PRACTICE.

3. How many square feet in a stock of 15 boards, 12 ft. 8′ in length, and 13′ wide? *Ans.* 205 ft. 10′.

4. What is the product of 371 ft. 2′ 6″ multiplied by 181 ft. 1′ 9″? *Ans.* 67242 ft. 10′ 1″ 4‴ 6⁗.

Note. Painting, plastering, paving, and some other kinds of work, are doue by the square yard. If the contents in square feet be divided by 9, the quotient, it is evident, will be square yards.

5. A man painted the walls of a room 8 ft. 2′ in height, and 72 ft. 4′ in compass; (that is, the measure of all its sides;) how many square yards did he paint?

Ans. 65 yds. 5 ft. 8′ 8″.

6. There is a room plastered, the compass of which is 47 ft. 3′, and the height 7 ft. 6′; what are the contents?

Ans. 39 yds. 3 ft. 4′ 6″.

7. How many cord feet of wood in a load 8 feet long, 4 feet wide, and 3 feet 6 inches high?

Note. It will be recollected, that 16 solid feet make a *cord foot.* *Ans.* 7 cord feet.

8. In a pile of wood 176 ft. in length, 3 ft. 9′ wide, and 4 ft. 3′ high, how many cords?

Ans. 21 cords, and $7\frac{5}{16}$ cord feet over.

9. How many feet of cord wood in a load 7 feet long, 3 feet wide, and 3 feet 4 inches high? and what will it come to at $ ‘40 per cord foot?

Ans. $4\frac{3}{8}$ cord feet, and it will come to $ 1‘75.

10. How much wood in a load 10 ft. in length, 3 ft. 9′ in width, and 4 ft. 8′ in height? and what will it cost at $ 1‘92 per cord?

Ans. 1 cord and $2\frac{15}{16}$ cord feet, and it will come to $ 2‘62½.

¶ **104.** *Remark.* By some surveyors of wood, dimensions are taken in feet and *decimals* of a foot. For this purpose, make a rule or scale 4 feet long, and divide it into feet, and each foot into ten equal parts. On one end of the rule,

for 1 foot, let each of these parts be divided into 10 other equal parts. The former division will be 10ths, and the latter 100ths of a foot. Such a rule will be found very convenient for surveyors of wood and of lumber, for painters, joiners, &c.; for the dimensions taken by it being in feet and decimals of a foot, the casts will be no other than so many operations in decimal fractions.

11. How many square feet in a hearth stone, which, by a rule, as above described, measures 4'5 feet in length, and 2'6 feet in width? and what will be its cost, at 75 cents per square foot? *Ans.* 11'7 feet; and it will cost $8'775.

12. How many cords in a load of wood 7'5 feet in length, 3'6 feet in width, and 4'8 feet in height? *Ans.* 1 cord, $1\frac{6}{10}$ ft.

13. How many cord feet in a load of wood 10 feet long, 3'4 feet wide, and 3'5 feet high? *Ans.* $7\frac{7}{16}$.

QUESTIONS.

1. What are duodecimals? 2. From what is the word *derived*? 3. Into how many parts is a foot usually divided, and what are the parts called? 4. What are the *other* denominations? 5. What is understood by the *indices* of the denominations? 6. In what are duodecimals chiefly used? 7. How are the contents of a *surface* bounded by straight lines found? 8. How are the contents of a *solid* found? 9. How is it known of what denomination is the product of any two denominations? 10. How may a scale or rule be formed for taking dimensions in feet and decimal parts of a foot?

INVOLUTION.

¶ **105.** Involution, or the raising of powers, is the multiplying any given number into itself continually a certain number of times. The products thus produced are called the *powers* of the given number. The number itself is called the *first* power, or *root*. If the *first* power be multiplied by *itself*, the product is called the *second* power or *square*; if the square be multiplied by the first power, the product is called the *third* power, or *cube*, &c.; thus,

5 is the root, or 1st power, of 5.
$5\times5=25$ is the 2d power, or square, of 5, $=5^2$
$5\times5\times5=125$ is the 3d power, or cube, of 5, $=5^3$.
$5\times5\times5\times5=625$ is the 4th power, or biquadrate, of 5, $=5^4$.

The number denoting the power is called the *index*, or *exponent*; thus, 5^4 denotes that 5 is raised or involved to the 4th power.

1. What is the square, or 2d power, of 7? *Ans.* 49.
2. What is the square of 30? *Ans.* 900.
3. What is the square of 4000? *Ans.* 16000000.
4. What is the cube, or 3d power, of 4? *Ans.* 64.
5. What is the cube of 800? *Ans.* 512000000.
6. What is the 4th power of 60? *Ans.* 12960000.
7. What is the square of 1? —— of 2? —— of 3? —— of 4? *Ans.* 1, 4, 9, and 16.
8. What is the cube of 1? —— of 2? —— of 3? —— of 4? *Ans.* 1, 8, 27, and 64.
9. What is the square of $\frac{2}{3}$? —— of $\frac{4}{5}$? —— of $\frac{7}{8}$? *Ans.* $\frac{4}{9}$, $\frac{16}{25}$, and $\frac{49}{64}$.
10. What is the cube of $\frac{2}{3}$? —— of $\frac{4}{5}$? —— of $\frac{7}{8}$? *Ans.* $\frac{8}{27}$, $\frac{64}{125}$, and $\frac{343}{512}$.
11. What is the square of $\frac{1}{2}$? —— the 5th power of $\frac{1}{2}$? *Ans.* $\frac{1}{4}$, and $\frac{1}{32}$.
12. What is the square of 1‘5? —— the cube? *Ans.* 2‘25, and 3‘375.
13. What is the 6th power of 1‘2? *Ans.* 2‘985984.
14. Involve $2\frac{1}{4}$ to the 4th power.

Note. A mixed number, like the above, may be reduced to an improper fraction before involving: thus, $2\frac{1}{4} = \frac{9}{4}$; or it may be reduced to a decimal; thus, $2\frac{1}{4} = 2‘25$.

Ans. $\frac{6561}{256} = 25\frac{161}{256}$.

15. What is the square of $4\frac{7}{8}$? *Ans.* $\frac{1521}{64} = 23\frac{49}{64}$.
16. What is the value of 7^4, that is, the 4th power of 7? *Ans.* 2401.
17. How much is 9^3? —— 6^5? —— 10^4? *Ans.* 729, 7776, 10000
18. How much is 2^7? —— 3^6? —— 4^5? —— 5^3? —— 6^5? —— 10^3? *Ans. to last*, 100000000.

The powers of the nine digits, from the first power to the fifth, may be seen in the following

TABLE.

Roots -	or 1st Powers	1	2	3	4	5	6	7	8	9
Squares	or 2d Powers	1	4	9	16	25	36	49	64	81
Cubes -	or 3d Powers	1	8	27	64	125	216	343	512	729
Biquadrates	or 4th Powers	1	16	81	256	625	1296	2401	4096	6561
Sursolids	or 5th Powers	1	32	243	1024	3125	7776	16807	32768	59049

EVOLUTION.

¶ **106.** Evolution, or the *extracting* of roots, is the method of finding the *root* of any power or number.

The *root*, as we have seen, is that number, which, by a continual multiplication into itself, produces the given power. The *square root* is a number which, being squared, will produce the given number; and the *cube*, or *third root*, is a number which, being cubed or involved to the 3d power, will produce the given number: thus, the *square root* of 144 is 12, because $12^2 = 144$; and the *cube root* of 343 is 7, because 7^3, that is, $7 \times 7 \times 7, = 343$; and so of other numbers.

Although there is no number which will not produce a perfect power by involution, yet there are many numbers of which *precise roots* can never be obtained. But, by the help of *decimals*, we can approximate, or approach, towards the root to any assigned degree of exactness. Numbers, whose precise roots cannot be obtained, are called *surd* numbers, and those, whose roots can be exactly obtained, are called *rational* numbers.

The square root is indicated by this character $\surd$ placed before the number; the other roots by the same character, with the index of the root placed over it. Thus, the square root of 16 is expressed $\sqrt{16}$; and the cube root of 27 is expressed $\sqrt[3]{27}$; and the 5th root of 7776, $\sqrt[5]{7776}$.

When the power is expressed by several numbers, with the sign + or — between them, a line, or *vinculum*, is drawn from the top of the sign over all the parts of it; thus, the square root of 21 — 5 is $\sqrt{21 - 5}$, &c.

EXTRACTION OF THE SQUARE ROOT.

¶ **107.** To extract the square root of any number is to find a number, which, being multiplied into itself, shall produce the given number.

1. Supposing a man has 625 yards of carpeting, a yard wide, what is the length of one side of a square room, the

floor of which the carpeting will cover? that is, what is one side of a square, which contains 625 square yards?

We have seen, (¶ 35,) that the contents of a square surface is found by multiplying the length of one side into itself, that is, by raising it to the second power; and hence, having the contents (625) given, we must extract its *square root* to find one side of the room.

This we must do by a sort of trial: and,

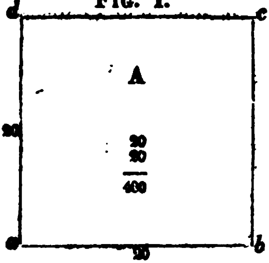
1st. We will endeavour to ascertain how many figures there will be in the root. This we can easily do, by pointing off the number, from units, into periods of two figures each; for the square of any root always contains just *twice* as many, or one figure *less* than twice as many figures, as are in the root; of which truth the pupil may easily satisfy himself by trial. Pointing off the number, we find, that the root will consist of *two* figures, a ten and a unit.

OPERATION.

$\dot{6}2\dot{5}$(2
4
———
225

2d. We will now seek for the first figure, that is, for the *tens* of the root, and it is plain, that we must extract it from the left hand period 6, (hundreds.) The greatest square in 6 (hundreds) we find, by trial, to be 4, (hundreds,) the root of which is 2, (tens, = 20;) therefore, we set 2 (tens) in the root. The *root*, it will be recollected, is *one side* of a square. Let us, then, form a square, (A, Fig. I.) each side of which shall be supposed 2 tens, = 20 yards, expressed by the root now obtained.

FIG. I.



The contents of this square are 20 × 20 = 400 yards, now disposed of, and which, consequently, are to be deducted from the whole number of yards, (625,) leaving 225 yards. This deduction is most readily performed by subtracting the square number 4, (hundreds,) or the square of 2, (the figure in the root already found,) from the period 6, (hundreds,) and bringing down the next period by the side of the remainder, making 225, as before.

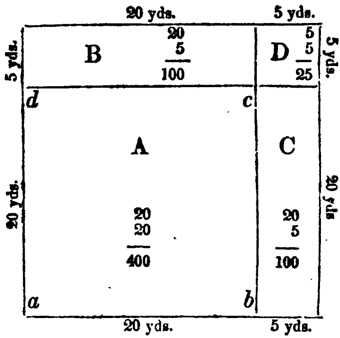
3d. The square A is now to be enlarged by the addition of the 225 remaining yards; and, in order that the figure may retain its *square form*, it is evident, the addition must be made on *two* sides. Now, if the 225 yards be divided by the *length* of the *two* sides, (20 + 20 = 40,) the quotient will be the *breadth* of this new addition of 225 yards to the sides *c d* and *b c* of the square A.

But our root already found, = 2 tens, is the length of *one* side of the figure A; we therefore take *double* this root, = 4 tens, for a divisor.

OPERATION—CONTINUED.

```
   6̇25̇(25
   4
   ---
45)225
   225
   ---
```

FIG. II.



The divisor, 4, (tens,) is in reality 40, and we are to seek how many times 40 is contained in 225, or, which is the same thing, we may seek how many times 4 (tens) is contained in 22, (tens,) rejecting the right hand figure of the dividend, because we have rejected the cipher in the divisor. We find our quotient, *that is*, the *breadth* of the addition, to be 5 yards; but, if we look at Fig. II., we shall perceive that this addition of 5 yards to the *two* sides does not complete the square; for there is still wanting, in the corner D, a small square, each side of which is equal to this last quotient, 5; we must, therefore, add this quotient, 5, to the divisor, 40, that is, place it at the right hand of the 4, (tens,) making it 45; and then the whole divisor, 45, multiplied by the quotient, 5, will give the contents of the whole addition around the sides of the figure A, which, in this case, being 225 yards, the same as our dividend, we have no remainder, and the work is done. Consequently, Fig. II. represents the floor of a square room, 25

yards on a side, which 625 square yards of carpeting will exactly cover.

The proof may be seen by adding together the several parts of the figure, thus :—

The square	A contains	400	yards.
The figure	B	100	....
....	C	100	....
....	D	25	....
	Proof,	625	....

Or we may prove it by involution, thus :— $25 \times 25 = 625$, as before.

From this example and illustration we derive the following general

RULE

FOR THE EXTRACTION OF THE SQUARE ROOT.

I. Point off the given number into periods of two figures each, by putting a dot over the units, another over the hundreds, and so on. These dots show the number of figures of which the root will consist.

II. Find the greatest square number in the left hand period, and write its root as a quotient in division. Subtract the square number from the left hand period, and to the remainder bring down the next period for a dividend.

III. Double the root already found for a divisor; seek how many times the divisor is contained in the dividend, excepting the right hand figure, and place the result in the root, and *also* at the right hand of the divisor; multiply the divisor, thus augmented, by the last figure of the root, and subtract the product from the dividend; to the remainder bring down the next period for a new dividend.

IV. Double the root already found for a new divisor, and continue the operation as before, until all the periods are brought down.

Note 1. If we double the right hand figure of the *last* divisor, we shall have the double of the root.

Note 2. As the value of figures, whether integers or decimals, is determined by their distance from the place of units, so we must always begin at unit's place to point off the given number, and, if it be a mixed number, we must point it off *both* ways from units, and if there be a deficiency in any period of decimals, it may be supplied by a cipher. It is plain, the *root* must always consist of so many integers

and decimals as there are periods belonging to each in the given number.

EXAMPLES FOR PRACTICE.

2. What is the square root of 10342656?

OPERATION.

```
 10342656 ( 3216, Ans.
  9
      ----
  62 ) 134
       124
      -----
 641 ) 1026
        641
       -----
6426 ) 38556
       38556
       -----
```

3. What is the square root of 43264?

OPERATION.

```
     43264 ( 208, Ans.
     4
    ----
408 ) 3264
      3264
      ----
```

4. What is the square root of 998001? *Ans.* 999
5. What is the square root of 234'09? *Ans.* 15'3.
6. What is the square root of 964'5192360241? *Ans.* 31'05671.
7. What is the square root of '001296? *Ans.* '036.
8. What is the square root of '2916? *Ans.* '54.
9. What is the square root of 36372961? *Ans.* 6031.
10. What is the square root of 164? *Ans.* 12'8 +.

¶ 108. In this last example, as there was a remainder, after bringing down all the figures, we continued the operation to decimals, by annexing two ciphers for a new period, and thus we may continue the operation to any assigned degree of exactness; but the pupil will readily perceive, that he can never, in this manner, obtain the *precise* root; for the last figure in each *dividend* will always be a cipher, and the

last figure in each *divisor* is the same as the last *quotient figure;* but no one of the nine digits, multiplied into itself, produces a number ending with a *cipher;* therefore, whatever be the quotient figure, there will still be a remainder.

11. What is the square root of 3? *Ans.* 1‘73 +.
12. What is the square root of 10? *Ans.* 3‘16 +.
13. What is the square root of 184‘2? *Ans.* 13‘57 +.
14. What is the square root of $\frac{4}{9}$?

Note. We have seen, (¶ 105, ex. 9,) that fractions are *squared* by squaring *both* the numerator and the denominator. Hence it follows, that the *square root* of a fraction is found by extracting the root of the numerator and of the denominator. The root of 4 is 2, and the root of 9 is 3.

Ans. $\frac{2}{3}$.

15. What is the square root of $\frac{4}{25}$? *Ans.* $\frac{2}{5}$.
16. What is the square root of $\frac{16}{100}$? *Ans.* $\frac{4}{10}$.
17. What is the square root of $\frac{81}{144}$? *Ans.* $\frac{9}{12} = \frac{3}{4}$.
18. What is the square root of $20\frac{1}{4}$? *Ans.* $4\frac{1}{2}$.

When the numerator and denominator are not *exact squares,* the fraction may be reduced to a decimal, and the *approximate* root found, as directed above.

19. What is the square root of $\frac{3}{4}$ = ‘75? *Ans.* ‘866 +.
20. What is the square root of $\frac{20}{24}$? *Ans.* ‘912 +.

SUPPLEMENT TO THE SQUARE ROOT.

QUESTIONS.

1. What is involution? 2. What is understood by a power? 3. —— the first, the second, the third, the fourth power? 4. What is the index, or exponent? 5. How do you involve a number to any required power? 6. What is evolution? 7. What is a root? 8. Can the precise root of all numbers be found? 9. What is a surd number? 10. —— a rational? 11. What is it to extract the square root of any number? 12. Why is the given sum pointed into periods of two figures each? 13. Why do we double the root for a divisor? 14. Why do we, in dividing, reject the right hand figure of the dividend? 15. Why do we place the quotient figure to the right hand of the divisor? 16. How may we

prove the work? 17. Why do we point off mixed numbers both ways from units? 18. When there is a remainder, how may we continue the operation? 19. Why can we never obtain the precise root of surd numbers? 20. How do we extract the square root of vulgar fractions?

EXERCISES.

1. A general has 4096 men; how many must he place in rank and file to form them into a square? *Ans.* 64.

2. If a square field contains 2025 square rods, how many rods does it measure on each side? *Ans.* 45 rods.

3. How many trees in each row of a square orchard containing 5625 trees? *Ans.* 75.

4. There is a circle, whose *area*, or superficial contents, is 5184 feet; what will be the length of the side of a square of equal area? $\sqrt{5184} = 72$ feet, *Ans.*

5. A has two fields, one containing 40 acres, and the other containing 50 acres, for which B offers him a square field containing the same number of acres as both of these; how many rods must each side of this field measure? *Ans.* 120 rods.

6. If a certain square field measure 20 rods on each side, how much will the side of a square field measure, containing 4 times as much? $\sqrt{20 \times 20 \times 4} = 40$ rods, *Ans.*

7. If the side of a square be 5 feet, what will be the side of one 4 times as large? —— 9 times as large? —— 16 times as large? —— 25 times as large? —— 36 times as large? *Answers,* 10 ft.; 15 ft.; 20 ft.; 25 ft.; and 30 ft.

8. It is required to lay out 288 rods of land in the form of a parallelogram, which shall be twice as many rods in length as it is in width.

Note. If the field be divided in the middle, it will form two *equal* squares.

Ans. 24 rods long, and 12 rods wide.

9. I would set out, at equal distances, 784 apple trees, so that my orchard may be 4 times as long as it is broad; how many rows of trees must I have, and how many trees in each row? *Ans.* 14 rows, and 56 trees in each row.

10. There is an oblong piece of land, containing 192 square rods, of which the width is $\frac{3}{4}$ as much as the length; required its dimensions. *Ans.* 16 by 12.

11. There is a circle, whose diameter is 4 inches; what is the diameter of a circle 9 times as large?

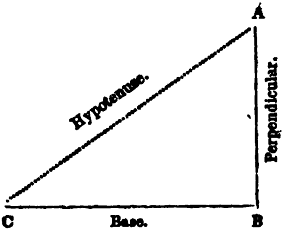
Note. The areas or contents of circles are in proportion to the *squares* of their *diameters*, or of their *circumferences.* Therefore, to find the diameter *required*, square the *given* diameter, multiply the square by the given ratio, and the square root of the product will be the diameter required.

$\sqrt{4 \times 4 \times 9} = 12$ inches, *Ans.*

12. There are two circular ponds in a gentleman's pleasure ground; the diameter of the less is 100 feet, and the greater is 3 times as large; what is its diameter? *Ans.* 173·2+ feet.

13. If the diameter of a circle be 12 inches, what is the diameter of one $\frac{1}{4}$ as large? *Ans.* 6 inches.

¶ **109.** 14. A carpenter has a large *wooden square;* one part of it is 4 feet long, and the other part 3 feet long; what is the length of a pole, which will just reach from one end to the other?



Note. A figure of 3 sides is called a triangle, and, if one of the corners be a *square corner*, or *right angle*, like the angle at B in the annexed figure, it is called a *right-angled triangle*, of which the square of the longest side, A C, (called the hypotenuse,) is equal to the *sum* of the squares of the other two sides, A B and B C.

$4^2 = 16$, and $3^2 = 9$; then, $\sqrt{9 + 16} = 5$ feet, *Ans.*

15. If, from the corner of a square room, 6 feet be measured off one way, and 8 feet the other way, along the sides of the room, what will be the length of a pole reaching from point to point? *Ans.* 10 feet.

16. A wall is 32 feet high, and a ditch before it is 24 feet wide; what is the length of a ladder that will reach from the top of the wall to the opposite side of the ditch?

Ans. 40 feet.

17. If the ladder be 40 feet, and the wall 32 feet, what is the width of the ditch? *Ans.* 24 feet.

18. The ladder and ditch given, required the wall.

Ans. 32 feet

19. The distance between the lower ends of two equal rafters is 32 feet, and the height of the ridge, above the beam on which they stand, is 12 feet; required the length of each rafter. *Ans.* 20 feet.

20. There is a building 30 feet in length and 22 feet in width, and the eaves project beyond the wall 1 foot on every side; the roof terminates in a point at the centre of the building, and is there supported by a post, the top of which is 10 feet above the beams on which the rafters rest; what is the distance from the foot of the post to the corners of the eaves? and what is the length of a rafter reaching to the middle of one *side*? —— a rafter reaching to the middle of one *end*? and a rafter reaching to the *corners* of the eaves?

Answers, in order, 20 ft.; 15‘62 + ft.; 18‘86 + ft.; and 22‘36 + ft.

21. There is a field 800 rods long and 600 rods wide; what is the distance between two opposite corners?

Ans. 1000 rods.

22. There is a square field containing 90 acres; how many rods in length is each side of the field? and how many rods apart are the opposite corners?

Answers, 120 rods; and 169‘7 + rods.

23. There is a square field containing 10 acres; what distance is the centre from each corner? *Ans.* 28‘28 + rods.

EXTRACTION OF THE CUBE ROOT.

¶ **110.** A solid body, having *six equal sides*, and each of the sides an *exact square*, is a CUBE, and the measure in length of one of its sides is the *root* of that cube; for the *length*, *breadth* and *thickness* of such a body are *all alike*; consequently, the length of one side, raised to the 3d power, gives the solid contents. (See ¶ 36.)

Hence it follows, that extracting the *cube root* of any number of feet is finding the length of one side of a cubic body, of which the whole contents will be equal to the given number of feet.

1. What are the solid contents of a cubic block, of which each side measures 2 feet? *Ans.* $2^3 = 2 \times 2 \times 2 = 8$ feet.

2. How many solid feet in a cubic block, measuring 5 feet on each side? *Ans.* $5^3 = 125$ feet.

3. How many feet in length is each side of a cubic block, containing 125 solid feet? *Ans.* $\sqrt[3]{125} = 5$ feet.

Note. The root may be found by trial.

4. What is the side of a cubic block, containing 64 solid feet?—— 27 solid feet?—— 216 solid feet?—— 512 solid feet? *Answers,* 4 ft.; 3 ft.; 6 ft.; and 8 ft.

5. Supposing a man has 13824 feet of timber, in separate blocks of 1 cubic foot each; he wishes to pile them up in a cubic pile; what will be the length of each side of such a pile?

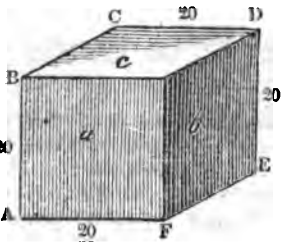
It is evident, the answer is found by extracting the cube root of 13824; but this number is so large, that we cannot so easily find the root by trial as in the former examples;—We will endeavour, however, to do it by a *sort of trial;* and,

1st. We will try to ascertain the number of figures, of which the root will consist. This we may do by pointing the number off into periods of *three* figures each (¶ 107, ex. 1.)

OPERATION.

$$\begin{array}{l} 1\dot{3}82\dot{4}(2 \\ \underline{\ 8\ \ \ \ } \\ \ 5824 \end{array}$$

FIG. I.



$$\begin{array}{r} 20 \\ \underline{20} \\ 400 \\ \underline{20} \\ 8000 \end{array}$$ *feet, Contents.*

Pointing off, we see, the root will consist of two figures, a *ten* and a *unit.* Let us, then, seek for the first figure, or tens of the root, which must be extracted from the left hand period, 13, (thousands.) The greatest cube in 13 (thousands) we find *by trial,* or by the *table of powers,* to be 8, (thousands,) the root of which is 2, (tens;) therefore, we place 2 (tens) in the root. The root, it will be recollected, is one side of a cube. Let us, then, form a cube, (Fig. I.) each side of which shall be supposed 20 feet, expressed by the root now obtained. The contents of this cube are $20 \times 20 \times 20 = 8000$ solid feet, which are now disposed of, and which, consequently, are to be deducted from the whole number of feet, 13824. 8000 taken from 13824 leave 5824 feet. This deduction is most readily performed by subtracting the cubic number, 8, or the cube of 2, (the figure of the root already found,) from

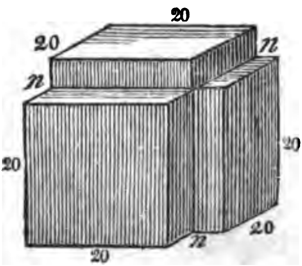
the period 13, (thousands,) and bringing down the next period by the side of the remainder, making 5824, as before.

2d. The cubic pile A D is now to be enlarged by the addition of 5824 solid feet, and, in order to preserve the cubic form of the pile, the addition must be made on one half of its sides, that is, on 3 sides, *a*, *b*, and *c*. Now, if the 5824 solid feet be divided by the square contents of these 3 equal sides, that is, by 3 times, $(20 \times 20 = 400) = 1200$, the quotient will be the thickness of the addition made to each of the sides *a*, *b*, *c*. But the root, 2, (tens,) already found, is the length of *one* of these sides; we therefore square the root, 2, (tens,) $= 20 \times 20 = 400$, for the *square contents* of *one* side, and multiply the product by 3, the *number* of sides, $400 \times 3 = 1200$; or, which is the same in effect, and more convenient in practice, we may square the 2, (tens,) and multiply the product by 300, thus, $2 \times 2 = 4$, and $4 \times 300 = 1200$, for the divisor, as before.

OPERATION—CONTINUED.

13824(24 *Root.*
8
Divisor, 1200)5824 *Dividend.*
4800
960
64
5824
0000

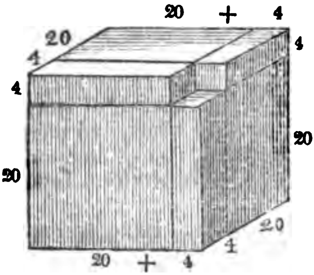
FIG. II.



The divisor, 1200, is contained in the dividend 4 times; consequently, 4 feet is the thickness of the addition made to each of the three sides, *a*, *b*, *c*, and $4 \times 1200 = 4800$, is the solid feet contained in these additions; but, if we look at Fig. II., we shall perceive, that this addition to the 3 sides does not complete the cube; for there are deficiencies in the 3 corners *n*, *n*, *n*. Now the *length* of each of these *deficiencies* is the same as the *length* of *each side*, that is, 2 (tens) $= 20$, and their *width* and *thickness* are each equal to the *last quotient* figure, (4); their contents, therefore, or the number of feet required to *fill* these deficiencies, will be found by multiplying the *square* of the last quotient figure, $(4^2) = 16$, by the length of *all* the deficiencies, that is, by 3 times

the length of *each* side, which is expressed by the former quotient figure, 2, (tens.) 3 times 2 (tens) are 6 (tens) = 60; or, what is the same in effect, and more convenient in practice, we may multiply the quotient figure, 2, (tens,) by 30, thus, 2 × 30 = 60, as before; then, 60 × 16 = 960, contents of the three deficiencies *n*, *n*, *n*.

Fig. III.



Looking at Fig. III., we perceive there is still a deficiency in the corner where the last blocks meet. This deficiency is a cube, each side of which is equal to the last quotient figure, 4. The cube of 4, therefore, (4 × 4 × 4 = 64,) will be the solid contents of this corner, which in Fig. IV. is seen filled.

Now, the sum of these several additions, viz. 4800 + 960 + 64 = 5824, will make the subtrahend, which, subtracted from the dividend, leaves no remainder, and the work is done.

Fig. IV.

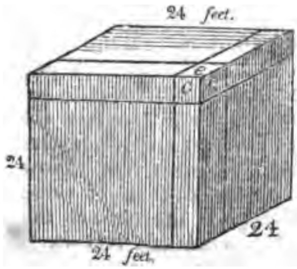


Fig. IV. shows the pile which 13824 solid blocks of one foot each would make, when laid together, and the root, 24, shows the length of one side of the pile. The correctness of the work may be ascertained by cubing the side now found, 24^3, thus, 24 × 24 × 24 = 13824, the given number; or it may be proved by adding together the contents of all the several parts, thus,

Feet.
8000 = contents of Fig. I.
4800 = addition to the sides *a*, *b*, and *c*, Fig. I.
960 = addition to fill the deficiencies *n*, *n*, *n*, Fig. II.
64 = addition to fill the corner *e*, *e*, *e*, Fig. IV.
13824 = contents of the whole pile, Fig. IV., 24 feet on each side

From the foregoing example and illustration we derive the following

RULE

FOR EXTRACTING THE CUBE ROOT.

I. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

II. Find the greatest cube in the left hand period, and put its root in the quotient.

III. Subtract the cube thus found from the said period, and to the remainder bring down the next period, and call this the *dividend.*

IV. Multiply the square of the quotient by 300, calling it the divisor.

V. Seek how many times the divisor may be had in the dividend, and place the result in the root; then multiply the divisor by this quotient figure, and write the product under the dividend.

VI. Multiply the square of this quotient figure by the former *figure* or *figures* of the root, and this product by 30, and place the product under the last; under all write the cube of this quotient figure, and call their amount the *subtrahend.*

VII. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on, till the whole is finished.

Note 1. If it happens that the divisor is not contained in the dividend, a cipher must be put in the root, and the next period brought down for a dividend.

Note 2. The same rule must be observed for continuing the operation, and pointing off for decimals, as in the square root.

Note 3. The pupil will perceive that the number which we call *the divisor*, when multiplied by the last quotient figure, does not produce so large a number as the real subtrahend; hence, the figure in the root must frequently be smaller than the quotient figure.

EXAMPLES FOR PRACTICE.

6. What is the cube root of 1860867?

OPERATION.

$$
\begin{array}{rl}
 & \dot{1}86\dot{0}86\dot{7}(123 \textit{ Ans.} \\
 & 1 \\
1^2 \times 300 = 300\ ) & \overline{860} \textit{ first Dividend.} \\
 & 600 \\
2^2 \times 1 \times 30 = & 120 \\
2^3 = & 8 \\
 & \overline{728} \textit{ first Subtrahend.} \\
12^2 \times 300 = 43200\ ) & \overline{132867} \textit{ second Dividend} \\
 & 129600 \\
3^2 \times 12 \times 30 = & 3240 \\
3^3 = & 27 \\
 & \overline{132867} \textit{ second Subtrahend.} \\
 & \overline{000000}
\end{array}
$$

7. What is the cube root of 373248? *Ans.* 72.
8. What is the cube root of 21024576? *Ans.* 276.
9. What is the cube root of 84'604519? *Ans.* 4'39.
10. What is the cube root of '000343? *Ans.* '07.
11. What is the cube root of 2? *Ans.* 1'25 +.
12. What is the cube root of $\frac{8}{27}$? *Ans.* $\frac{2}{3}$.

Note. See ¶ 105, ex. 10, and ¶ 108, ex. 14.

13. What is the cube root of $\frac{125}{216}$? *Ans.* $\frac{5}{6}$
14. What is the cube root of $\frac{343}{1728}$? *Ans.* $\frac{7}{12}$
15. What is the cube root of $\frac{1}{500}$? *Ans.* '125 +.
16. What is the cube root of $\frac{1}{125}$? *Ans.* $\frac{1}{5}$.

SUPPLEMENT TO THE CUBE ROOT.

QUESTIONS.

1. What is a cube? 2. What is understood by the cube root? 3. What is it to extract the cube root? 4. Why is the square of the quotient multiplied by 300 for a divisor? 5. Why, in finding the subtrahend, do we multiply the square of the last quotient figure by 30 times the former figure of the root? 6. Why do we cube the quotient figure? 7. How do we prove the operation?

EXERCISES.

1. What is the side of a cubical mound, equal to one 288 feet long, 216 feet broad, and 48 feet high? *Ans.* 144 feet.

2. There is a cubic box, one side of which is 2 feet; how many solid feet does it contain? *Ans.* 8 feet.

3. How many cubic feet in one 8 times as large? and what would be the length of one side?

Ans. 64 solid feet, and one side is 4 feet.

4. There is a cubical box, one side of which is 5 feet; what would be the side of one containing 27 times as much? —— 64 times as much? —— 125 times as much?

Ans. 15, 20, and 25 feet.

5. There is a cubical box, measuring 1 foot on each side; what is the side of a box 8 times as large? —— 27 times? —— 64 times? *Ans.* 2, 3, and 4 feet.

¶ 111. Hence we see, that the *sides* of cubes are as the *cube roots* of their *solid contents*, and, consequently, their *contents* are as the *cubes* of their *sides*. The same proportion is true of the *similar sides*, or of the *diameters* of *all* solid figures of similar forms.

6. If a ball, weighing 4 pounds, be 3 inches in diameter, what will be the diameter of a ball of the same metal, weighing 32 pounds? $4 : 32 :: 3^3 : 6^3$ *Ans.* 6 inches.

7. If a ball, 6 inches in diameter, weigh 32 pounds, what will be the weight of a ball 3 inches in diameter? *Ans.* 4 lbs.

8. If a globe of silver, 1 inch in diameter, be worth \$6, what is the value of a globe 1 foot in diameter?

Ans. \$10368.

9. There are two globes; one of them is 1 foot in diameter, and the other 40 feet in diameter; how many of the smaller globes would it take to make 1 of the larger?

Ans. 64000.

10. If the diameter of the sun is 112 times as much as the diameter of the earth, how many globes like the earth would it take to make one as large as the sun? *Ans.* 1404928.

11. If the planet Saturn is 1000 times as large as the earth, and the earth is 7900 miles in diameter, what is the diameter of Saturn? *Ans.* 79000 miles.

12. There are two planets of equal density; the diameter of the less is to that of the larger as 2 to 9; what is the ratio of their solidities? *Ans.* $\frac{8}{729}$; or, as 8 to 729.

Note. The roots of most powers may be found by the square and cube root only: thus, the biquadrate, or 4th root, is the square root of the square root; the 6th root is the cube root of the square root; the 8th root is the square root of the 4th root; the 9th root is the cube root of the cube root, &c. Those roots, viz. the 5th, 7th, 11th, &c., which are not resolvable by the square and cube roots, seldom occur, and, when they do, the work is most easily performed by logarithms; for, if the logarithm of any number be divided by the index of the root, the quotient will be the logarithm of the root itself.

ARITHMETICAL PROGRESSION.

¶ **112.** Any rank or *series* of numbers, more than two, *increasing* or *decreasing* by a constant difference, is called an Arithmetical *Series*, or *Progression*.

When the numbers are formed by a continual *addition* of the common difference, they form an *ascending series;* but when they are formed by a continual *subtraction* of the common difference, they form a *descending series.*

Thus, { 3, 5, 7, 9, 11, 13, 15, &c. is an *ascending* series.
15, 13, 11, 9, 7, 5, 3, &c. is a *descending* series.

The numbers which form the series are called the *terms* of the series. The *first* and *last* terms are the *extremes*, and the other terms are called the *means.*

There are five things in arithmetical progression, any *three* of which being given, the other *two* may be found:—

1st. The *first* term.
2d. The *last* term.
3d. The *number* of terms.
4th. The *common difference.*
5th. The *sum* of all the terms.

1. A man bought 100 yards of cloth, giving 4 cents for the first yard, 7 cents for the second, 10 cents for the third, and so on, with a common difference of 3 cents; what was the cost of the last yard?

As the *common difference*, 3, is added to every yard except the last, it is plain the *last* yard must be 99 × 3, = 297 cents, *more* than the *first* yard. *Ans.* 301 cents.

Hence, *when the first term, the common difference, and the number of terms, are given, to find the last term,*—Multiply the number of terms, less 1, by the common difference, and add the first term to the product for the *last term.*

2. If the first term be 4, the common difference 3, and the number of terms 100, what is the last term? *Ans.* 301.

3. There are, in a certain triangular field, 41 rows of corn; the first row, in 1 corner, is a single hill, the second contains 3 hills, and so on, with a common difference of 2; what is the number of hills in the last row? *Ans.* 81 hills.

4. A man puts out \$1, at 6 per cent. simple interest, which, in 1 year, amounts to \$1'06, in 2 years to \$1'12, and so on, in arithmetical progression, with a common difference of \$'06; what would be the amount in 40 years? *Ans.* \$3'40.

Hence we see, that the yearly amounts of any sum, at simple interest, form an arithmetical series, of which the *principal* is the *first term*, the *last amount* is the *last* term, the yearly *interest* is the *common difference*, and the number of *years* is 1 less than the *number* of terms.

5. A man bought 100 yards of cloth in arithmetical progression; for the first yard he gave 4 cents, and for the last 301 cents; what was the common increase of the price on each succeeding yard?

This question is the reverse of example 1; therefore, $301 - 4 = 297$, and $297 \div 99 = 3$, common difference.

Hence, when the extremes and number of terms are given, to find the common difference,—Divide the difference of the extremes by the number of terms, less 1, and the quotient will be the common difference.

6. If the extremes be 5 and 605, and the number of terms 151, what is the common difference? *Ans.* 4.

7. If a man puts out \$1, at simple interest, for 40 years, and receives, at the end of the time, \$3'40, what is the rate?

If the extremes be 1 and 3'40, and the number of terms 41, what is the common difference? *Ans.* '06.

8. A man had 8 sons, whose ages differed alike; the youngest was 10 years old, and the eldest 45; what was the common difference of their ages? *Ans.* 5 years.

9. A man bought 100 yards of cloth in arithmetical series; he gave 4 cents for the *first* yard, and 301 cents for the *last* yard; what was the average price per yard, and what was the amount of the whole?

Since the price of each succeeding yard increases by a *constant excess*, it is plain, the *average* price is as much *less* than the price of the *last* yard, as it is *greater* than the price of the *first* yard; therefore, one half the sum of the first and last price is the *average price*.

One half of 4 cts. + 301 cts. = 152½ cts. = average price; and the price, 152½ cts. × 100 = 15250 cts. = \$152'50, whole cost. } *Ans.*

Hence, when *the extremes and the number of terms are given, to find the sum of all the terms*,—Multiply ½ the sum of the extremes by the number of terms, and the product will be the answer.

10. If the extremes be 5 and 605, and the number of terms 151, what is the sum of the series? *Ans.* 46055.

11. What is the sum of the first 100 numbers, in their natural order, that is, 1, 2, 3, 4, &c.? *Ans.* 5050.

12. How many times does a common clock strike in 12 hours? *Ans.* 78.

13. A man rents a house for \$ 50, annually, to be paid at the close of each year; what will the rent amount to in 20 years, allowing 6 per cent., simple interest, for the use of the money?

The last year's rent will evidently be \$ 50 without interest, the last but *one* will be the amount of \$ 50 for 1 year, the last but *two* the amount of \$ 50 for 2 years, and so on, in arithmetical series, to the first, which will be the amount of \$ 50 for 19 years = \$ 107.

If the first term be 50, the last term 107, and the number of terms 20, what is the sum of the series? *Ans.* \$ 1570.

14. What is the amount of an annual pension of \$ 100, being in arrears, that is, remaining unpaid, for 40 years, allowing 5 per cent. *simple interest*? *Ans.* \$ 7900.

15. There are, in a certain triangular field, 41 rows of corn; the first row, being in 1 corner, is a single hill, and the last row, on the side opposite, contains 81 hills; how many hills of corn in the field? *Ans.* 1681 hills.

16. If a triangular piece of land, 30 rods in length, be 20 rods wide at one end, and come to a point at the other, what number of square rods does it contain? *Ans.* 300.

17. A debt is to be discharged at 11 several payments, in arithmetical series, the first to be $5, and the last $75; what is the whole debt? —— the common difference between the several payments?

Ans. whole debt, $440; common difference, $7.

18. What is the sum of the series 1, 3, 5, 7, 9, &c., to 1001? *Ans.* 251001.

Note. By the reverse of the rule under ex. 5, the *difference of the extremes* 1000, divided by the *common difference* 2, gives a quotient, which, increased by 1, is the *number of terms* = 501.

19. What is the sum of the arithmetical series 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c., to the 50th term inclusive? *Ans.* $712\frac{1}{2}$.

20. What is the sum of the decreasing series 30, $29\frac{2}{3}$, $29\frac{1}{3}$, 29, $28\frac{2}{3}$, &c., down to 0?

Note. $30 \div \frac{1}{3} + 1 = 91$, number of terms. *Ans.* 1365.

QUESTIONS.

1. What is an arithmetical progression? 2. When is the series called *ascending*? 3. —— when *descending*? 4. What are the numbers, forming the progression, called? 5. What are the first and last terms called? 6. What are the other terms called? 7. When the *first term*, common difference, and number of terms, are given, how do you find the *last term*? 8. How may arithmetical progression be applied to simple interest? 9. When the extremes and number of terms are given, how do you find the common difference? 10. —— how do you find the sum of all the terms?

GEOMETRICAL PROGRESSION.

¶ **113.** Any series of numbers, continually increasing by a constant multiplier, or decreasing by a constant divisor, is called a *Geometrical Progression.* Thus, 1, 2, 4, 8, 16, &c. is an increasing geometrical series, and 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, &c. is a decreasing geometrical series.

As in arithmetical, so also in geometrical progression, there are five things, any *three* of which being given, the other *two* may be found :—

1st. The *first* term.
2d. The *last* term.
3d. The *number* of terms.
4th. The *ratio.*
5th. The *sum* of all the terms.

The *ratio* is the *multiplier* or *divisor*, by which the series is formed.

1. A man bought a piece of silk, measuring 17 yards, and, by agreement, was to give what the last yard would come to, reckoning 3 cents for the first yard, 6 cents for the second, and so on, doubling the price to the last; what did the piece of silk cost him?

$3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 196608$ cents, $=$ \$1966'08, *Answer.*

In examining the process by which the last term (196608) has been obtained, we see, that it is a product, of which the *ratio* (2) is sixteen times a factor, that is, *one time less* than the number of terms. The last term, then, is the sixteenth power of the *ratio*, (2,) multiplied by the *first term* (3.)

Now, to raise 2 to the 16th power, we need not produce all the *intermediate powers;* for $2^4 = 2 \times 2 \times 2 \times 2 = 16$, is a product of which the *ratio* 2 is 4 times a factor; now, if 16 be multiplied by 16, the product, 256, evidently contains the same factor (2) 4 times $+$ 4 times, $=$ 8 times; and $256 \times 256 = 65536$, a product of which the ratio (2) is 8 times $+$ 8 times, $=$ 16 times, factor; it is, therefore, the 16th power of 2, and, multiplied by 3, the first term, gives 196608, the last term, as before. Hence,

When the first term, ratio, and number of terms, are given, to find the last term,—

I. Write down a few leading powers of the *ratio* with their *indices* over them.

II. Add together the most *convenient indices*, to make an index *less* by *one* than the number of the term sought.

III. Multiply together the *powers* belonging to those *indices*, and their product, multiplied by the *first term*, will be the term sought.

2. If the first term be 5, and the ratio 3, what is the 8th term?

Powers of the ratio, with their indices over them. $\begin{cases} 1 \quad 2 \quad 3 \quad + \quad 4 = \quad 7 \\ 3,\ 9,\ 27, \times 81 = 2187 \times 5 \text{ first} \\ \quad \text{term}, = 10935, \textit{Answer}. \end{cases}$

3. A man plants 4 kernels of corn, which, at harvest, produce 32 kernels; these he plants the second year; now, supposing the annual increase to continue 8 fold, what would be the produce of the 16th year, allowing 1000 kernels to a pint? *Ans.* 2199023255'552 bushels.

4. Suppose a man had put out one cent at compound interest in 1620, what would have been the amount in 1824, allowing it to double once in 12 years?

$2^{17} = 131072$. *Ans.* $1310'72.

5. A man bought 4 yards of cloth, giving 2 cents for the first yard, 6 cents for the second, and so on, in 3 fold ratio; what did the whole cost him?

$2 + 6 + 18 + 54 = 80$ cents. *Ans.* 80 cents.

In a long series, the process of adding in this manner would be tedious. Let us try, therefore, to devise some shorter method of coming to the same result. If all the terms, *excepting the last*, viz. $2 + 6 + 18$, be multiplied by the ratio, 3, the product will be the series $6 + 18 + 54$ subtracting the *former* series from the *latter*, we have, for the remainder, $54 - 2$, that is, the *last term*, less the *first term*, which is evidently as many times the first series $(2 + 6 + 18)$ as is expressed by the *ratio*, *less* 1: hence, if we *divide* the difference of the extremes $(54 - 2)$ by the ratio, *less* 1, $(3 - 1,)$ the quotient will be the sum of all the terms, *excepting the last*, and, adding the last term, we shall have the *whole amount*. Thus, $54 - 2 = 52$, and $3 - 1 = 2$; then, $52 \div 2 = 26$, and 54 added, makes 80, *Answer*, as before.

Hence, *when the extremes and ratio are given, to find the sum of the series*,—Divide the *difference* of the *extremes* by the *ratio*, *less* 1, and the quotient, *increased* by the *greater term*, will be the *answer*.

6. If the extremes be 4 and 131072, and the ratio 8, what is the whole amount of the series?

$$\frac{131072 - 4}{8 - 1} + 131072 = 149796 \text{ Answer.}$$

7. What is the sum of the descending series 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c., *extended to infinity*?

It is evident the last term must become 0, or indefinitely near to nothing; therefore, the extremes are 3 and 0, and the ratio 3. *Ans.* $4\frac{1}{2}$.

8. What is the value of the infinite series $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$, &c.? *Ans.* $1\frac{1}{3}$.

9. What is the value of the infinite series, $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000}$, &c., or, what is the same, the decimal '11111, &c., continually repeated? *Ans.* $\frac{1}{9}$.

10. What is the value of the infinite series, $\frac{2}{100} + \frac{2}{10000}$, &c., descending by the ratio 100, or, which is the same, the repeating decimal '020202, &c.? *Ans.* $\frac{2}{99}$.

11. A gentleman, whose daughter was married on a new year's day, gave her a dollar, promising to triple it on the first day of each month in the year; to how much did her portion amount?

Here, before finding the amount of the series, we must find the *last term*, as directed in the rule after ex. 1.

Ans. \$265'720.

The *two processes* of finding the *last term*, and the *amount*, may, however, be conveniently reduced to *one*, thus:—

When the first term, the ratio, and the number of terms, are given, to find the sum or amount of the series,—Raise the *ratio* to a power whose index is equal to the *number of terms*, from which subtract 1; divide the *remainder* by the *ratio, less* 1, and the quotient, multiplied by the *first term*, will be the *answer*.

Applying this rule to the last example, $3^{12} = 531441$, and $\frac{531441 - 1}{3 - 1} \times 1 = 265720$. *Ans.* \$265'720, as before.

12. A man agrees to serve a farmer 40 years without any other reward than 1 kernel of corn for the first year, 10 for the second year, and so on, in 10 fold ratio, till the end of the time; what will be the amount of his wages, allowing 1000 kernels to a pint, and supposing he sells his corn at 50 cents per bushel?

$$\frac{10^{40} - 1}{10 - 1} \times 1 = \left\{ \begin{array}{l} 1,111,111,111,111,111,111,111,111, \\ \quad 111,111,111,111,111 \text{ kernels.} \end{array} \right.$$

Ans. \$8,680,555,555,555,555,555,555,555,555,555,555 '555$\frac{555}{1000}$.

13. A gentleman, dying, left his estate to his 5 sons, to the youngest \$1000, to the second \$1500, and ordered, that each son should exceed the younger by the ratio of $1\frac{1}{2}$; what was the amount of the estate?

Note. Before finding the power of the ratio $1\frac{1}{2}$, it may be reduced to an improper fraction $= \frac{3}{2}$, or to a decimal, 1'5.

$\frac{\frac{3}{2}^5 - 1}{\frac{3}{2} - 1} \times 1000 =$ \$$13187\frac{1}{2}$; or, $\frac{1'5^5 - 1}{1'5 - 1} \times 1000 =$ \$13187'50, *Answer.*

Compound Interest by Progression.

¶ **114.** 1. What is the amount of \$4, for 5 years, at 6 per cent. compound interest?

We have seen, (¶ 92,) that *compound interest* is that, which arises from adding the interest to the principal at the close of each year, and, for the next year, casting the interest on that *amount*, and so on. The amount of \$1 for 1 year is \$1'06; if the *principal*, therefore, be multiplied by 1'06, the product will be its *amount for* 1 *year*; this *amount*, multiplied by 1'06, will give the amount (compound interest) for 2 years; and this *second amount*, multiplied by 1'06, will give the amount for 3 years; and so on. Hence, the several *amounts*, arising from any sum at compound interest, form a *geometrical series*, of which the *principal* is the *first term*; the *amount of* \$1 *or* 1 £., &c., at the *given rate per cent.*, is the *ratio*; the *time, in years*, is 1 less than the *number of terms*; and the *last amount* is the *last term*.

The last question may be resolved into this:—If the first term be 4, the number of terms 6, and the ratio 1'06, what is the last term?

$1'06^5 = 1'338$, and $1'338 \times 4 =$ \$5'352+. *Ans.* \$5'352.

Note 1. The powers of the amounts of \$1, at 5 and at 6 per cent., may be taken from the table, under ¶ 91. Thus, opposite 5 years, under 6 per cent., you find 1'338, &c.

Note 2. The several processes may be conveniently exhibited by the use of letters; thus:—

Let P. represent the Principal.
..... R. .... the Ratio, or the amount of \$1, &c. for 1 year.
..... T. .... the Time, in *years*.
..... A. .... the Amount.

When two or more letters are *joined together*, like a word,

they are to be *multiplied together*. Thus PR. implies, that the *principal* is to be multiplied by the *ratio*. When one letter is *placed above another*, like the index of a power, the *first* is to be *raised to a power*, whose index is *denoted* by the *second*. Thus $R^{T.}$ implies, that the ratio is to be raised to a power, whose index shall be equal to the *time*, that is, the number of *years*.

2. What is the amount of 40 dollars for 11 years, at 5 per cent. compound interest?

$R^{T.} \times P. = A.$; therefore, $1‘05^{11} \times 40 = 68‘4$.

Ans. $68‘40.

3. What is the amount of $6 for 4 years, at 10 per cent. compound interest? *Ans.* $8‘78$\frac{6}{10}$.

4. If the amount of a certain sum for 5 years, at 6 per cent. compound interest, be $5‘352, what is that sum, or principal?

If the number of terms be 6, the ratio 1‘06, and the last term 5‘352, what is the first term?

This question is the reverse of the last; therefore,

$\frac{A.}{R^{T.}} = P.$; or, $\frac{5‘352}{1‘338} = 4$. *Ans.* $4.

5. What principal, at 10 per cent. compound interest, will amount, in 4 years, to $8‘7846? *Ans.* $6.

6. What is the present worth of $68‘40, due 11 years hence, discounting at the rate of 5 per cent. compound interest? *Ans.* $40.

7. At what rate per cent. will $6 amount to $8‘7846 in 4 years?

If the first term be 6, the last term 8‘7846, and the number of terms 5, what is the ratio?

$\frac{A.}{P.} = R^{T.}$, that is, $\frac{8‘7846}{6} = 1‘4641 =$ the 4th power of the ratio; and then, by extracting the 4th *root*, we obtain 1‘10 for the ratio. *Ans.* 10 per cent.

8. In what time will $6 amount to $8‘7846, at 10 per cent. compound interest?

$\frac{A.}{P.} = R^{T.}$, that is, $\frac{8‘7846}{6} = 1‘4641 = 1‘10^{4}$; therefore, if we divide 1‘4641 by 1‘10, and then divide the quotient thence arising by 1‘10, and so on, till we obtain a quotient that will not contain 1‘10, the *number* of these divisions will be the *number of years*. *Ans.* 4 years.

9. At 5 per cent. compound interest, in what time will \$40 amount to \$68'40?

Having found the power of the ratio 1'05, as before, which is 1'71, you may look for this number in the *table*, under the given rate, 5 per cent., and against it you will find the number of years. *Ans.* 11 years.

10. At 6 per cent. compound interest, in what time will \$4 amount to \$5'352? *Ans.* 5 years.

Annuities at Compound Interest.

¶ **115.** It may not be amiss, in this place, briefly to show the application of compound interest, in computing the amount and present worth of *annuities.*

An ANNUITY is a sum payable at *regular periods*, of one year each, either for a *certain number of years*, or *during the life* of the pensioner, or *forever.*

When annuities, rents, &c. are not paid at the time they *become due*, they are said to be in *arrears.*

The sum of all the annuities, rents, &c. remaining unpaid, together with the *interest* on each, for the time they have remained due, is called the *amount.*

1. What is the amount of an annual pension of \$100, which has remained unpaid 4 years, allowing 6 per cent. compound interest?

The *last* year's pension will be \$100, without interest; the last but *one* will be the *amount* of \$100 for 1 year; the last but *two* the amount (compound interest) of \$100 for 2 years, and so on; and the sum of these several amounts will be the answer. We have then a *series of amounts*, that is, a *geometrical series*, (¶ 114,) to find the sum of all the terms.

If the first term be 100, the number of terms 4, and the ratio 1'06, what is the sum of all the terms?

Consult the rule, under ¶ 113, ex. 11.

$$\frac{1'06^4 - 1}{'06} \times 100 = 437'45. \qquad \textit{Ans. } \$437'45.$$

Hence, *when the annuity, the time, and rate per cent. are given, to find the amount,*—RAISE the *ratio* (the amount of

$ 1, &c. for 1 year) to a power denoted by the number of years; from this power subtract 1; then divide the remainder by the ratio, *less* 1, and the quotient, multiplied by the annuity, will be the amount.

Note. The powers of the amounts, at 5 and 6 per cent. up to the 24th, may be taken from the *table*, under ¶ 91.

2. What is the amount of an annuity of $ 50, it being *in arrears* 20 years, allowing 5 per cent. compound interest?

Ans. $ 1653'29.

3. If the annual rent of a house, which is $ 150, be in arrears 4 years, what is the amount, allowing 10 per cent. compound interest? *Ans.* $ 696'15.

4. To how much would a salary of $ 500 per annum amount in 14 years, the money being improved at 6 per cent. compound interest? —— in 10 years? —— in 20 years? —— in 22 years? —— in 24 years?

Ans. to the last, $ 25407'75.

¶ 116. If the annuity is paid in advance, or if it be bought at the beginning of the first year, the sum which ought to be given for it is called the *present worth.*

5. What is the present worth of an annual pension of $ 100, to continue 4 years, allowing 6 per cent. compound interest?

The present worth is, evidently, a sum which, at 6 per cent. compound interest, would, in 4 years, produce an *amount* equal to the *amount of the annuity* in arrears the same time.

By the *last rule*, we find the *amount* = $ 437'45, and by the directions under ¶ 114, ex. 4, we find the *present* worth = $ 346'51. *Ans.* $ 346'51.

Hence, *to find the present worth of any annuity,*—First find its *amount* in arrears for the whole time; this *amount*, divided by that *power of the ratio* denoted by the number of years, will give the *present worth.*

6. What is the present worth of an annual salary of $ 100 to continue 20 years, allowing 5 per cent.? *Ans.* $ 1246'22.

The operations under this rule being somewhat tedious, we subjoin a

TABLE,

Showing the present worth of $1, or 1 £. annuity, at 5 and 6 per cent. compound interest, for any number of years from 1 to 34.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0'95238	0'94339	18	11'68958	10'8276
2	1'85941	1'83339	19	12'08532	11'15811
3	2'72325	2'67301	20	12'46221	11'46992
4	3'54595	3'4651	21	12'82115	11'76407
5	4'32948	4'21236	22	13'163	12'04158
6	5'07569	4'91732	23	13'48807	12'30338
7	5'78637	5'58238	24	13'79864	12'55035
8	6'46321	6'20979	25	14'09394	12'78335
9	7'10782	6'80169	26	14'37518	13'00316
10	7'72173	7'36008	27	14'64303	13'21053
11	8'30641	7'88687	28	14'89813	13'40616
12	8'86325	8'38384	29	15'14107	13'59072
13	9'39357	8'85268	30	15'37245	13'76483
14	9'89864	9'29498	31	15'59281	13'92908
15	10'37966	9'71225	32	15'80268	14'08398
16	10'83777	10'10589	33	16'00255	14'22917
17	11'27407	10'47726	34	16'1929	14'36613

It is evident, that the present worth of $2 annuity is 2 times as much as that of $1; the present worth of $3 will be 3 times as much, &c. Hence, *to find the present worth of any annuity, at 5 or 6 per cent.*,—Find, in this table, the present worth of $1 annuity, and multiply it by the *given annuity*, and the product will be the *present worth.*

7. What ready money will purchase an annuity of $150, to continue 30 years, at 5 per cent. compound interest?

The present worth of $1 annuity, by the table, for 30 years, is $15'37245; therefore, 15'37245 × 150 = $2305'867, *Ans.*

8. What is the present worth of a yearly pension of $40, to continue 10 years, at 6 per cent. compound interest? —— at 5 per cent.? —— to continue 15 years? —— 20 years? —— 25 years? —— 34 years?

Ans. to last, $647'716.

U*

When annuities *do not commence* till a certain period of time has elapsed, or till some particular event has taken place, they are said to be *in reversion.*

9. What is the present worth of $100 annuity, to be continued 4 years, but not to commence till 2 years hence, allowing 6 per cent. compound interest?

The present worth is evidently a sum which, at 6 per cent. compound interest, would in 2 years produce an *amount* equal to the *present worth* of the annuity, were it to *commence immediately.* By the last rule, we find the present worth of the annuity, to *commence immediately,* to be $346'51, and, by directions under ¶ 114, ex. 4, we find the present worth of $346'51 for 2 years, to be $308'393. *Ans.* $308'393.

Hence, *to find the present worth of any annuity taken in reversion, at compound interest,*—First, find the present worth, to *commence immediately,* and *this sum,* divided by the power of the ratio, denoted by the *time in reversion,* will give the answer.

10. What ready money will purchase the reversion of a ease of $60 *per annum,* to continue 6 years, but not to commence till the end of 3 years, allowing 6 per cent. compound interest to the purchaser?

The present worth, to commence immediately, we find to be, $295'039, and $\frac{295'039}{1'06^3} = 247'72$. *Ans.* $247'72.

It is plain, the same result will be obtained by finding the present worth of the annuity, to commence immediately, and to *continue to the end of the time,* that is, $3 + 6 = 9$ years, and then *subtracting* from *this sum* the present worth of the annuity, continuing for the *time of reversion,* 3 years. Or, we may find the present worth of $1 for the *two times by the table,* and multiply their difference by the given annuity. Thus, by the table,

The whole time,	9 years, =	6'80169
The time in reversion,	3 years, =	2'67301
	Difference, =	4'12868
		60
		$247'72080 *Ans.*

11. What is the present worth of a lease of $100 to continue 20 years, but not to commence till the end of 4 years,

allowing 5 per cent.? —— what, if it be 6 years in reversion? —— 8 years? —— 10 years? —— 14 years?

Ans. to last, \$629'426.

¶ 117. 12. What is the worth of a freehold estate, of which the yearly rent is \$60, allowing to the purchaser 6 per cent.?

In this case, the annuity continues *forever*, and the estate is evidently worth a sum, of which the yearly *interest* is equal to the yearly *rent* of the estate. The principal *multiplied* by the rate gives the interest; therefore, the interest *divided* by the rate will give the principal; $60 \div {}'06 = 1000$.

Ans. \$1000.

Hence, *to find the present worth of an annuity, continuing forever*,—Divide the annuity by the *rate per cent.*, and the quotient will be the present worth.

Note. The worth will be the same, whether we reckon *simple* or *compound* interest; for, since a *year's interest of the price* is the *annuity*, the profits arising from *that price* can neither be *more nor less* than the profits arising from the *annuity*, whether they be employed at simple or compound interest.

13. What is the worth of \$100 annuity, to continue forever, allowing to the purchaser 4 per cent.? —— allowing 5 per cent.? —— 8 per cent.? —— 10 per cent.? —— 15 per cent.? —— 20 per cent.? *Ans. to last*, \$500.

14. Suppose a freehold estate of \$60 *per annum*, to commence 2 years hence, be put on sale; what is its value, allowing the purchaser 6 per cent.?

Its present worth is a sum which, at 6 per cent. compound interest, would, in 2 years, produce an amount equal to the worth of the estate *if entered on immediately.*

$\frac{60}{'06} = \$1000 =$ the worth, if entered on immediately,

and $\frac{\$1000}{1'06^2} = \$889'996$, the present worth.

The same result may be obtained by subtracting from the *worth* of the estate, *to commence immediately*, the *present worth* of the *annuity* 60, *for* 2 *years*, *the time of* REVERSION. Thus, by the *table*, the present worth of \$1 for 2 years is $1'83339 \times 60 = 110'0034 =$ present worth of \$60 for 2 years, and $\$1000 - \$110'0034 = \$889'9966$, *Ans.* as before.

15. What is the present worth of a perpetual annuity of \$ 100, to commence 6 years hence, allowing the purchaser 5 per cent. compound interest? —— what, if 8 years in reversion? —— 10 years? —— 4 years? —— 15 years? —— 30 years? *Ans. to last*, \$ 462'755.

The foregoing examples, in compound interest, have been confined to *yearly* payments; if the payments are *half* yearly, we may take *half* the *principal or annuity*, *half* the *rate per cent.*, and *twice* the *number of years*, and work as before, and so for *any* other part of a *year*.

QUESTIONS.

1. What is a geometrical progression or series? 2. What is the ratio? 3. When the first term, the ratio, and the number of terms, are given, how do you find the *last term*? 4. When the extremes and ratio are given, how do you find the *sum of all the terms*? 5. When the first term, the ratio, and the number of terms, are given, how do you find the amount of the series? 6. When the *ratio is a fraction*, how do you proceed? 7. What is *compound interest*? 8. How does it appear that the *amounts*, arising by compound interest, form a *geometrical series*? 9. What is the *ratio*, in compound interest? —— the number of *terms*? —— the *first term*? —— the *last term*? 10. When the rate, the time, and the principal, are given, how do you find the *amount*? 11. When A. R. and T. are given, how do you find P.? 12. When A. P. and T. are given, how do you find R.? 13. When A. P. and R. are given, how do you find T.? 14. What is an *annuity*? 15. When are annuities said to be *in arrears*? 16. What is the *amount*? 17. In a geometrical series, to what is the *amount* of an annuity equivalent? 18. How do you find the *amount* of an annuity, at compound interest? 19. What is the *present worth* of an annuity? —— how computed at compound interest? —— how found by the table? 20. What is understood by the term *reversion*? 21. How do you find the present worth of an annuity, taken *in reversion*? —— by the table? 22. How do you find the present worth of a *freehold estate*, or a *perpetual annuity*? —— the same taken *in reversion*? —— by the table?

PERMUTATION.

¶ **118.** Permutation is the method of finding how many different ways the order of any number of things may be varied or changed.

1. Four gentlemen agreed to dine together so long as they could sit, every day, in a different order or position; how many days did they dine together?

Had there been but *two* of them, *a* and *b*, they could sit only in 2 times 1 ($1 \times 2 = 2$) different positions, thus, *a b*, and *b a*. Had there been *three*, *a*, *b*, and *c*, they could sit in $1 \times 2 \times 3 = 6$ different positions; for, beginning the order with *a*, there will be 2 positions, viz. *a b c*, and *a c b*; next, beginning with *b*, there will be 2 positions, *b a c*, and *b c a*; lastly, beginning with *c*, we have *c a b*, and *c b a*, that is, in all, $1 \times 2 \times 3 = 6$ different positions. In the same manner, if there be *four*, the different positions will be $1 \times 2 \times 3 \times 4 = 24$. *Ans.* 24 days.

Hence, *to find the number of different changes or permutations, of which any number of different things are capable*,—Multiply continually together all the terms of the natural series of numbers, from 1 up to the given number, and the last product will be the answer.

2. How many variations may there be in the position of the nine digits? *Ans.* 362880.

3. A man bought 25 cows, agreeing to pay for them 1 cent for every different order in which they could all be placed; how much did the cows cost him?

Ans. $ 15511210043330985984000.

4. Christ Church, in Boston, has 8 bells; how many changes may be rung upon them? *Ans.* 40320.

MISCELLANEOUS EXAMPLES.

¶ **119.** 1. $\overline{4+6} \times \overline{7-1} = 60$.

A line, or *vinculum*, drawn over several numbers, signifies, that the numbers under it are to be taken jointly, or as one whole number.

2. $\overline{9-8+4}\times\overline{8+4-6}=$ how many? *Ans.* 30.

3. $\overline{7+4-\overline{2+3}+40}\times 5=$ how many? *Ans.* 230.

4. $\dfrac{\overline{3+6-2}\times\overline{4-2}}{2\times 2}=$ how many? *Ans.* $3\frac{1}{2}$.

5. There are two numbers; the greater is 25 times 78, and their difference is 9 times 15; their sum and product are required.

Ans. 3765 is their sum; 3539250 their product.

6. What is the difference between thrice five and thirty, and thrice thirty-five? $35\times 3-\overline{5\times 3+30}=60$, *Ans.*

7. What is the difference between six dozen dozen, and half a dozen dozen? *Ans.* 792.

8. What number divided by 7 will make 6488?

9. What number multiplied by 6 will make 2058?

10. A gentleman went to sea at 17 years of age; 8 years after he had a son born, who died at the age of 35; after whom the father lived twice 20 years; how old was the father at his death? *Ans.* 100 years.

11. What number is that, which being multiplied by 15 the product will be $\frac{3}{4}$? $\frac{3}{4}\div 15=\frac{1}{20}$, *Ans.*

12. What decimal is that, which being multiplied by 15, the product will be ‘75? ‘75 ÷ 15 = ‘05, *Ans.*

13. What is the decimal equivalent to $\frac{1}{35}$?

Ans. ‘0285714.

14. What fraction is that, to which if you add $\frac{2}{5}$, the sum will be $\frac{5}{6}$? *Ans.* $\frac{13}{30}$.

15. What number is that, from which if you take $\frac{3}{5}$, the remainder will be $\frac{1}{8}$? *Ans.* $\frac{29}{40}$.

16. What number is that, which being divided by $\frac{3}{4}$, the quotient will be 21? *Ans.* $15\frac{3}{4}$.

17. What number is that, which multiplied by $\frac{2}{3}$ produces $\frac{1}{4}$? *Ans.* $\frac{3}{8}$.

18. What number is that, from which if you take $\frac{2}{5}$ of itself, the remainder will be 12? *Ans.* 20.

19. What number is that, to which if you add $\frac{2}{5}$ of $\frac{5}{3}$ of itself, the whole will be 20? *Ans.* 12.

20. What number is that, of which 9 is the $\frac{2}{3}$ part?

Ans. $13\frac{1}{2}$.

21. A farmer carried a load of produce to market: he sold 780 lbs. of pork, at 6 cents per lb.; 250 lbs. of cheese, at 8 cents per lb.; 154 lbs. of butter, at 15 cents per lb.:

in pay he received 60 lbs. of sugar, at 10 cents per lb.; 15 gallons of molasses, at 42 cents per gallon; ½ barrel of mackerel, at $3'75; 4 bushels of salt, at $1'25 per bushel; and the balance in money: how much money did he receive?

Ans. $68'85.

22. A farmer carried his grain to market, and sold

75 bushels of wheat, at $1'45 per bushel.
64 .... rye, ... $ '95 ....
142 .... corn, ... $ '50 ....

In exchange he received sundry articles:—

8 pieces of cloth, each
containing 31 yds., at $1'75 per yd.
2 quintals of fish, ... $2'30 per quin.
8 hhds. of salt, ... $4'30 per hhd.

and the balance in money.

How much money did he receive? *Ans.* $38'80.

23. A man exchanges 760 gallons of molasses, at 37½ cents per gallon, for 66½ cwt. of cheese, at $4 per cwt.; how much will be the balance in his favour? *Ans.* $19.

24. Bought 84 yards of cloth, at $1'25 per yard; how much did it come to? How many bushels of wheat, at $1'50 per bushel, will it take to pay for it?

Ans. to the last, 70 bushels.

25. A man sold 342 pounds of beef, at 6 cents per pound, and received his pay in molasses, at 37½ cents per gallon; how many gallons did he receive? *Ans.* 54'72 gallons.

26. A man exchanged 70 bushels of rye, at $'92 per bushel, for 40 bushels of wheat, at $1'37½ per bushel, and received the balance in oats, at $'40 per bushel; how many bushels of oats did he receive? *Ans.* 23½.

27. How many bushels of potatoes, at 1 s. 6 d. per bushel, must be given for 32 bushels of barley, at 2 s. 6 d. per bushel? *Ans.* 53⅓ bushels.

28. How much salt, at $1'50 per bushel, must be given in exchange for 15 bushels of oats, at 2 s. 3 d. per bushel?

Note. It will be recollected that, when the price and cost are given, to find the quantity, they must both be reduced to the same denomination before dividing. *Ans.* 3¾ bushels.

29. How much wine, at $2'75 per gallon, must be given in exchange for 40 yards of cloth, at 7 s. 6 d. per yard?

Ans. 18$\frac{2}{11}$ gallons.

30. A had 41 cwt. of hops, at 30 s. per cwt., for which B gave him 20 £. in money, and the rest in prunes, at 5 d. per lb.; how many prunes did A receive?

Ans. 17 cwt. 3 qrs. 4 lbs.

31. A has linen cloth worth $ '30 per yard; but, in bartering, he will have $ '35 per yard; B has broadcloth worth $ 3'75 ready money; at what price ought the broadcloth to be rated in bartering with A?

'30 : '35 : : 3'75 : $ 4'375, *Ans.* Or, $\frac{'35}{'30}$ of 3'75 = $ 4'37½, *Ans.* The two operations will be seen to be exactly alike.

32. If cloth, worth 2 s. per yard, cash, be rated in barter at 2 s. 6 d., how should wheat, worth 8 s. cash, be rated in exchanging for the cloth? *Ans.* 10 s., or $ 1'666⅔.

33. If 4 bushels of corn cost $ 2, what is it per bushel?

Ans. $ '50.

34. If 9 bushels of wheat cost $ 13'50, what is that per bushel? *Ans.* $ 1'50.

35. If 40 sheep cost $ 100, what is that per head?

Ans. $ 2'50.

36. If 3 bushels of oats cost 7 s. 6 d., how much are they per bushel? *Ans.* 2 s. 6 d., = $ '41⅔.

37. If 22 yards of broadcloth cost 21 £. 9 s., what is the price per yard? *Ans.* 19 s. 6 d., = $ 3'25.

38. At $ '50 per bushel, how much corn can be bought for $ 2'00? *Ans.* 4 bushels.

39. A man, having $ 100, would lay it out in sheep, at $ 2'50 apiece; how many can he buy? *Ans.* 40.

40. If 20 cows cost $ 300, what is the price of 1 cow? —— of 2 cows? —— of 5 cows? —— of 15 cows?

Ans. to the last, $ 225.

41. If 7 men consume 24 lbs. of meat in one week, how much would 1 man consume in the same time? —— 2 men? —— 5 men? —— 10 men? *Ans. to the last,* 34⅖ lbs.

Note. Let the pupil also perform these questions by the rule of proportion.

42. If I pay $ 6 for the use of $ 100, how much must I pay for the use of $ 75? *Ans.* $ 4'50.

43. What premium must I pay for the insurance of my house against loss by fire, at the rate of ½ per cent., that is, ½ dollar on a hundred dollars, if my house be valued at $ 2475? *Ans.* $ 12'375.

44. What will be the insurance, per annum, of a store and contents, valued at $ 9876'40, at 1½ per centum?

Ans. $ 148'146.

45. What commission must I receive for selling $ 478 worth of books, at 8 per cent.? *Ans.* $ 38'24.

46. A merchant bought a quantity of goods for $ 734, and sold them so as to gain 21 per cent.; how much did he gain? and for how much did he sell his goods?

Ans. to the last, $ 888'14.

47. A merchant bought a quantity of goods at Boston, for $ 500, and paid $ 43 for their transportation; he sold them so as to gain 24 per cent. on the whole cost; for how much did he sell them? *Ans.* $ 673'32.

48. Bought a quantity of books for $ 64, but for cash a discount of 12 per cent. was made; what did the books cost? *Ans.* $ 56'32.

49. Bought a book, the price of which was marked $ 4'50, but for cash the bookseller will sell it at 33⅓ per cent. discount; what is the cash price? *Ans.* $ 3'00.

50. A merchant bought a cask of molasses, containing 120 gallons, for $ 42; for how much must he sell it to gain 15 per cent.? how much per gallon? *Ans. to last,* $ '40¼.

51. A merchant bought a cask of sugar, containing 740 pounds, for $ 59'20; how must he sell it per pound, to gain 25 per cent.? *Ans.* $ '10.

52. What is the interest, at 6 per cent., of $ 71'02 for 17 months 12 days? *Ans.* $ 6'178 +.

53. What is the interest of $ 487'003 for 18 months?

Ans. $ 43'83 +.

54. What is the interest of $ 8'50 for 7 months?

Ans. $ '297½.

55. What is the interest of $ 1000 for 5 days?

Ans. $ '833⅓.

56. What is the interest of $ '50 for 10 years?

Ans. $ '30.

57. What is the interest of $ 84'25 for 15 months and 7 days, at 7 per cent.? *Ans.* $ 7'486 +.

58. What is the interest of $ 154'01 for 2 years, 4 months and 3 days, at 5 per cent.? *Ans.* $ 18'082.

59. What sum, put to interest at 6 per cent., will, in 2 years and 6 months, amount to $ 150?

Note. See ¶ 85. *Ans.* $ 130'434 +.

60. I owe a man $ 475'50, to be paid in 16 months with-

X

out interest; what is the present worth of that debt, the use of the money being worth 6 per cent.? *Ans.* $440'277 +.

61. What is the present worth of $1000 payable in 4 years and 2 months, discounting at the rate of 6 per cent.? *Ans.* $800.

62. A merchant bought articles to the amount of $500, and sold them for $575; how much did he gain?

What per cent. was his gain? that is, How many dollars did he gain on each $100 which he laid out? If $500 gain $75, what does $100 gain? *Ans.* 15 per cent.

63. A merchant bought cloth at $3'50 per yard, and sold it at $4'25 per yard; how much did he gain per centum? *Ans.* $21\frac{3}{7}$ per cent.

64. A man bought a cask of wine, containing 126 gallons, for $283'50, and sold it out at the rate of $2'75 per gallon; how much was his *whole* gain? how much per gallon? how much per cent.?

Ans. His whole gain, $63'00; per gallon, $'50; which is $22\frac{2}{9}$ per centum.

65. If $100 gain $6 in 12 months, in what time will it gain $4? —— $10? —— $14?

Ans. to the last, 28 months.

66. In what time will $54'50, at 6 per cent., gain $2'18? *Ans.* 8 months.

67. 20 men built a certain bridge in 60 days, but, it being carried away in a freshet, it is required how many men can rebuild it in 50 days.

days.		days.		men.		
50	:	60	: :	20	:	24 men, *Ans.*

68. If a field will feed 7 horses 8 weeks, how long will it feed 28 horses? *Ans.* 2 weeks.

69. If a field, 20 rods in length, must be 8 rods in width to contain an acre, how much in width must be a field, 16 rods in length, to contain the same? *Ans.* 10 rods.

70. If I purchase for a cloak 12 yards of plaid $\frac{5}{8}$ of a yard wide, how much bocking $1\frac{1}{2}$ yards wide must I have to line it? *Ans.* 5 yards.

71. If a man earn $75 in 5 months, how long must he work to earn $460? *Ans.* $30\frac{2}{3}$ months.

72. A owes B $540, but, A not being worth so much money, B agrees to take $'75 on a dollar; what sum must B receive for the debt? *Ans.* $405.

73. A cistern, whose capacity is 400 gallons, is supplied

by a pipe which lets in 7 gallons in 5 minutes; but there is a leak in the bottom of the cistern which lets out 2 gallons in 6 minutes; supposing the cistern empty, in what time would it be filled?

In 1 minute $\frac{7}{5}$ of a gallon is admitted, but in the same time $\frac{2}{6}$ of a gallon leaks out. *Ans.* 6 hours, 15 minutes.

74. A ship has a leak which will fill it so as to make it sink in 10 hours; it has also a pump which will clear it in 15 hours: now, if they begin to pump when it begins to leak, in what time will it sink?

In 1 hour the ship would be $\frac{1}{10}$ filled by the leak, but in the same time it would be $\frac{1}{15}$ emptied by the pump.

Ans. 30 hours.

75. A cistern is supplied by a pipe which will fill it in 40 minutes; how many pipes, of the same bigness, will fill it in 5 minutes? *Ans.* 8.

76. Suppose I lend a friend \$500 for 4 months, he promising to do me a like favour; some time afterward, I have need of \$300; how long may I keep it to balance the former favour? *Ans.* $6\frac{2}{3}$ months.

77. Suppose 800 soldiers were in a garrison with provisions sufficient for 2 months; how many soldiers must depart, that the provisions may serve them 5 months?

Ans. 480.

78. If my horse and saddle are worth \$84, and my horse be worth 6 times as much as my saddle, pray what is the value of my horse? *Ans.* \$72.

79. Bought 45 barrels of beef, at \$3'50 per barrel, among which are 16 barrels, whereof 4 are worth no more than 3 of the others; how much must I pay? *Ans.* \$143'50.

80. Bought 126 gallons of rum for \$110; how much water must be added to reduce the first cost to \$'75 per gallon?

Note. If \$'75 buy 1 gallon, how many gallons will \$110 buy? *Ans.* $20\frac{2}{3}$ gallons.

81. A thief, having 24 miles start of the officer, holds his way at the rate of 6 miles an hour; the officer pressing on after him at the rate of 8 miles an hour, how much does he gain in 1 hour? how long before he will overtake the thief?

Ans. 12 hours.

82. A hare starts 12 rods before a hound, but is not perceived by him till she has been up $1\frac{1}{4}$ minutes; she scuds away at the rate of 36 rods a minute, and the dog, on view,

makes after, at the rate of 40 rods a minute; how long will the course hold? and what distance will the dog run?

Ans. $14\frac{1}{4}$ minutes, and he will run 570 rods.

83. The hour and minute hands of a watch are exactly together at 12 o'clock; when are they next together?

In 1 hour the minute hand passes over 12 spaces, and the hour hand over 1 space; that is, the minute hand gains upon the hour hand 11 spaces in 1 hour; and it must gain 12 spaces to coincide with it. *Ans.* 1 h. 5 m. $27\frac{3}{11}$ s.

84. There is an island 20 miles in circumference, and three men start together to travel the same way about it; A goes 2 miles per hour, B 4 miles per hour, and C 6 miles per hour; in what time will they come together again?

Ans. 10 hours.

85. There is an island 20 miles in circumference, and two men start together to travel around it; A travels 2 miles per hour, and B 6 miles per hour; how long before they will again come together?

B gains 4 miles per hour, and must gain 20 miles to overtake A; A and B will therefore be together once in every 5 hours.

86. In a river, supposing two boats start at the same time from places 300 miles apart; the one proceeding up stream is retarded by the current 2 miles per hour, while that moving down stream is accelerated the same; if both be propelled by a steam engine, which would move them 8 miles per hour in still water, how far from each starting place will the boats meet?

Ans. $112\frac{1}{2}$ miles from the lower place, and $187\frac{1}{2}$ miles from the upper place.

87. A man bought a pipe (126 gallons) of wine for \$275; he wishes to fill 10 bottles, 4 of which contain 2 quarts, and 6 of them 3 pints each, and to sell the remainder so as to make 30 per cent. on the first cost; at what rate per gallon must he sell it? *Ans.* \$2'936 +.

88. Thomas sold 150 pine apples at \$'$33\frac{1}{3}$ apiece, and received as much money as Harry received for a certain number of watermelons at \$'25 apiece; how much money did each receive, and how many melons had Harry?

Ans. \$50, and 200 melons.

89. The third part of an army was killed, the fourth part taken prisoners, and 1000 fled; how many were in this army?

This and the eighteen following questions are usually

wrought by a rule called *Position*, but they are more easily solved on general principles. Thus, $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ of the army; therefore, 1000 is $\frac{5}{12}$ of the whole number of men; and, if 5 twelfths be 1000, how much is 12 twelfths, or the whole? *Ans.* 2400 men.

90. A farmer, being asked how many sheep he had, answered, that he had them in 5 fields; in the first were $\frac{1}{4}$ of his flock, in the second $\frac{1}{6}$, in the third $\frac{1}{8}$, in the fourth $\frac{1}{12}$, and in the fifth 450; how many had he? *Ans.* 1200.

91. There is a pole, $\frac{1}{4}$ of which stands in the mud, $\frac{1}{3}$ in the water, and the rest of it out of the water; required the part out of the water. *Ans.* $\frac{5}{12}$.

92. If a pole be $\frac{1}{3}$ in the mud, $\frac{3}{5}$ in the water, and 6 feet out of the water, what is the length of the pole? *Ans.* 90 feet.

93. The amount of a certain school is as follows: $\frac{1}{16}$ of the pupils study grammar, $\frac{3}{8}$ geography, $\frac{3}{10}$ arithmetic, $\frac{3}{20}$ learn to write, and 9 learn to read: what is the number of each?

Ans. 5 in grammar, 30 in geography, 24 in arithmetic; 12 learn to write, and 9 learn to read.

94. A man, driving his geese to market, was met by another, who said, "Good morrow, sir, with your hundred geese;" says he, "I have not a hundred; but if I had, in addition to my present number, one half as many as I now have, and $2\frac{1}{2}$ geese more, I should have a hundred:" how many had he?

100 — $2\frac{1}{2}$ is what part of his present number?

Ans. He had 65 geese.

95. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches, and 40, cherries; how many trees does the orchard contain? *Ans.* 1200.

96. In a certain village, $\frac{1}{2}$ of the houses are painted white, $\frac{1}{4}$ red, $\frac{1}{6}$ yellow, 3 are painted green, and 7 are unpainted; how many houses in the village? *Ans.* 120.

97. Seven eighths of a certain number exceed four fifths of the same number by 6; required the number.

$\frac{7}{8} - \frac{4}{5} = \frac{3}{40}$; consequently, 6 is $\frac{3}{40}$ of the required number. *Ans.* 80.

98. What number is that, to which if $\frac{1}{5}$ of itself be added, the sum will be 30? *Ans.* 25.

99. What number is that, to which if its $\frac{1}{2}$ and $\frac{1}{4}$ be added, the sum will be 84?

$84 = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ times the required number. *Ans.* 48.

100. What number is that, which, being increased by $\frac{3}{5}$ and $\frac{2}{3}$ of itself, and by 22 more, will be made three times as much?

The number, being taken 1, $\frac{2}{3}$, and $\frac{3}{5}$ times, will make $2\frac{4}{15}$ times, and 22 is evidently what that wants of 3 times.

Ans. 30.

101. What number is that, which being increased by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itself, the sum will be $234\frac{3}{4}$? *Ans.* 90.

102. A, B, and C, talking of their ages, B said his age was once and a half the age of A, and C said his age was twice and one tenth the age of both, and that the sum of their ages was 93; what was the age of each?

Ans. A 12 years, B 18 years, C 63 years old.

103. A schoolmaster, being asked how many scholars he had, said, "If I had as many more as I now have, $\frac{3}{4}$ as many, $\frac{1}{2}$ as many, $\frac{1}{4}$ and $\frac{1}{8}$ as many, I should then have 435;" what was the number of his pupils? *Ans.* 120.

104. A and B commenced trade with equal sums of money; A gained a sum equal to $\frac{1}{5}$ of his stock, and B lost \$200; then A's money was double that of B's; what was the stock of each?

By the condition of the question, one half of $\frac{6}{5}$, that is, $\frac{3}{5}$ of the stock, is equal to $\frac{5}{5}$ of the stock, less \$200; consequently, \$200 is $\frac{2}{5}$ of the stock. *Ans.* \$500.

105. A man was hired 50 days on these conditions,—that, for every day he worked, he should receive \$'75, and, for every day he was idle, he should forfeit \$'25; at the expiration of the time, he received \$27'50; how many days did he work, and how many was he idle?

Had he worked every day, his wages would have been \$'75 × 50 = \$37'50, that is, \$10 more than he received; but every day he was idle lessened his wages \$'75 + \$'25 = \$1; consequently he was idle 10 days.

Ans. He wrought 40, and was idle 10 days.

106. A and B have the same income; A saves $\frac{1}{8}$ of his; but B, by spending \$30 per annum more than A, at the end of 8 years finds himself \$40 in debt; what is their income, and what does each spend per annum?

Ans. Their income, \$200 per annum; A spends \$175, and B \$205 per annum.

107. A man, lying at the point of death, left to his three sons his property; to A $\frac{1}{2}$ wanting \$20, to B $\frac{1}{3}$, and to C

the rest, which was $10 less than the share of A; what was each one's share? *Ans.* $80, $50 and $70.

108. There is a fish, whose head is 4 feet long; his tail is as long as his head and ½ the length of his body, and his body is as long as his head and tail; what is the length of the fish?

The pupil will perceive, that the length of the body is ½ the length of the fish. *Ans.* 32 feet.

109. A can do a certain piece of work in 4 days, and B can do the same work in 3 days; in what time would both, working together, perform it? *Ans.* $1\frac{5}{7}$ days.

110. Three persons can perform a certain piece of work in the following manner: A and B can do it in 4 days, B and C in 6 days, and A and C in 5 days: in what time can they all do it together? *Ans.* $3\frac{9}{37}$ days.

111. A and B can do a piece of work in 5 days; A can do it in 7 days; in how many days can B do it? *Ans.* 17½ days.

112. A man died, leaving $1000 to be divided between his two sons, one 14, and the other 18 years of age, in such proportion, that the share of each, being put to interest at 6 per cent., should amount to the same sum when they should arrive at the age of 21; what did each receive?

Ans. The elder, $546'153+; the younger, $453'846+.

113. A house being let upon a lease of 5 years, at $60 per annum, and the rent being in arrear for the whole time, what is the sum due at the end of the term, simple interest being allowed at 6 per cent.? *Ans.* $336.

114. If 3 dozen pair of gloves be equal in value to 40 yards of calico, and 100 yards of calico to three pieces of satinet of 30 yards each, and the satinet be worth 50 cents per yard, how many pair of gloves can be bought for $4? *Ans.* 8 pair.

115. A, B, and C, would divide $100 between them, so as that B may have $3 more than A, and C $4 more than B; how much must each man have?

Ans. A $30, B $33, and C $37.

116. A man has pint bottles, and half pint bottles; how much wine will it take to fill 1 of each sort? —— how much to fill 2 of each sort? —— how much to fill 6 of each sort?

117. A man would draw off 30 gallons of wine into 1 pint and 2 pint bottles, of each an equal number; how

many bottles will it take, of each kind, to contain the 30 gallons? *Ans.* 80 of each.

118. A merchant has canisters, some holding 5 pounds, some 7 pounds, and some 12 pounds; how many, of each an equal number, can be filled out of 12 cwt. 3 qrs. 12 lbs. of tea? *Ans.* 60.

119. If 18 grains of silver make a thimble, and 12 pwts. make a teaspoon, how many, of each an equal number, can be made from 15 oz. 6 pwts. of silver? *Ans.* 24 of each.

120. Let 60 cents be divided among three boys, in such a manner that, as often as the first has 3 cents, the second shall have 5 cents, and the third 7 cents; how many cents will each receive? *Ans.* 12, 20, and 28 cents.

121. A gentleman, having 50 shillings to pay among his labourers for a day's work, would give to every boy 6 d., to every woman 8 d., and to every man 16 d.; the number of boys, women, and men, was the same; I demand the number of each. *Ans.* 20.

122. A gentleman had 7 £. 17 s. 6 d. to pay among his labourers; to every boy he gave 6 d., to every woman 8 d., and to every man 16 d.; and there were for every boy three women, and for every woman two men; I demand the number of each. *Ans.* 15 boys, 45 women, and 90 men.

123. A farmer bought a sheep, a cow, and a yoke of oxen for \$82'50; he gave for the cow 8 times as much as for the sheep, and for the oxen 3 times as much as for the cow; how much did he give for each?

Ans. For the sheep \$2'50, the cow \$20, and the oxen \$60.

124. There was a farm, of which A owned $\frac{2}{7}$, and B $\frac{11}{21}$. the farm was sold for \$1764; what was each one's share of the money? *Ans.* A's \$504, and B's \$1260.

125. Four men traded together on a capital of \$3000, of which A put in $\frac{1}{2}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D $\frac{1}{12}$; at the end of 3 years they had gained \$2364; what was each one's share of the gain?

Ans. A's \$1182. B's \$ 591. C's \$ 394. D's \$ 197.

126. Three merchants accompanied; A furnished $\frac{2}{5}$ of the capital, B $\frac{3}{8}$, and C the rest; they gain \$1250; what

part of the capital did C furnish, and what is each one's share of the gain?

Ans. C furnished $\frac{3}{16}$ of the capital; and A's share of the gain was \$500, B's \$468'75, and C's \$281'25.

127. A, B, and C, traded in company; A put in \$500, B \$350, and C 120 yards of cloth; they gained \$332'50, of which C's share was \$120; what was the value of C's cloth per yard, and what was A and B's shares of the gain?

Note. C's gain, being \$120, is $\frac{12000}{33250} = \frac{48}{133}$ of the whole gain: hence the gain of A and B is readily found; also the price at which C's cloth was valued per yard.

Ans. { C's cloth, per yard, \$4.
A's share of the gain, \$125.
B's do. \$87'50. }

128. Three gardeners, A, B, and C, having bought a piece of ground, find the profits of it amount to 120£. per annum. Now the sum of money which they laid down was in such proportion, that, as often as A paid 5£., B paid 7£., and as often as B paid 4£., C paid 6£. I demand how much each man must have per annum of the gain.

Note. By the question, so often as A paid 5£., C paid $\frac{6}{4}$ of 7£. *Ans.* A 26£. 13 s. 4 d., B 37£. 6 s. 8 d., C 56£.

129. A gentleman divided his fortune among his sons, giving A 9£. as often as B 5£., and C 3£. as often as B 7£.; C's dividend was 1537$\frac{5}{8}$£.; to what did the whole estate amount? *Ans.* 11583£. 8 s. 10 d.

130. A and B undertake a piece of work for \$54, on which A employed 3 hands 5 days, and B employed 7 hands 3 days; what part of the work was done by A, what part by B, and what was each one's share of the money?

Ans. A $\frac{5}{12}$ and B $\frac{7}{12}$; A's money \$22'50, B's \$31'50.

131. A and B trade in company for one year only; on the first of January, A put in \$1200, but B could not put any money into the stock until the first of April; what did he then put in, to have an equal share with A at the end of the year? *Ans.* \$1600.

132. A, B, C, and D, spent 35 s. at a reckoning, and, being a little dipped, agreed that A should pay $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D $\frac{1}{8}$; what did each pay in this proportion?

Ans. A 13 s. 4 d., B 10 s., C 6 s. 8 d., and D 5 s.

133. There are 3 horses, belonging to 3 men, employed to draw a load of plaster from Boston to Windsor for \$26'45;

A and B's horses together are supposed to do $\frac{3}{4}$ of the work, A and C's $\frac{9}{10}$, B and C's $\frac{13}{20}$; they are to be paid proportionally; what is each one's share of the money?

Ans. { A's \$ 11'50 ($=\frac{10}{23}$.)
B's \$ 5'75 ($=\frac{5}{23}$.)
C's \$ 9'20 ($=\frac{8}{23}$.)

Proof, - - - \$ 26'45.

134. A person, who was possessed of $\frac{2}{5}$ of a vessel, sold $\frac{5}{8}$ of his share for 375£.; what was the vessel worth?

Ans. 1500£.

135. A gay fellow soon got the better of $\frac{2}{7}$ of his fortune; he then gave 1500£. for a commission, and his profusion continued till he had but 450£. left, which he found to be just $\frac{3}{8}$ of his money, after he had purchased his commission; what was his fortune at first? *Ans.* 3780£.

136. A younger brother received 1560£., which was just $\frac{7}{22}$ of his elder brother's fortune, and $5\frac{3}{8}$ times the elder brother's fortune was $\frac{3}{8}$ as much again as the father was worth; pray, what was the value of his estate? *Ans.* 19165£. 14 s. $3\frac{3}{7}$ d.

137. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in three months; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him nine months longer, when he had only 537£. left; what was the sum bequeathed him by his father?

Ans. 2082£. 18 s. $2\frac{2}{11}$ d.

138. A cannon ball, at the first discharge, flies about a mile in *eight* seconds; at this rate, how long would a ball be in passing from the earth to the sun, it being 95173000 miles distant?

Ans. 24 years, 46 days, 7 hours, 33 minutes, 20 seconds.

139. A general, disposing his army into a square battalion, found he had 231 over and above, but, increasing each side with one soldier, he wanted 44 to fill up the square; of how many men did his army consist? *Ans.* 19000.

140. A and B cleared, by an adventure at sea, 45 guineas, which was 35£. *per cent.* upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; what money did each adventure?

Ans. A 104£. 4 s. $2\frac{10}{19}$ d., B 75£. 15 s. $9\frac{9}{19}$ d.

141. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot? what would be the weight of such a tube, which would extend across the

Atlantic from Boston to London, estimating the distance at 3000 miles? *Ans.* 1 lb. 8 oz. 6 pwts. $3\frac{9}{13}$ grs.

142. A military officer drew up his soldiers in rank and file, having the number in rank and file equal; on being reinforced with three times his first number of men, he placed them all in the same form, and then the number in rank and file was just double what it was at first; he was again reinforced with three times his whole number of men, and, after placing them all in the same form as at first, his number in rank and file was 40 men each; how many men had he at first? *Ans.* 100 men.

143. Supposing a man to stand 80 feet from a steeple, and that a line reaching from the belfry to the man is just 100 feet in length; the top of the spire is 3 times as high above the ground as the steeple is; what is the height of the spire? and the length of a line reaching from the top of the spire to the man? *See* ¶ 109.

Ans. to last, 197 feet, nearly.

144. Two ships sail from the same port; one sails directly east, at the rate of 10 miles an hour, and the other directly south, at the rate of $7\frac{1}{2}$ miles an hour; how many miles apart will they be at the end of 1 hour? —— 2 hours? —— 24 hours? —— 3 days? *Ans. to last,* 900 miles.

145. There is a square field, each side of which is 50 rods; what is the distance between opposite corners?

Ans. 70·71 + rods.

146. What is the area of a square field, of which the opposite corners are 70·71 rods apart? and what is the length of each side? *Ans. to last,* 50 rods, nearly.

147. There is an oblong field, 20 rods wide, and the distance of the opposite corners is $33\frac{1}{3}$ rods; what is the length of the field? —— its area?

Ans. Length, $26\frac{2}{3}$ rods; area, 3 acres, 1 rood, $13\frac{1}{3}$ rods.

148. There is a room 18 feet square; how many yards of carpeting, 1 yard wide, will be required to cover the floor of it? $18^2 = 324$ ft. $= 36$ yards, *Ans.*

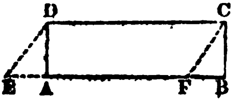
149. If the floor of a square room contain 36 square yards, how many feet does it measure on each side?

Ans. 18 feet.

When *one side* of a square is given, how do you find its *area*, or superficial contents?

When the *area*, or superficial contents, of a square is given, how do you find *one side*?

150. If an oblong piece of ground be 80 rods long and 20 rods wide, what is its area?



Note. A *Parallelogram*, or *Oblong*, has its opposite sides *equal* and *parallel*, but the adjacent sides unequal. Thus A B C D is a parallelogram, and also E F C D, and it is easy to see, that the *contents* of both are equal. *Ans.* 1600 rods, = 10 acres.

151. What is the length of an oblong, or parallelogram, whose area is 10 acres, and whose breadth is 20 rods?

Ans. 80 rods.

152. If the area be 10 acres, and the length 80 rods, what is the other side?

When the *length* and *breadth* are given, how do you find the *area* of an oblong, or parallelogram?

When the *area* and *one side* are given, how do you find the *other* side?

153. If a board be 18 inches wide at one end, and 10 inches wide at the other, what is the *mean* or *average* width of the board? *Ans.* 14 inches.

When the *greatest* and *least* width are given, how do you find the *mean* width?

154. How many square feet in a board 16 feet long, 1‘8 feet wide at one end, and 1‘3 at the other?

Mean width, $\frac{1‘8 + 1‘3}{2} = 1‘55$; and $1‘55 \times 16 = 24‘8$ feet, *Ans.*

155. What is the number of square feet in a board 20 feet long, 2 feet wide at one end, and running to a point at the other? *Ans.* 20 feet.

How do you find the contents of a straight edged board, when one end is wider than the other?

If the length be in *feet*, and the breadth in feet, in what denomination will the product be?

If the length be *feet*, and the breadth *inches*, what *parts of a foot* will be the product?

156. There is an oblong field, 40 rods long and 20 rods wide; if a straight line be drawn from one corner to the opposite corner, it will be divided into two equal *right-angled triangles*; what is the area of each?

Ans. 400 square rods = 2 acres, 2 roods.

157. What is the area of a triangle, of which the *base* is 30 rods, and the *perpendicular* 10 rods? *Ans.* 150 rods.

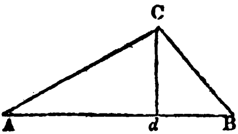
158. If the area be 150 rods, and the base 30 rods, what is the perpendicular? *Ans.* 10 rods.

159. If the perpendicular be 10 rods, and the area 150 rods, what is the base? *Ans.* 30 rods.

When the *legs* (the *base* and *perpendicular*) of a right-angled triangle are given, how do you find its *area*?

When the area and *one* of the legs are given, how do you find the *other* leg?

Note. Any triangle may be divided into two *right-angled* triangles, by drawing a perpendicular from one corner to the opposite side, as may be seen by the annexed figure.



Here A B C is a triangle, divided into two right-angled triangles, A *d* C, and *d* B C; therefore the whole *base*, A B, multiplied by *one half* the *perpendicular* *d* C, will give the area of the whole. If A B = 60 feet, and *d* C = 16 feet, what is the area? *Ans.* 480 feet.

160. There is a triangle, each side of which is 10 feet; what is the length of a perpendicular from one angle to its opposite side? and what is the area of the triangle?

Note. It is plain, the perpendicular will divide the opposite side into two *equal parts.* See ¶ 109.

Ans. Perpendicular, 8'66 + feet; area, 43'3 + feet.

161. What is the solid contents of a cube measuring 6 feet on each side? *Ans.* 216 feet.

When *one side* of a cube is given, how do you find its *solid contents*?

When the *solid contents* of a cube are given, how do you find *one side* of it?

162. How many cubic inches in a brick which is 8 inches long, 4 inches wide, and 2 inches thick? —— in 2 bricks? —— in 10 bricks? *Ans. to last*, 640 cubic inches.

163. How many bricks in a cubic foot? —— in 40 cubic feet? —— in 1000 cubic feet? *Ans. to last*, 27000.

164. How many bricks will it take to build a wall 40 feet in length, 12 feet high, and 2 feet thick? *Ans.* 25920.

165. If a wall be 150 bricks, = 100 feet, in length, and 4 bricks, = 16 inches, in thickness, how many bricks will lay one course? —— 2 courses? —— 10 courses? If the

wall be 48 courses, = 8 feet, high, how many bricks will build it? 150 × 4 = 600, and 600 × 48 = 28800, *Ans.*

166. The river Po is 1000 feet broad, and 10 feet deep, and it runs at the rate of 4 miles an hour; in what time will it discharge a cubic mile of water (reckoning 5000 feet to the mile) into the sea? *Ans.* 26 days, 1 hour.

167. If the country, which supplies the river Po with water, be 380 miles long, and 120 broad, and the whole land upon the surface of the earth be 62,700,000 square miles, and if the quantity of water discharged by the rivers into the sea be every where proportional to the extent of land by which the rivers are supplied; how many times greater than the Po will the whole amount of the rivers be? *Ans.* 1375 times.

168. Upon the same supposition, what quantity of water, altogether, will be discharged by all the rivers into the sea in a year, or 365 days? *Ans.* 19272 cubic miles.

169. If the proportion of the sea on the surface of the earth to that of land be as $10\frac{1}{2}$ to 5, and the mean depth of the sea be a quarter of a mile; how many years would it take, if the ocean were empty, to fill it by the rivers running at the present rate? *Ans.* 1708 years, 17 days, 12 hours.

170. If a cubic foot of water weigh 1000 oz. avoirdupois, and the weight of mercury be $13\frac{1}{2}$ times greater than of water, and the height of the mercury in the barometer (the weight of which is equal to the weight of a column of air on the same base, extending to the top of the atmosphere) be 30 inches; what will be the weight of the air upon a square foot? —— a square mile? and what will be the whole weight of the atmosphere, supposing the size of the earth as in questions 166 and 168?

Ans. 2109'375 lbs. weight on a square foot.
52734375000 .... mile.
10249980468750000000 .... of the whole atmosphere.

171. If a circle be 14 feet in diameter, what is its circumference?

Note. It is found by calculation, that the *circumference* of a circle measures about $3\frac{1}{7}$ times as much as its *diameter*, or, more accurately, in decimals, 3'14159 times. *Ans.* 44 feet.

172. If a wheel measure 4 feet across from side to side, how many feet around it? *Ans.* $12\frac{4}{7}$ feet.

173. If the diameter of a circular pond be 147 feet, what is its circumference? *Ans.* 462 feet.

174. What is the diameter of a circle, whose circumference is 462 feet? *Ans.* 147 feet.

175. If the distance through the centre of the earth, from side to side, be 7911 miles, how many miles around it?

7911 × 3'14159 = 24853 square miles, nearly, *Ans.*

176. What is the area or contents of a circle, whose diameter is 7 feet, and its circumference 22 feet?

Note. The *area* of a circle may be found by multiplying ½ *the diameter* into ½ *the circumference.* *Ans.* 38½ square feet.

177. What is the area of a circle, whose circumference is 176 rods? *Ans.* 2464 rods.

178. If a circle is drawn within a square, containing 1 square rod, what is the area of that circle?

Note. The diameter of the circle being 1 rod, the circumference will be 3'14159. *Ans.* '7854 of a square rod, nearly.

Hence, if we *square the diameter* of any circle, and multiply the *square* by '7854, the product will be the *area* of the circle.

179. What is the area of a circle whose diameter is 10 rods? 10^2 × '7854 = 78'54. *Ans.* 78'54 rods.

180. How many square inches of leather will cover a ball 3½ inches in diameter?

Note. The *area* of a *globe* or *ball* is 4 times as much as the area of a *circle* of the same diameter, and may be found, therefore, by multiplying the *whole circumference* into the *whole diameter.* *Ans.* 38½ square inches.

181. What is the number of square miles on the surface of the earth, supposing its diameter 7911 miles?

7911 × 24853 = 196,612,083, *Ans.*

182. How many solid inches in a ball 7 inches in diameter?

Note. The *solid contents* of a globe are found by multiplying its *area* by ⅙ part of its diameter.

Ans. 179⅔ solid inches.

183. What is the number of cubic miles in the earth, supposing its diameter as above?

Ans. 259,233,031,435 miles.

184. What is the capacity, in cubic inches, of a hollow globe 20 inches in diameter, and how much wine will it contain, 1 gallon being 231 cubic inches?

Ans. 4188'8 + cubic inches, and 18'13 + gallons.

185. There is a round log, all the way of a bigness; the areas of the circular ends of it are each 3 square feet;

how many solid feet does 1 foot in *length* of this log contain? —— 2 feet in length? —— 3 feet? —— 10 feet? A solid of this form is called a *Cylinder*.

How do you find the solid content of a cylinder, when the *area of one end*, and the *length* are given?

186. What is the solid content of a round stick, 20 feet long and 7 inches through, that is, the ends being 7 inches in diameter?

Find the area of *one end*, as before taught, and multiply it by the *length*. *Ans.* 5'347 + cubic feet.

If you multiply *square inches* by *inches in length*, what parts of a *foot* will the product be? —— if *square inches* by *feet* in length, what part?

187. A bushel measure is 18'5 inches in diameter, and 8 inches deep; how many cubic inches does it contain?

Ans. 2150'4 +.

It is plain, from the above, that the solid content of all bodies, which are of uniform bigness throughout, whatever may be the form of the ends, is found by *multiplying the area of one end into its height or length.*

Solids which decrease gradually from the base till they come to a point, are generally called *Pyramids*. If the base be a square, it is called a *square pyramid;* if a triangle, a *triangular pyramid;* if a circle, a *circular pyramid*, or a *cone*. The point at the top of a pyramid is called the *vertex*, and a line, drawn from the *vertex* perpendicular to the *base*, is called the *perpendicular height* of the pyramid.

The *solid content* of any pyramid may be found by multiplying the area of the *base* by $\frac{1}{3}$ of the perpendicular *height*.

188. What is the solid content of a pyramid whose base is 4 feet square, and the perpendicular height 9 feet?

$4^2 \times \frac{9}{3} = 48.$ *Ans.* 48 feet.

189. There is a *cone*, whose height is 27 feet, and whose *base* is 7 feet in diameter; what is its content?

Ans. 346$\frac{1}{2}$ feet.

190. There is a cask, whose head diameter is 25 inches, bung diameter 31 inches, and whose length is 36 inches; how many wine gallons does it contain? —— how many beer gallons?

Note. The *mean* diameter of the cask may be found by adding 2 thirds, or, if the staves be but little curving, 6 tenths, of the difference between the head and bung diame-

tess, to the head diameter. The cask will then be reduced to a cylinder.

Now, if the square of the mean diameter be multiplied by '7854, (ex. 177,) the product will be the area of one end, and that, multiplied by the length, in inches, will give the solid content, in cubic inches, (ex. 185,) which, divided by 231, (note to table, wine meas.) will give the content in wine gallons, and, divided by 282, (note to table, beer meas.) will give the content in ale or beer gallons.

In this process we see, that the square of the mean diameter will be multiplied by '7854, and divided, for wine gallons, by 231. Hence we may contract the operation by only multiplying by their quotient ($\frac{'7854}{231}$ = '0034;) that is, by '0034, (or by 34, pointing off 4 figures from the product for decimals.) For the same reason we may, for *beer* gallons, multiply by ($\frac{'7854}{282}$ = '0028, *nearly*,) '0028, &c.

Hence this concise RULE, *for guaging or measuring casks,—Multiply the square of the mean diameter by the length; multiply this product by* 34 *for wine, or by* 28 *for beer, and, pointing off four decimals, the product will be the content in gallons and decimals of a gallon.*

In the above example, the bung diameter, 31 in. — 25 in. the head diameter = 6 in. difference, and $\frac{2}{3}$ of 6 = 4 inches; 25 in. + 4 in. = 29 in. mean diameter.

Then, 29^2 = 841, and 841 × 36 in. = 30276.

Then, $\begin{cases} 30276 \times 34 = 1029384. & \textit{Ans. } 102'9384 \text{ wine gals.} \\ 30276 \times 28 = 847728. & \textit{Ans. } 84'7728 \text{ beer gals.} \end{cases}$

191. How many wine gallons in a cask whose bung diameter is 36 inches, head diameter 27 inches, and length 45 inches? *Ans.* 166'617.

192. There is a *lever* 10 feet long, and the *fulcrum*, or prop, on which it turns, is 2 feet from one end; how many pounds *weight* at the end, 2 feet from the prop, will be balanced by a *power* of 42 pounds at the other end, 8 feet from the prop?

Note. In turning around the prop, the end of the lever 8 feet from the prop will evidently pass over a *space* of 8 inches, while the end 2 feet from the prop passes over a *space* of 2 inches. Now, it is a fundamental principle in mechanics, that the *weight* and *power* will exactly *balance each other*, when they are *inversely* as the *spaces* they pass over. Hence, in this example, 2 pounds, 8 feet from the prop, will balance

8 pounds 2 feet from the prop; therefore, if we *divide the distance of the* POWER *from the prop by the distance of the* WEIGHT *from the prop, the quotient will always express the ratio of the* WEIGHT *to the* POWER; $\frac{8}{2} = 4$, that is, the *weight* will be 4 times as much as the *power*. $42 \times 4 = 168$.

Ans. 168 lbs.

193. Supposing the lever as above, what *power* would it require to raise 1000 pounds? *Ans.* $\frac{1000}{4} = 250$ pounds.

194. If the weight to be raised be 5 times as much as the power to be applied, and the distance of the weight from the prop be 4 feet, how far from the prop must the power be applied? *Ans.* 20 feet.

195. If the greater distance be 40 feet, and the less $\frac{1}{2}$ of a foot, and the power 175 pounds, what is the *weight*?

Ans. 14000 pounds.

196. Two men carry a kettle, weighing 200 pounds; the kettle is suspended on a pole, the bale being 2 feet 6 inches from the hands of one, and 3 feet 4 inches from the hands of the other; how many pounds does each bear?

Ans. $\begin{cases} 114\frac{2}{7} \text{ pounds.} \\ 85\frac{5}{7} \text{ pounds.} \end{cases}$

197. There is a windlass, the wheel of which is 60 inches in diameter, and the axis, around which the rope coils, is 6 inches in diameter; how many pounds on the axle will be balanced by 240 pounds at the wheel?

Note. The *spaces* passed over are evidently as the *diameters*, or the *circumferences;* therefore, $\frac{60}{6} = 10$, ratio.

Ans. 2400 pounds.

198. If the diameter of the wheel be 60 inches, what must be the diameter of the axle, that the *ratio* of the weight to the power may be 10 to 1? *Ans.* 6 inches.

Note. This calculation is on the supposition, that there is no *friction*, for which it is usual to add $\frac{1}{3}$ to the power which is to work the machine.

199. There is a screw, whose threads are 1 inch asunder, which is turned by a lever 5 feet, = 60 inches, long; what is the ratio of the weight to the power?

Note. The power applied at the end of the lever will describe the circumference of a circle $60 \times 2 = 120$ inches in diameter, while the weight is raised 1 inch; therefore, the *ratio* will be found *by dividing the circumference of a circle, whose diameter is twice the length of the lever, by the distance*

between the threads of the screw. $120 \times 3\frac{1}{7} = 377\frac{1}{7}$ circumference, and $\frac{377\frac{1}{7}}{1} = 377\frac{1}{7}$, ratio, *Ans.*

200. There is a screw, whose threads are $\frac{1}{4}$ of an inch asunder; if it be turned by a lever 10 feet long, what *weight* will be balanced by 120 pounds power? *Ans.* 30171 pounds.

201. There is a machine, in which the power moves over 10 feet, while the weight is raised 1 inch; what is the power of that machine, that is, what is the ratio of the *weight* to the *power*? *Ans.* 120.

202. A man put 20 apples into a wine gallon measure, which was afterwards filled by pouring in 1 quart of water; required the contents of the apples in cubic inches.

Ans. $173\frac{1}{4}$ inches.

203. A rough stone was put into a vessel, whose capacity was 14 wine quarts, which was afterwards filled with $2\frac{1}{2}$ quarts of water; what was the cubic content of the stone?

Ans. $664\frac{1}{8}$ inches.

FORMS OF NOTES, BONDS, RECEIPTS, AND ORDERS.

NOTES.

No. I.

Overdean, Sept. 17, 1802.

For value received, I promise to pay to OLIVER BOUNTIFUL, or order, sixty-three dollars fifty-four cents, on demand, with interest after three months. WILLIAM TRUSTY.

Attest, TIMOTHY TESTIMONY.

No. II.

Bilfort, Sept. 17, 1802.

For value received, I promise to pay to O. R., or bearer, ——— dollars ——— cents, three months after date.

PETER PENCIL.

No. III.

By two Persons.

Arian, Sept. 17, 1802.

For value received, we, jointly and severally, promise to pay to C. D., or order, ——— dollars ——— cents, on demand, with interest.

ALDEN FAITHFUL.

Attest, CONSTANCE ADLEY. JAMES FAIRFACE.

Observations.

1. No note is negotiable unless the words "*or order*," otherwise "*or bearer*," be inserted in it.

2. If the note be written to pay him "*or order*," (No. I.) then Oliver Bountiful may endorse this note, that is, write his name on the backside, and sell it to A, B, C, or whom he pleases. Then A, who buys the note, calls on William Trusty for payment, and if he neglects, or is unable to pay, A may recover it of the endorser.

3. If a note be written to pay him "*or bearer*," (No. II.) then any person, who holds the note, may sue and recover the same of Peter Pencil.

4. The rate of interest, established by law, being *six per cent. per annum*, it becomes unnecessary, in writing notes, to mention the rate of interest; it is sufficient to write them for the payment of such a sum, with interest, for it will be understood legal interest, which is six per cent.

5. All notes are either payable on demand, or at the expiration of a certain term of time agreed upon by the parties, and mentioned in the note, as three months, a year, &c.

6. If a bond or note mention no time of payment, it is always on demand, whether the words "*on demand*" be expressed or not.

7. All notes, payable at a certain time, are on interest as soon as they become due, though in such notes there be no mention made of interest.

This rule is founded on the principle, that every man ought to receive his money when due, and that the non-payment of it at that time is an injury to him. The law, therefore, to do him justice, allows him interest from the time the money becomes due, as a compensation for the injury.

8. Upon the same principle, a note, payable on demand, without any mention made of interest, is on interest after a

demand of payment, for upon demand such notes immediately become due.

9. If a note be given for a specific article, as rye, payable in one, two, or three months, or in any certain time, and the signer of such note suffers the time to elapse without delivering such article, the holder of the note will not be obliged to take the article afterwards, but may demand and recover the value of it in money.

BONDS.

A Bond, with a Condition, from one to another.

Know all men by these presents, that I, C. D. of, &c., in the county of, &c., am held and firmly bound to E. F., of, &c., in two hundred dollars, to be paid to the said E. F., or his certain attorney, his executors, administrators, or assigns, to which payment, well and truly to be made, I bind myself, my heirs, executors and administrators, firmly by these presents. Sealed with my seal. Dated the eleventh day of ———, in the year of our Lord one thousand eight hundred and two.

The Condition of this obligation is such, that, if the above-bound C. D., his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid, unto the above-named E. F., his executors, administrators, or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the eleventh day of ——— next ensuing the date hereof,—then this obligation to be void, or otherwise to remain in full force and virtue.

Signed, &c.

A Condition of a Counter Bond, or Bond of Indemnity, where one man becomes bound for another.

The condition of this obligation is such, that whereas the above-named A. B., at the special instance and request, and for the only proper debt of the above-bound C. D., together with the said C. D., is, and by one bond or obligation bearing equal date with the obligation above-written, held and firmly bound unto E. F., of, &c., in the penal sum of ——— dollars, conditioned for the payment of the sum of, &c., with legal interest for the same, on the ——— day of ———

next ensuing the date of the said in part recited obligation, as in and by the said in part recited bond, with the condition thereunder written, may more fully appear;—if, therefore, the said C. D., his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid, unto the said E. F., his executors, administrators, or assigns, the said sum of, &c., with legal interest of the same, on the said ——— day of, &c., next ensuing the date of the said in part recited obligation, according to the true intent and meaning, and in full discharge and satisfaction of the said in part recited bond or obligation,—then, &c.—otherwise, &c.

Note. The principal difference between a note and a bond is, that the latter is an instrument of more solemnity, being given under seal. Also, a note may be controlled by a special agreement, different from the note, whereas, in case of a bond, no special agreement can in the least control what appears to have been the intention of the parties, as expressed by the words in the condition of the bond.

RECEIPTS.

Sitgrieves, Sept. 19, 1802.

Received from Mr. Durance Adley ten dollars in full of all accounts.

Orvand Constance.

Sitgrieves, Sept. 19, 1802.

Received of Mr. Orvand Constance five dollars in full of all accounts.

Durance Adley.

Receipt for Money received on a Note.

Sitgrieves, Sept. 19, 1802.

Received of Mr. Simpson Eastly (by the hand of Titus Trusty) sixteen dollars twenty-five cents, which is endorsed on his note of June 3, 1802.

Peter Cheerful.

A Receipt for Money received on Account.

Sitgrieves, Sept. 19, 1802.

Received of Mr. Orand Landike fifty dollars on account.

Eldro Slackley.

Receipt for Money received for another Person.

Salem, Aug. 10, 1827.

Received from P. C. one hundred dollars for account of J. B. ELI TRUMAN.

Receipt for Interest due on a Note.

Amherst, July 6, 1827.

Received of I. S. thirty dollars, in full of one year's interest of $500, due to me on the ——— day of ——— last, on note from the said I. S. SOLOMON GRAY.

Receipt for Money paid before it becomes due.

Hillsborough, May 3, 1827.

Received of T. Z. ninety dollars, advanced in full for one year's rent of my farm, leased to the said T. Z., ending the first day of April next, 1828. HONESTUS JAMES.

Note. There is a distinction between receipts given in full of *all accounts*, and others in full of *all demands*. The former cut off accounts *only;* the latter cut off not only accounts, but all obligations and right of action.

ORDERS.

Archdale, Sept. 9, 1802.

Mr. STEPHEN BURGESS. For value received, pay to A. B., or order, ten dollars, and place the same to my account.

SAMUEL SKINNER.

Pittsburgh, Sept. 9, 1821.

Mr. JAMES ROBOTTOM. Please to deliver Mr. L. D. such goods as he may call for, not exceeding the sum of twenty-five dollars, and place the same to the account of your humble servant, NICHOLAS REUBENS.

BOOK-KEEPING.

It is necessary that every man should have some regular, uniform method of keeping his accounts. What this method shall be, the law does not prescribe; but, in cases of dispute, it requires that the book, or that on which the charges were *originally* made, be produced in open court, when he will be required to answer to the following questions:—

Is this your book, and the method in which you keep your accounts?

Did you make the charges, now in dispute, at the time they purport to have been made?

Are they just and true?

Have you received pay for them, or any part? if so, how much?

An answer in the affirmative, *under oath*, to the above questions, (the last only excepted,) is all that is required to substantiate his claim.

For farmers and mechanics, the following method will be found both convenient and easy. It consists in having one single book, entering the name of the person, with whom an account is to be opened, at the top of the *left* hand page, *Dr.*, and at the top of the *right* hand page, *Cr.*, as follows:—

Dr.	***James Macknight.***			***James Macknight.***			**Cr.**
1827.		*dolls.*	*cts.*	1827.		*dolls.*	*cts.*
Jan. 5.	To 5 cords of Wood, at $ 1'75,	8	75	April 8.	By one Plough, - -	9	25
May 16.	To one day's work, self and oxen, - - -	1	50	May 10.	By repairing Cart Wheels,	1	50
July 23.	To 4 bushels of Rye, at 75 cts. delivered by your order to C. D. - - -	3	00	Sept. 12.	By Cash to balance, -	2	50
		13	25			13	25